Name Solutions

ECE 340
Fall 2011
Exam 1

September 29, 2011

1. This exam is closed book and closed notes. No talking. No calculators.
2. Useful tables and equations are attached to the end of the test.
3. Show intermediate steps in order to get partial credit if you make an error.

1. (5 Points) Let $x(t) = \cos(4\pi t) + 5\sin(6\pi t - 0.5\pi)$.

a) Is $x(t)$ periodic? Justify your answer. If $x(t)$ is periodic, what is its period?

b) Is $x(t)$ an energy signal or a power signal? Calculate either the energy or the power as appropriate.

a) $x(t) = \cos(2\pi 2t) + 5\sin(2\pi 3t - \frac{1}{2}\pi)$

$f_1 = 2$, $f_2 = 3$ $\Rightarrow$ $f_0 = 1$, $T_0 = \frac{1}{f_0} = 1$; $f_1 = 2f_0$, $f_2 = 3f_0$; Yes, periodic

$$\boxed{T_0 = 1 \text{ sec}}$$

b) Periodic signal w/ infinite energy

$\Rightarrow$ Power Signal

$$P_x = \frac{(1)^2}{2} + \frac{(5)^2}{2} = \frac{26}{2} = \boxed{13}$$

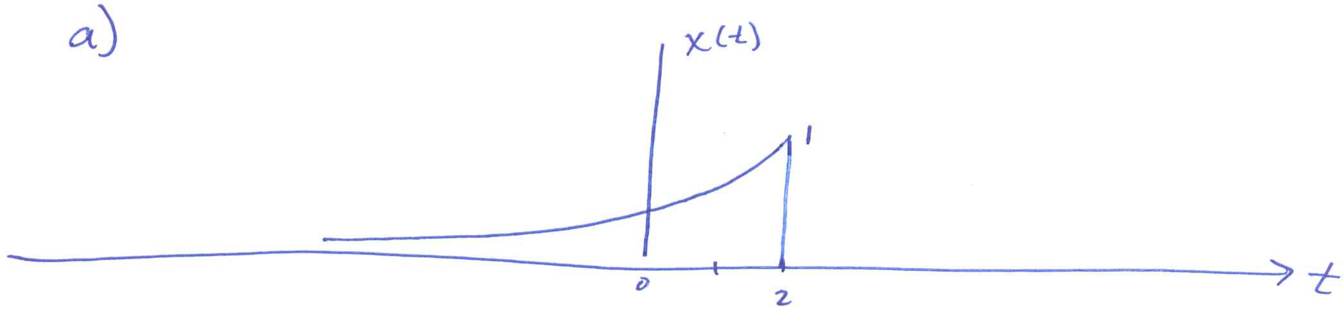
2. (6 Points) A signal is described by $x(t)=e^{(t-2)}u(-t+2)$. Sketch the following:

