\[ V_0 = \frac{2 \times 10^{-9} \times 1 \times 10^4 \times 3 \times 10^3}{1.7 \times 10^{-15} \times 10^3} = 4.72 \times 10^5 \]
\[ C_j = A \sqrt{\frac{\frac{3}{8} + \sqrt{\frac{3}{8} + \frac{3}{8}}}{V_0 - V_w}} \]

\[ \frac{1}{x_0} = \frac{\frac{3}{8} (\frac{1}{V_0 - V_w})}{\frac{3}{8} (V_0 - V_w)^2} \]

\[ \epsilon A = C_j \]

\[ \frac{1}{\sqrt{y'_0}} = \frac{1}{x_0} \]
\[ V_f = 2.24 \times 10^{-19} \times 2 \times 10^{16} = 2.91 \times 10^{-3} \]

\[ V = \frac{1}{2} \left( 2.91 \times 10^{-3} \right)^{1/2} \]

\[ C = A \left[ \frac{1}{2} \left( \frac{1}{V} \right)^{1/2} \right]^{1/2} \]

Continued on Next Page
The semiconductor in silicon, $T = 300K$, the doping doping is in the same everywhere with $N_D = 10^{17} \text{cm}^{-3}$ and $G_D = 10^{15} \text{cm}^{-3} \sec$ in all parts inside the semiconductor. Also, the statement of the problem implies equilibrium conditions exist for $x > 0$.

**Step 1—Characterize the system under equilibrium conditions.**

In Si at room temperature $n = 10^{10} \text{cm}^{-3}$. Since $N_D = N_A$, $n = N_D = 10^{15} \text{cm}^{-3}$ and $p_D = n_D/n = 10^{15} \text{cm}^{-3}$. With the doping doping uniform, the equilibrium $n$ and $p$ values are the same everywhere throughout the semiconductor.

**Step 2—Solve the problem qualitatively.**

Prior to $t = 0$, equilibrium conditions prevail and $\delta n = 0$. Starting at $t = 0$ the light creates additional carriers both $n$ and $\delta n$ will begin to increase. The growing excess carrier numbers, however, in turn loosen an increased indirect thermal recombination rate which is proportional to $\delta n$. Consequently, as $\delta n$ grows as a result of photo-generation, more and more of the excess holes are eliminated per second by recombination through $R^{-1}$ centers. Eventually, a level is reached where the carriers assimilated per second by indirect thermal recombination balance the carriers created per second by the light, and a steady state condition is attained.

**Step 3—Perform quantitative analysis.**

The minority carrier diffusion equation is the starting point for most first-order quantitative analyses. After examining the problem for obvious conditions that would invalidate the use of the given $\delta n$, the appropriate minority carrier diffusion equation is written down, the equation is simplified, and a solution is sought subject to boundary conditions stated or implied in the problem.
For the problem under consideration a customary inspection reveals that all simplifying assumptions involved in deriving the diffusion equation are roughly satisfied. Specifically, only minority carrier concentration is of interest; the equilibrium carrier concentrations are set as a function of position, indirect thermal generation is assumed in the absence of light and there are no "other processes" except for photonemission. Because the photogeneration is uniform throughout the semiconductor, the perturbed carrier concentrations are also position-independent and the electric field must closely be zero in the perturbed system. Finally, a drain current €I_T = 10^4 amperes = 10^9 picocoulombs is consistent with low-level injection prevailing at all times.

With no obstruction to utilizing the diffusion equation, the desired quantitative solution can now be obtained by solving

$$\frac{\Delta N}{dt} = G_T \frac{\Delta N}{dx^2} - \Delta N + G_L \tag{3.55}$$

subject to the boundary condition

$$\Delta N(x, t = 0) = 0 \tag{3.56}$$

Since $G_T$ is a function of position, the diffusion equation becomes an ordinary differential equation and simplifies to

$$\frac{\Delta N}{dt} + \frac{\Delta N}{\tau_p} = G_L \tag{3.57}$$

The general solution of Eq. (3.57) is

$$\Delta N(t) = G_L \tau_p \left( 1 - e^{-\frac{t}{\tau_p}} \right) \tag{3.58}$$

Applying the boundary conditions yields

$$A = -G_L \tau_p \tag{3.59}$$

and

$$\Delta N(t) = G_L \tau_p \left( 1 - e^{-\frac{t}{\tau_p}} \right) \leq \text{solution} \tag{3.60}$$

Step 5—Examine the solution.

Failing to examine the mathematical solution is a serious problem in growing vegetables and thus failing to use the produce. Relative to the Eq. (3.60) result, $G_L \tau_p$ has the dimensions of a concentration (number/cm^3) and the solution is at least dimensionally correct. A plot of the Eq. (3.60) result is shown in Fig. 3.25. Note that, in agreement with qualitative predictions, $\Delta N(t)$ starts from zero at $t = 0$ and eventually becomes at $G_L \tau_p$, after a few $\tau_p$.

