The text for examination is 1 hr 15 min. Students are allowed to use 2 sheets of notes. Please show your work, partial credit may be given. At each problem state explicitly all assumptions that you make in the solution process (please make explicit references, if needed, to formulae that you use). Show your work with care, where applicable. Please write on the paper provided. Use additional sheets only if necessary.

**Useful Data:**
- Electron charge: \( e = 1.6 \times 10^{-19} [\text{C}] \)
- Boltzmann constant: \( k_B = 8.616 \times 10^{-5} [\text{eV} / \text{K}] \)
- Ideal gas constant: \( R = 8.314 [\text{J/K/mol}] \)
- Room temp. dep. data: \( N_e = 1.07 \times 10^{16} [\text{cm}^3] \)
- \( kT = 0.0259 [\text{eV}] \)
- \( E_g = 1.12 [\text{eV}] \)

1. (20 pts.) A Si device is in the equilibrium conditions at \( T = 300 \, \text{°K} \). The electronic field, \( E_x \), inside the device is sketched below. The doping levels are: \( N_A = 10^{17} [\text{cm}^3] \), \( N_D = 10^{17} [\text{cm}^3] \), \( x_L \leq x \leq x_r \).

Using the relations:

\[
\varepsilon_x = \frac{1}{x} \frac{dE_x}{dx} = \frac{1}{x} \frac{dE_x}{dx} = \frac{dE_x}{dx}
\]

we find the energy band diagram for the device. Include \( E_x, N_A, N_D \). Please explain principles/relations used in your work.

![Energy Band Diagram](image)

a) Sketch the energy band diagram for the device. Include \( E_x, N_A, N_D \). Please explain principles/relations used in your work.

b) Determine the position of Fermi-level.

For \( \alpha < x \leq x_A \), we have: \( \rho = N_A^+ \) and \( \rho = n^+ \varepsilon.x \)

Thus, \( E_F - E_x = \epsilon kT \ln \frac{N_D}{N_A} \Rightarrow E_F - E_x = 0.0259 \, kT \ln \frac{N_D}{N_A} \), \( = 0.34 \, \text{eV} \)

For \( x_L \leq x < x_A \), we have \( \rho = N_D^+ \) and \( \rho = n^+ \varepsilon.x \)

which yields \( E_F - E_x = \epsilon kT \ln \frac{N_D}{N_A} \Rightarrow E_F - E_x = 0.34 \, \text{eV} \).
2. (20 points) Consider a silicon p-n junction in thermal equilibrium at room temperature (300 K).

with the uniform concentration of donors \( N_d = 10^{14} \text{ cm}^{-3} \) and acceptors \( N_a = 10^{17} \text{ cm}^{-3} \) in

respective n- and p-regions. Assume abrupt (step) junction and depletion approximation.

a) (6 points) Calculate the built-in voltage \( V_b \) :

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{N_d}{N_a} \right) = 0.0259 \text{ V} (1.05 \times 10^{20}) = 0.0259 \text{ V} \cdot 10^{20} = 0.647 \text{ V}
\]

\[
V_{bi} = V_{nb} = 0.647 \text{ V}
\]

d) (5 points) Calculate coordinates \( x_n, x_p \) :

\[
x_n = \frac{1}{\frac{2e}{kT} \ln \left( \frac{N_d}{N_a N_a/kT} \right)} = \frac{1}{\frac{2e}{kT} \ln \left( \frac{N_d}{N_a N_a/kT} \right)}
\]

\[
x_n = 1.305 \times 10^3 \text{ cm} \quad \text{or} \quad 1.305 \times 10^{-3} \text{ cm}
\]

\[
x_n = 2.83 \times 10^{-3} \text{ cm}
\]

\[
x_p = 10^{-3} x_n \quad \Rightarrow \quad x_p = 2.83 \times 10^{-3} \text{ cm}
\]

\[
\left[ x_p = 2.83 \times 10^{-3} \text{ cm} \right] \quad \text{or} \quad 2.83 \times 10^{-3} \text{ cm}
\]

c) (6 points) Determine maximum electric field in the depletion region.

\[
\text{Max. field occurs at } x = 0
\]

\[
\varepsilon(x) = -\frac{\varepsilon}{\varepsilon_m} x = -\frac{\varepsilon}{\varepsilon_m} x
\]

\[
\varepsilon(0) = \frac{-1.5 \times 10^{-12} \text{ Cm}}{1.05 \times 10^{-12} \text{ Cm}} = -1.5 \times 10^{-2} \text{ Cm}
\]

\[
\varepsilon(0) = -1.5 \times 10^{-2} \text{ Cm} = -1.5 \times 10^{-2} \text{ Cm}
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\[
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\]
3. (23 points) Two identical $p-n$ junction diodes were manufactured with differences in doping. Manufacturing of the first diode resulted in $N_{c1} = 10^{17}$ $[1/cm^3]$ and $N_{e1} = 10^{14}$ $[1/cm^3]$, while the manufacturing of the second diode yielded $N_{c2} = 10^{17}$ $[1/cm^3]$ and $N_{e2} = 10^{13}$ $[1/cm^3]$. The diodes operate in room temperature $(300[^{o}K])$. Assume that diffusivities and life times of holes (minority carriers) in $n$-type neutral regions are the same in both diodes.

a) (15 points) Compare the two diodes using model based on abrupt junction and depletion approximation.