a) $x(t)$

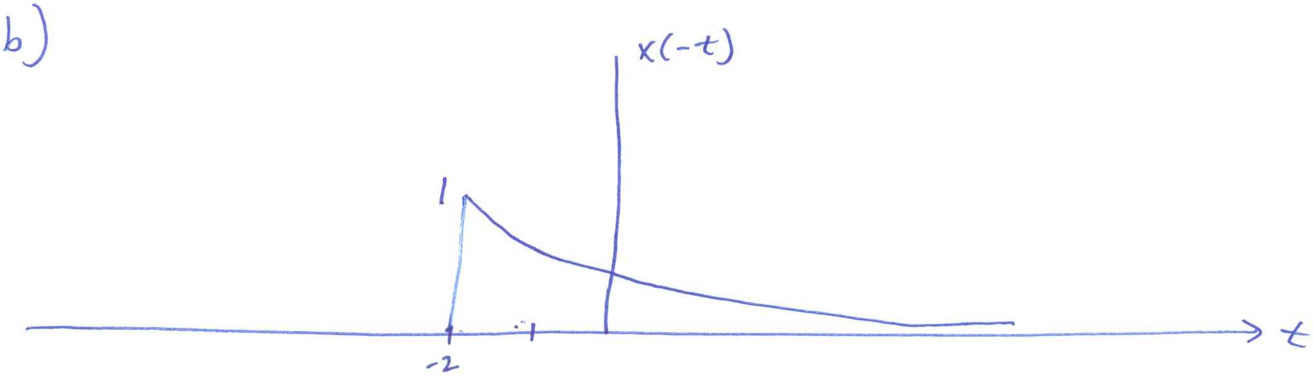
b) $x(-t)$

c) $2x(-t-3)$

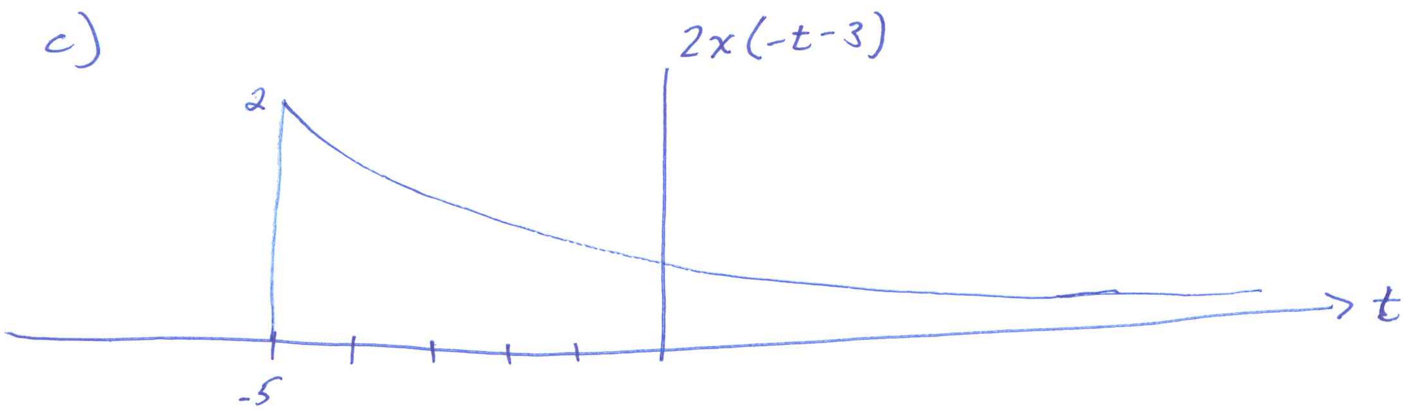
a)



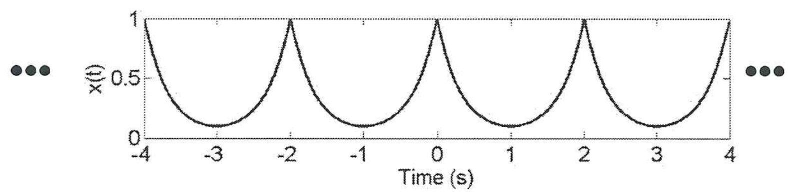
b)



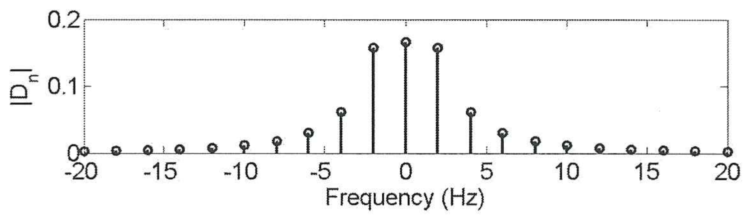
c)



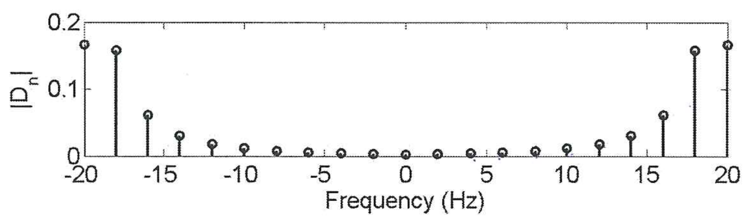
3. (4 Points) Which Fourier Series amplitude spectrum shown below is the correct spectrum for the periodic $x(t)$ shown?



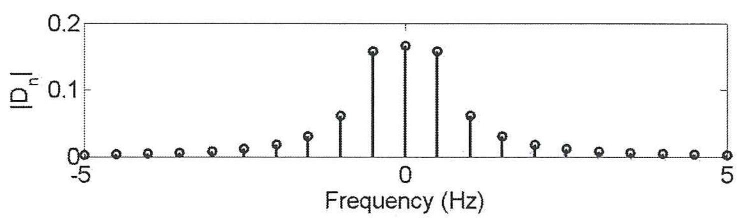
A)?