Exaggeration—we would be remiss if we did not point out the contrast between the hypothetical problem just completed and the photoconductive decay measurement described in subsection 3.3.4. Light output from the microscope used in the experiment can be modeled to first order by the pulse train pictured in Fig. 3.36a. The Eq. (3.60) solution therefore approximately describes the carrier buildup during a light pulse. It should be noted, however, that the microscope light pulses have a duration of $\alpha \approx 1 \mu s$ compared to a minority carrier lifetime of $\tau_p = 50 \mu s$. With $\alpha \tau_p \ll 1$, one uses only the very

![Figure 3.25 Solution to Sample Problem No. 1: Photocarrier-induced increase in the excess carrier concentration as a function of time.](image-url)
The semiconductor is again silicon semi-transparent doped with an \( N_D \sim 10^{15} \text{cm}^{-3} \). Steady state equations are inferred from the statement of the problem, since we are asked for \( \Delta P(x) \) and not \( \Delta P(x, t) \). Moreover, as \( x \rightarrow \infty \), \( \Delta P(x, t) \rightarrow 0 \) and \( \Delta P \rightarrow 0 \). The latter boundary condition follows from the semi-transparent nature of the bar.

The photogeneration rate at \( x = 0 \) in the nonphotoreginating light cannot possibly extend to \( x = \infty \).

The recombination rates of the light allow us to set \( C_0 = 0 \) for \( x > 0 \). From the problem statement this is to express the temperature of operation. When this happens, it is reasonable to assume an insulating \( T = 300 \text{ K} \).

If the light were reemitted, the silicon bar in Sample Problem 2 maintained at 300 K would cool to an equilibrium condition identical to that described in Sample Problem 1. Under equilibrium conditions, both \( N_D \sim 10^{15} \text{cm}^{-3} \) and \( N_A \sim 10^{15} \text{cm}^{-3} \), and the carrier concentrations are constant throughout the semiconductor bar.

Qualitatively it is a simple matter to predict the expected effect of the nonphotoreginating light on the silicon bar. The light first creates excess carriers right at \( x = 0 \). With more carriers, a recombination front is expected to move from the bar.

The diagram shows the expected behavior of the nonphotoreginating light. The concentration decreases exponentially with distance from the interface due to recombination.

**Figure 2.26** Approximate model for the light capture in the photoregenerating device. (a) Time after the light-off \( \Delta P(x) \) at the photoregenerating device. Initially, the concentration is equal to zero and \( \Delta P(x, t) \) starts from a finite value. Over time, the concentration decreases exponentially, as shown in (b).

**Solution to Sample Problem 2**

For steady-state conditions, the concentration \( \Delta P(x) \) at steady state is given by (2.56):
In preparation for obtaining a solution of the diffusion equation, we observe that the system under consideration is one-dimensional, the model is restricted to the majority carrier hole, the equilibrium carrier concentration is position independent, and thermal R−D behavior, there are no “other processes” for x > 0, and low-level injection condition clearly proves (D_i, min = D_i, max = 10^13 cm^2/Vs < η = 10^16 cm^2/Vs). The only question that might be raised concerning the use of the diffusion equation is the starting point for the quantitative analysis -- another η = 0. With the light on, a random distribution of holes and associated distribution of positive charge will appear near x = 0. The excess hole pile-up, however, is very small (D_i, min = D_i, max) and the associated electric field is therefore expected to be correspondingly small. Moreover, in problems of this type it is found that the majority carriers, to any charged electrons in the given problem, recombine in such a way as to partly cancel the minority carrier charge. This experience indicates that the η = 0 assumption is reasonable, and use of the minority carrier diffusion equation to be justified.

Under steady state conditions with η = 0 for x > 0 the hole diffusion equation reduces to the form

\[ \frac{D_i}{x^2} \frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\tau_i} \phi = 0 \quad \text{for} \quad x > 0 \]  

(3.52)

which is to be solved subject to the boundary conditions

\[ \phi(x = 0) = \phi(x = L) = 0 \]  

(3.65)

and

\[ \phi(x = L) = 0 \]  

(3.64)

Equation (3.52) should be recognized as one of the simplified diffusion equations cited in Table 3.2, with the general solution

\[ \phi(x, t) = Ae^{-x^2 t} + Be^{x^2 t} \]  

(3.65)

where

\[ \phi_0 = \sqrt{\frac{\lambda_i}{2\pi}} \]  

(3.66)

Because exp(x^2 t) \to \infty \quad \text{as} \quad t \to \infty, \quad \text{the only way that the Eq. (3.64) boundary condition can be satisfied is for} \quad \phi \quad \text{to be identically zero. With} \quad \phi = 0, \quad \text{application of the Eq. (3.63) boundary condition yields}

\[ \lambda_i = \frac{D_i}{\tau_i} \]  

(3.67)

and

\[ \phi(x, t) = Ae^{-x^2 t} \quad \text{or} \quad \phi(x, t) = Be^{x^2 t} \]  

(3.68)

The Eq. (3.68) result is illustrated in Fig. 3.2B. In agreement with qualitative arguments, the represnting empirically gives rise to a monotonically decreasing \( \Delta N / N \) starting from \( \Delta N / N = 0 \) at \( x = 0 \) and decreasing to \( \Delta N / N = 0 \) as \( x \to \infty \). Note that no precise functional form of the fall off in the excess carrier concentration is exponential with a characteristic decay length equal to \( \lambda_i \).

Figure 3.2B: Solution to Sample Problem No. 2 showing the excess hole concentration inside the base as a function of position.