The time for the examination is 1 hr 15 min. Students are allowed to use one sheet of notes. Please show your work, partial credit may be given. At each problem state explicitly all assumptions that you make in the solution process (please make explicit references, if needed, to formulas that you use). Show your work with units, where applicable. Please write on the paper provided. Use additional sheets only if necessary.

Useful Data: Room temp: intrinsic carrier density: \( n_i = 1.67 \times 10^{16} \text{ cm}^{-3} \)

Electron charge: \( q = 1.6 \times 10^{-9} \text{ Coul} \)

Boltzmann constant: \( k = 8.616 \times 10^{-5} \text{ eV/}^\circ \text{K} \)

\[ [\text{eV}] = 1.6 \times 10^{-19} \text{ [Joule]} = 1.6 \times 10^{-8} \text{ [eV]} \]

1. (15 pts.) Silicon wafer was carefully cut to have a surface oriented to lie along crystallographic plane denoted by Miller indices \( (100) \). The wafer has a mark indicating the orientation of crystal unit cells using the Miller directions \( (011) \).

a) (10 points) What is the crystallographic plane used for the orientation of the wafer surface? Sketch the plane in the enclosed system of axes.

b) (5 points) Explain the orientation of unit cells (show the appropriate axes) on the sketch of wafer with arrow indicating \( (011) \) directions.
of electron density given by \( n_e(x) = \frac{\alpha}{(x+y)^2} \) along the length of sample axis designated by the \( x \)-

with zero at the left end of the sample (assume \( n_e(x) \gg n_i \) in the range of \( x \)). Note: the sample is not biased, i.e. it is not connected to any circuit. The sample is in the room temperature.

a) (15 points) Using continuity equations and neglecting G/R term derive the expression for the built-in electric field caused by this non-uniform doping.

Majority carriers are electrons and the density much larger than \( n_i \), therefore we use the continuity equation for electrons. The current density is zero because the sample is isolated. Thus:

\[
\mathbf{J} = e_n n_e \mathbf{E}(x) + e_i n_i \mathbf{E}(x) = 0 \implies \mathbf{E}(x) = -\frac{e_i n_i}{e_n n_e} \frac{d\mathbf{E}}{dx}
\]

Using the formula for \( n_e(x) \) we get:

\[
\frac{d\mathbf{E}}{dx} = -\frac{2e\beta(x+y)}{(x+y)^2} = -\frac{2e\alpha}{(x+y)^2}
\]

Substituting above deric equation for \( \mathbf{E}(x) \) we obtain:

\[
\mathbf{E}(x) = -\frac{2e\alpha}{(x+y)^2} \times \left( -\frac{2e\alpha}{(x+y)^2} \right) = \frac{D_n}{\varepsilon_0} \frac{2e\alpha}{x+y}
\]

Because \( \frac{D_n}{\varepsilon_0} = \frac{k}{\varepsilon} \) we get:

\[
\mathbf{E}(x) = \frac{k}{\varepsilon} \frac{2e\alpha}{x+y}
\]

b) (5 points) Calculate the electric field at the distance of 1 [cm] from the origin. Assume \( \varepsilon = 1 \) [permittivity of vacuum].

For \( k = 2\pi \alpha \) and \( x = 2 \) [cm] we obtain:

\[
\mathbf{E}(x=2) = 2.5 \left[ \frac{\text{mV}}{\text{cm}} \right]
\]
3. (30 points) A thin, uniformly doped n-type slab remains for a long time at room temperature. At time \( t = 0 \) the slab begins to be illuminated with the light of constant intensity (see the schematic illustration below). The energy delivered to the slab via this illumination causes excitation resulting in generation of electron pairs with a given rate, \( A \), per cubic cm. Determine the time behavior of minority carriers.

\[ \text{hv} \]

a) (15 pts) Formulate the equation governing the behavior of minority carriers in time from the time \( t = 0 \).

Assume one-dimensional modeling (width and thickness of the slab are negligible with respect to the length) and low level injection. Additionally, assume light generates electron-hole pairs with a uniform rate. Consequently, there are no variations in \( x \).

From the Boltzmann equation for holes, \( \frac{d(n_p-n_o)}{dt} = \frac{1}{\tau} \left( \frac{P}{v_k} + A - n_p \right) \)

Thus we get the equation (for holes) \( \frac{d(n_p-n_o)}{dt} = A - \frac{P}{v_k} \)

b) (8 pts) Determine the initial condition for the problem.

The initial condition is determined by thermal equilibrium. Thus \( n_p(0) = n_o \).

c) (7 pts) Solve the governing equation giving the analytical formula for the minority carrier density.