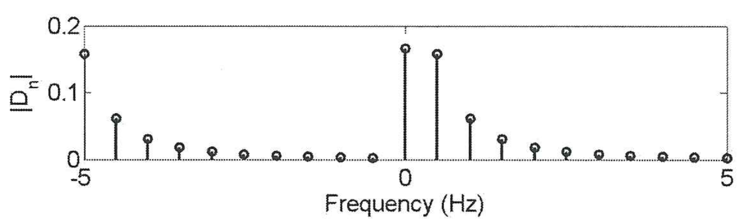
B)?



C)?



D)?



4. (4 Points) Let $x(t)=5te^{-0.1t}\left[\delta(t+0.5)-\delta(t-5)\right]$.

Calculate $\int_0^{100} x(t)\,dt$

$$\int_0^{100} 5te^{-0.1t}\left[\underbrace{\delta(t+0.5)}_{t=-0.5}-\delta(t-5)\right]dt$$

falls outside the limits of integration

$$\Rightarrow = \int_0^{100} -5te^{-0.1t}\delta(t-5)\,dt = -5te^{-0.1t}\Big|_{t=5}$$

$$= \boxed{-25e^{-0.5}}$$

5. (9 Points) A system is characterized by the input-output relationship

$$y(t) = ctx(t) + dx(t-1)$$

where c and d are constants.

Determine and state whether the system is linear, time-invariant, and/or causal. For full credit, show enough steps or provide enough explanation to justify your answer.

A) $y_1(t) = ctx_1(t) + dx_1(t-1)$

$y_2(t) = ct\,x_2(t) + dx_2(t-1)$

$a_1y_1(t) + a_2y_2(t) = a_1ct\,x_1(t) + a_2ctx_2(t)$
$+ a_1dx_1(t-1) + a_2dx_2(t-1)$

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$y_3(t) = ct\,x_3(t) + dx_3(t-1)$

$= ct(a_1x_1(t) + a_2x_2(t)) + d(a_1x_1(t) + a_2x_2(t))$

$= a_1y_1(t) + a_2y_2(t)$ from above $\therefore$ Linear

B) $y(t-T) = c(t-T)x(t-T) + dx(t-T-1)$

Let $x_1(t) = x(t-T)$

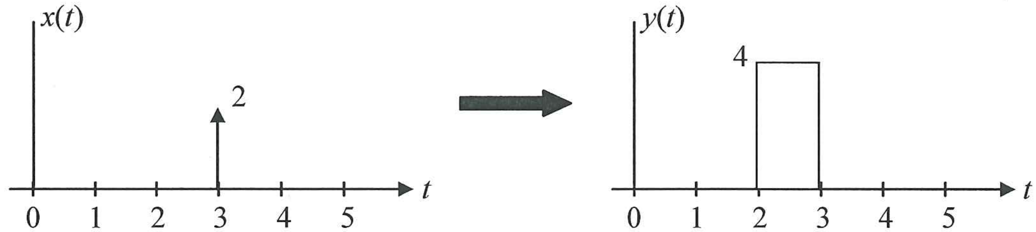
$y_1(t) = ct\,x_1(t) + dx_1(t-1)$

$= ct\,x(t-T) + dx(t-T-1) \neq y(t-T)$

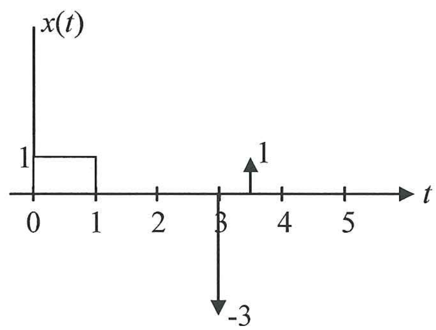
$\therefore$ Not time invariant

c) Causal because all inputs are @ current time or delayed

6. (11 Points) A linear, time-invariant, continuous (LTIC) system has the following input-output pair.