Using the formula $I_{sc} = \frac{2N_{c}^2}{N_{e}} \beta \frac{D_{p}}{L_{p}} \frac{D_{n}}{L_{n}}$, the diode 1 we have,

$$I_{sc1} = \frac{2N_{c1}^2}{N_{e1}} \beta \frac{D_{p1}}{L_{p1}} \frac{D_{n1}}{L_{n1}},$$

Thus for the diode $2$ we have

$$I_{sc2} = \frac{2N_{c2}^2}{N_{e2}} \beta \frac{D_{p2}}{L_{p2}} \frac{D_{n2}}{L_{n2}},$$

Consequently

$$\frac{I_{sc1}}{I_{sc2}} = \frac{N_{c1}^2}{N_{c2}^2} = 10,$$

Because $\frac{D_{p1}}{L_{p1}} = \frac{D_{p2}}{L_{p2}}$.

b) (8 points) Sketch (qualitatively) the diodes $I-V$ characteristics on the same system of axes (given below).

$$I_1 = I_{sc1} A \left( e^{\frac{V_0}{D_{p1} L_{p1}}} - 1 \right)$$

$$I_2 = I_{sc2} A \left( e^{\frac{V_0}{D_{p2} L_{p2}}} - 1 \right)$$

or else

$$I_1 = 10 I_{sc2} A \left( e^{\frac{V_0}{D_{p2} L_{p2}}} - 1 \right) = 10 I_2$$

- $-A_{sc2}$
- $-10 A_{sc2}$
4. (20 points) Computer analysis of p - n junction diode operating in steady state conditions at room temperature \(300 [^\circ K]\) determined the carrier concentrations sketched below.

![Diode diagram](image)

a) (5 points) Is the diode forward biased, reverse biased or in thermal equilibrium?

The diode is forward biased because excess carrier densities at the depletion boundary exceed the equilibrium densities \(n_o, p_o\).

b) (5 points) Are Low Level Injection conditions satisfied?

The Low Level Injection conditions are satisfied because minority carrier densities are much smaller than majority carrier density \(n_d < n_o\) and \(p_d < p_o\).

c) (10 points) Determine the applied voltage, \(V_a\).

Note: From the graph \(n_d = \frac{-7}{(2x)}\) and \(p_d = 10^5\) [\(\text{cm}^{-3}\)]

Using the junction law (for example \(x = x_n\)):

\[
\frac{n_d(x_n)}{p_d(x_n)} = \frac{p_d(x_n)}{n_d(x_n)} = \frac{10^5}{10^5} = \frac{10^5}{10^5}
\]

Thus we get:

\[
\frac{10^5}{10^5} = e^{\frac{V_a}{kT}} \Rightarrow V_a = \frac{kT \ln 10^5}{q} = 0.298 [\text{V}]
\]

Alternatively:

\[
\frac{n_d(x_n)}{p_d(x_n)} = e^{rac{V_a}{kT}} \Rightarrow V_a = \frac{kT \ln (p_d(x_n)/n_d(x_n))}{q}
\]

\[
V_a = 0.298 [\text{V}]
\]
5. (17 points) A silicon $p-n$ junction diode operating in room temperatures is characterized by the following energy-band diagram.

Assume that $E_l(-\infty) = E_v(+\infty)$ and $x_n + x_p = 2 \times 10^{-6}$ [cm]. The cross-section of the structure is $A = 10^{-3}$ [cm$^2$].

a) (9 points) Determine the magnitude of the reverse-bias voltage, $V_r$, applied to the diode.

From the graph:

\[ V_r = \frac{1}{2} \left( E_F - E_F \right)_p \]

\[ V_r = \frac{1}{2} \times \frac{1}{2} \times 2.11 \text{ eV} = \frac{1}{2} \times 1.12 \text{ V} \]

\[ V_r = 0.56 \text{ V} \]

b) (8 points) What is the value of built-in voltage, $V_b$?

From the graph:

\[ V_b = \frac{1}{2} \left[ (E_l - E_F)_p \text{ side} + (E_F - E_v)_n \text{ side} \right] \]

\[ V_b = \frac{1}{2} \times \frac{1}{2} \times 2.11 \text{ eV} = \frac{1}{2} \times 1.12 \text{ V} \]

Thus

\[ V_b = 0.56 \text{ V} \]