GEOMETRICAL IMAGING

First-order image is “perfect”

- scaled (by magnification) version of object
  
magnification = image distance/object distance

- no blurring
OPTICAL IMAGE FORMATION

Real image is blurred by two optical effects:

- **Aberrations**
  - e.g., spherical aberration, astigmatism, coma

- **Diffraction**

High quality, aberration-free system is diffraction-limited.

Optical systems can be designed with little aberration, but diffraction cannot be avoided or reduced due to the wave nature of light and finite apertures of optical systems.
Physical Basis

- Even a "perfect" system, from a geometrical optics viewpoint, will not form a point image of a point source.
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- Optical system apertures describe the wave nature of light, unlike geometrical optics.
- Optical system apertures truncate radiation wavefronts.
- Entrance and exit pupils are virtual apertures, i.e., images of the physical aperture.
- Even a "perfect" system, from a geometrical optics viewpoint, will not form a point image of a point source.

Physical Basis

Optical Image Formation
Imaging System as a Linear System

- Consider an optical imaging system as a "black box," with an input wavefront from the object, an entrance pupil, exit pupil, and an output wavefront forming the image.
- For incoherent light, the system is linear in intensity (square of complex field amplitude).
- Consider an optical imaging system as a "black box," with an input wavefront from the object, an entrance pupil, exit pupil, and an output wavefront forming the image.

**System f-number N = f/D**

- Imaging System as a Linear System

**OPTICAL IMAGE FORMATION**
OPTICAL IMAGE FORMATION

• Impulse response called the Point Spread Function (PSF)

\[
PSF(x, y) = g(x, y) \ast \delta(x, y)
\]

where

\[
f(x, y) = \text{geometrical image}
\]

\[
g(x, y) = \text{diffraction-limited image}
\]

General LSI Imaging equation for optical diffraction

Image of a point source (delta function input)

Impulse response called the Point Spread Function (PSF)
OPTICAL IMAGE FORMATION

- 2-D Fourier transform of the PSF called the Optical Transfer Function (OTF)

\[ \mathcal{S}\int \psi(x,y) e^{-j2\pi(f_x x + f_y y)} df_x df_y = \text{OTF} \]

ALWAYS A LOW-PASS FILTER (LPF)
Diffraction-Limited PSF

• Incoherent light, circular aperture with diameter D

The first zero occurs at \( r' = \frac{\lambda f}{N} \)

\[
\text{exit pupil diameter} = D
\]

\[
\text{where } J_1 \text{ is the Bessel function of the first kind and the normalized radius } r' \text{ is,}
\]

\[
\text{exit pupil function} = \left( \frac{r'}{D} \right)_0^\infty
\]

PSF is Fourier-transform, scaled and squarwed and

\[
\left( \frac{r'}{D} \right)_0^\infty = (r')_0^\infty
\]

OPTICAL IMAGE FORMATION
OPTICAL IMAGE FORMATION

The central bright region, to first-zero ring, is called the "Airy disk". The "Airy pattern" refers to the radial profile of the point spread function (PSF) normalized by the radius of the Airy disk.
Example calculation of PSF size

system specs:

- \( D = 1 \text{ cm} \)
- \( f = 50 \text{ mm} \)
- \( N = \frac{f}{D} = 5 \)
- \( f = 50 \text{ mm} \)
- \( D = 1 \text{ cm} \)

radius of PSF = \( 1.22 \lambda N = 3.36 \mu \text{m} \)

very small!


\[
\text{wavelength of light} = \lambda \\
\text{f-number} = N \\
\text{focal length} = f \\
\text{aperture diameter} = D
\]

where

\[
\frac{N\lambda}{f} = \frac{f\lambda}{D} = \psi D
\]

and the cutoff frequency is given by

\[
\psi D/d = \psi
\]

where the normalized radial spatial frequency is given by

\[
[\sqrt{\frac{\cos d}{d}} - 1]^2 \psi_d - (\psi d) \psi = (\psi d)^2 OTF
\]

- Incoherent light, circular aperture with diameter D

**Diffraction-Limited OTF**
OPTICAL IMAGE FORMATION

• General LSI imaging equation for optical diffraction where

- $f(x,y) = \text{geometrical image}$
- $g(x,y) = \text{diffraction-limited image}$

Take Fourier transform where

- $F(u,v) = \text{spatial frequency spectrum of geometrical image}$
- $G(u,v) = \text{spatial frequency spectrum of diffraction image}$
- $\text{OTF}(u,v) = \text{Optical Transfer Function}$