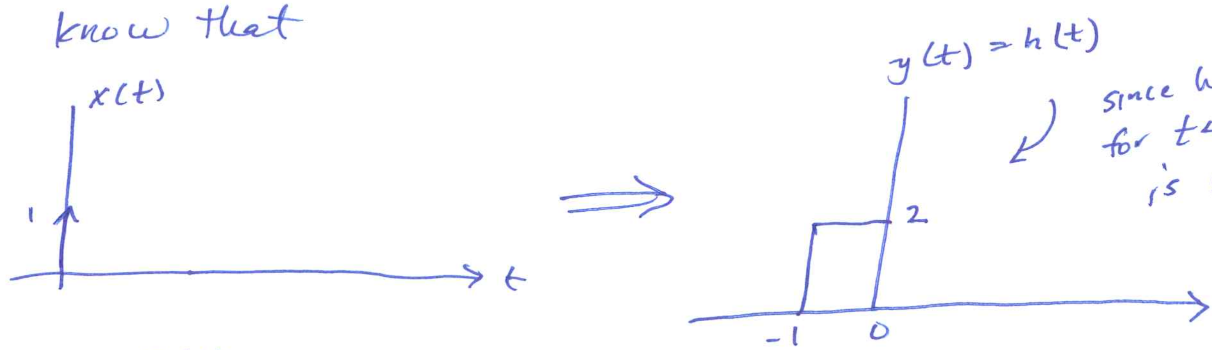


a) Draw the output of the system when the input is:



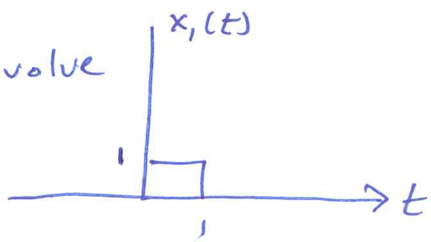
b) Is this system causal or non-causal?

a) Using the given pair, plus linearity and time invariance, we know that

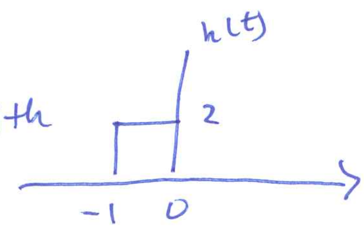


since $h(t) \neq 0$ for $t<0$, system is non-causal

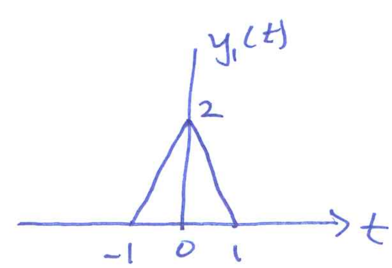
Convolve



with



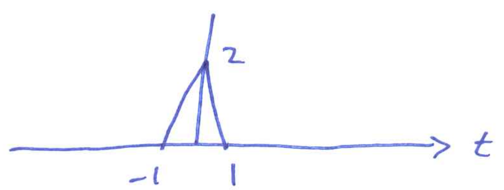
=



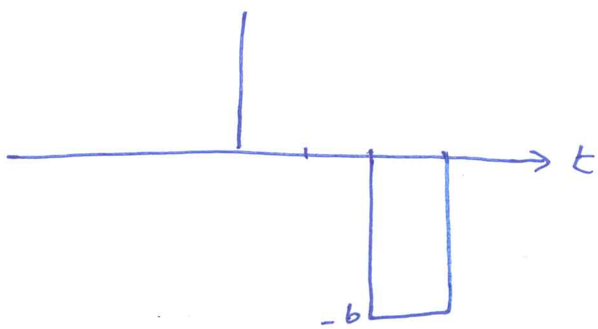
Also $\quad -3\delta(t-3) * h(t) = y_2(t) = -3h(t-3)$

and $\quad \delta(t-3.5) * h(t) = y_3(t) = h(t-3.5)$

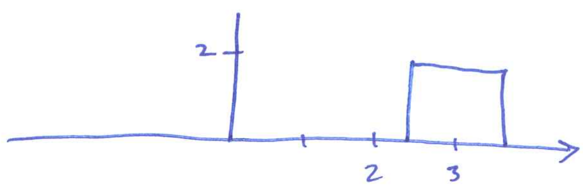
$\therefore \; y(t) = y_1(t) + y_2(t) + y_3(t) =$