Designate: \( \Delta n = n_p - n_o \)

The differential equation is: \( \frac{d(\Delta n)}{dt} + \frac{1}{\tau} (\Delta n) = A \)

The IC is: \( \Delta n(0) = 0 \)

The solution is:

\( \Delta n(t) = A \left( 1 - e^{-\frac{t}{\tau}} \right) \)

or else:

\( n_p(t) = n_o + A \left( 1 - e^{-\frac{t}{\tau}} \right) \)

Note; for \( t = 0 \) we get and for \( t \to \infty \) we get:

\( n_p(0) = n_o \)

\( n_p(\infty) = n_o + A \)
4. (20 points) A silicon slab of length $l = 5 \text{ mm}$ and a square cross-section $(0.5 \text{ mm}) \times (0.5 \text{ mm})$ is connected to the voltage source of $10 \text{ V}$. The slab was fabricated from silicon doped with Phosphorus, such that the dopant density is $10^{16}$ $\text{cm}^{-3}$ Calculate the current flowing in the circuit, which is in the room temperature. Treat silicon-metal contacts as ideal (no contact resistance). Mobility plot is given below.

![Mobility plot](image_url)

**a) (10 points) Calculate the conductivity of the slab material.**

Phosphorus is a donor, hence the material is n-type ($n = 10^{18} \text{ cm}^{-3}$) and the conductivity is: $\sigma = \mu_n \cdot q \cdot n$, because $\mu < \mu_n$.

From the plot we read: $\mu_n = 900 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$.

Thus $\sigma = 1.6 \cdot 10^{-19} \text{ cm}^2 \text{ V}^{-1} \cdot 10^{16} \text{ cm}^{-3} \cdot 900 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} = 1.52 \text{ S cm}^{-1}$.

**b) (5 points) Calculate the current density.**

$$J = \frac{\sigma \cdot E}{\mu_n} = \frac{10 \text{ V}}{0.5 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}} = 20 \text{ A cm}^{-2}$$

Thus $J = 1.52 \text{ A cm}^{-2} \cdot 10 \text{ V} \Rightarrow J = 30.4 \text{ A cm}^{-2}$.

**c) (5 points) Calculate the current.**

The current (assuming uniform current density) is

$$I = J \cdot A = 30.4 \text{ A cm}^{-2} \cdot 0.05 \text{ cm} \times 0.05 \text{ cm} \Rightarrow I = 0.76 \text{ A cm}^{-2}$$

$A = 0.05 \text{ cm} \times 0.05 \text{ cm}$

$$I = 0.076 \text{ A}$$
5. (15 points) Consider a bulk silicon in thermal equilibrium at room temperature (27°C) with the uniform 
concentration of acceptors \( N_a^- = 9.05 \times 10^{17} \text{ cm}^{-3} \) and donors \( N_d^- = 9.10 \times 10^{17} \text{ cm}^{-3} \).

a) (7 points) Determine the majority carriers and their density?

Because \( N_a^- > N_d^- \), 
using the charge neutrality: \( \rho_{a^-} + \rho_{d^-} = \rho_{a^+} \rho_{d^+} \) and \( \rho_{d^-} \rho_{a^-} = \eta_e \), 
we get \( \rho_{a^-} = \frac{\eta_e}{\rho_{d^-}} \) and from charge neutrality: \( \frac{\eta_e}{\rho_{a^-}} + \rho_{a^-} = \rho_{a^+} \rho_{d^+} \) 
or else
\[
\rho_{a^-}^2 - (\rho_{d^-} - \rho_{d^+}) \rho_{a^-} - \rho_{d^-}^2 = 0
\]
which yields two solutions: 
\[
\rho_{a^-} = \frac{\rho_{d^-} - \rho_{d^+} \pm \sqrt{(\rho_{d^-} - \rho_{d^+})^2 + 4 \rho_{d^-}^2}}{2}
\]
The second soln. is non-physical and it is rejected.
Thus we get
\[
\rho_{a^-} = \left( 5 \times 10^{15} \right)^2 - \frac{2 \times 10^{15} - 4 \times 10^2}{2}
\]
Finally
\[
\rho_{a^-} = 5 \times 10^{15} \text{ cm}^{-1}
\]
b) (2 points) What is the minority carrier density?

\[
\rho_{d^-} = \frac{1.07 \times 10^{-15}}{5 \times 10^{15}} = 2.29 \times 10^{-4} \text{ cm}^{-1}
\]
c) (6 points) Calculate the charge density of fixed (immobile) charges?

\[
\rho_{\text{net}} = q \left( \rho_{d^-} - \rho_{a^-} \right)
\]
\[
\rho_{\text{net}} = 1.6 \times 10^{-19} \left( \text{C mol}^{-1} \right) - 5 \times 10^{-15} \left( \text{C cm}^{-2} \right)
\]

After arithmetical
\[
\rho_{\text{net}} = 8 \times 10^{-4} \left( \text{C cm}^{-2} \right)
\]