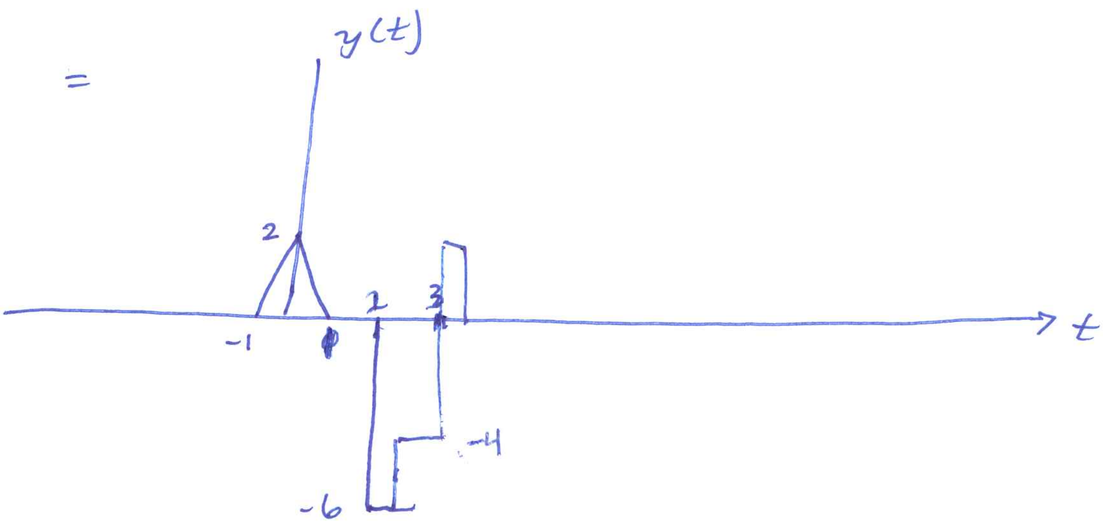
+



+



=



7. (11 Points) Let the impulse response of a causal LTIC system be:

$$h(t)=\left[e^{-\lambda_1 t}+e^{-\lambda_2 t}+2e^{-\lambda_3 t}\right]u(t)$$

where $\lambda_1=-2+3j$, $\lambda_2=-2-3j$, and $\lambda_3=-1$.

a) Is this system BIBO stable?

b) What is the transfer function, $H(s)$, of this system?

c) Suppose the input to this system is $x(t)=e^{(-1+j3)t}$. Write an expression for the output $y(t)$ due to this input.

a) $\left[e^{\lambda_1 t}+e^{\lambda_2 t}+2e^{\lambda_3 t}\right]u(t) =$

$$H(s)=\int_0^\infty h(t)e^{-st}\,dt=\int_0^\infty\left[e^{-(s-\lambda_1)t}+e^{-(s-\lambda_2)t}+2e^{-(s-\lambda_3)t}\right]dt$$

$$=\frac{1}{s-\lambda_1}+\frac{1}{s-\lambda_2}+\frac{2}{s-\lambda_3}$$

$\therefore$ Roots are, λ_1, λ_2, and λ_3

- All have negative real parts. $\therefore$ roots are in the LHP. System is stable

b) From above $H(s)=\dfrac{1}{s-\lambda_1}+\dfrac{1}{s-\lambda_2}+\dfrac{2}{s-\lambda_3}$

c) $$H(s)\Big|_{s=-1+j3}=\frac{1}{-1+j3-(-2+3j)}+\frac{1}{-1+j3-(-2-3j)}+\frac{2}{-1+j3-(-1)}$$

$$=\frac{1}{1+0j}+\frac{1}{1+6j}+\frac{2}{j3}$$

$$=1-\frac{2}{3}j+\left(\frac{(1-6j)}{(1+6j)(1-6j)}\right)=1-\frac{2}{3}j+\frac{1}{37}-\frac{6j}{37}$$

$$=\left(1+\frac{1}{37}\right)-\left(\frac{2}{3}+\frac{6}{37}\right)j=K$$

$\approx$ $1-0.8j$

$\boxed{y(t)=K\cdot x(t)}$

Alt:

a) $$H(s) = \int_0^\infty \left[e^{-(s+\lambda_1)t} + e^{-(s+\lambda_2)t} + 2e^{-(s+\lambda_3)t} \right] dt$$

$$= \frac{1}{s+\lambda_1} + \frac{1}{s+\lambda_2} + \frac{2}{s+\lambda_3} = H(s)$$

Roots are $-\lambda_1, -\lambda_2$, and $-\lambda_3$ which are all in RHP.

$\therefore$ system is <u>unstable</u>

b) $$H(s) = \frac{1}{s+\lambda_1} + \frac{1}{s+\lambda_2} + \frac{2}{s+\lambda_3}$$

c) $$H(s)\Big|_{s=-1+j3} = \frac{1}{-1+j3+(-2+3j)} + \frac{1}{-1+j3+(-2-3j)} + \frac{2}{-1+j3+(-1)}$$

$$= \frac{1}{-3+6j} + \frac{1}{-3} + \frac{2}{-2+j3}$$

$$= \frac{-3-6j}{45} - \frac{1}{3} + \frac{-4-j6}{13}$$

$$= \left(-\frac{1}{15} - \frac{1}{3} - \frac{4}{13}\right) + j\left(-\frac{2}{15} - \frac{6}{13}\right) = K$$