Note, all three quantities are functions of spatial frequency in the image space, i.e.,

$$g_{xy}(x,y) = G_{uv}(u,v) \cdot F_{uv}(u,v) \cdot \text{OTF}_{uv}(u,v)$$

where

$$G_{uv}(u,v) = F_{uv}(u,v) \cdot \text{OTF}_{uv}(u,v)$$

Take Fourier transform where

- $g(x,y) = \text{diffraction-limited image}$
- $F(x,y) = \text{geometrical image}$

where

$$g_{x,y}(x,y) = \text{PSF}_x \ast \ast (\lambda', \lambda') \ast f = \text{PSF}_x \ast \ast (\lambda', \lambda') \ast f$$

General LSI imaging equation for optical diffraction

OPTICAL IMAGE FORMATION
\[ \varphi_0 = \varphi_c \]

**OTF**

- **OTF** is a low-pass filter of spatial frequencies.
- Nearly a "cone" function of \((u,v)\)
- At zero frequency
- By convention, always normalized to 1

**Spatial Frequencies**

- **OTF** is a low-pass filter of spatial frequencies.
OPTICAL IMAGE FORMATION

• Example calculation of OTF cutoff frequency

System specs:
• \( D = 1 \text{cm} \)
• \( f = 50 \text{mm} \)
• \( N = \frac{f}{D} = 5 \)
• \( \lambda = 0.55 \mu \text{m} \) (green)

Cutoff frequency = \( \frac{1}{\lambda N} \approx 0.363 \text{cycles/\mu m} = 363 \text{cycles/mm} \)

Very high! (the human vision system has a cutoff frequency of about 10-12 cycles/mm at normal viewing distance.)

Optical Image Formation
Modulation Transfer Function (MTF)

Optical Image Formation

For sine wave, $A + B \sin (2\pi u_0 x)$, modulation is a measure of signal contrast.

$$\text{signal modulation} = \frac{B}{A}$$

$$\text{signal modulation} = \frac{\text{max} + \text{min}}{\text{max} - \text{min}} = \frac{A + B}{A}$$

$$\text{signal modulation} = \left(\frac{A}{\lambda} \cdot n\right)_{\text{MTF}} = \left|\left(\frac{A}{\lambda} \cdot n\right)_{\text{OTF}}\right| = \left(\frac{A}{\lambda} \cdot n\right)_{\text{MTF}}$$
OPTICAL IMAGE FORMATION

• Use MTF to predict image contrast, given object and system

Image modulation = \frac{A}{B} \cdot (\circ n) x (\partial W)

\text{MTF} = \text{Input modulation at spatial frequency } u_0

- corresponds to the geometrical image formed by the optics

\text{Output is}

\text{System input to LSI}

\text{corresponds to the diffraction image formed by the optics}

\text{Output} = (x) \text{input modulation} \times \text{MTF(u,v)}

\text{For sinusoidal input to LSI system}
For imaging system
typical MTF
What is going on here?

Input pattern: “chirp” function

spatial frequency, \( u \)

greyscale pattern

maps spatial frequency into a spatial representation

\[ u = \frac{2\pi}{\cos(2\pi x^2)} \]

spatial frequency increases linearly with distance

Profile

DIFFRACTION IMAGING EXAMPLE

OPTICAL IMAGE FORMATION
OPTICAL IMAGE FORMATION

Diffraction-limited optics with a circular aperture

Optical Image Formation

System Response

Output Pattern

Convolution

Envelope of output pattern amplitude is the MTF

(cutoff-frequency, \( u_{c} \))

(because of pattern used)
OPTICAL IMAGE FORMATION

LINE AND EDGE SPREAD FUNCTIONS

• PSF is often difficult to measure because of insufficient energy

• Use a line source (slit) or edge (step) source to increase energy for measurement

• PSF is often difficult to measure because of insufficient energy

• Use a line source (slit) or edge (step) source to increase energy for measurement
OPTICAL IMAGE FORMATION

**Line Spread Function (LSF)**

- **Integrate PSF along direction of line source**
- **Line spread function (LSF)**
- **If PSF is not rotationally symmetric, LSF is different in different directions**
- **If PSF does not have zeros like PSF**

\[
\int_{-\infty}^{\infty} \psi_p(\xi, x) \psi_d(x) dx = (\xi)^{LSF}
\]

\[
\int_{-\infty}^{\infty} \xi \psi_p(\xi, x) \psi_d(x) dx = (x)^{LSF}
\]
OPTICAL IMAGE FORMATION

Edge Spread Function (ESF)

- ESF is a monotonic function of distance.
- Equivalent to step response in electronic system.

\[
\int_{-\infty}^{\infty} \text{ESF}(x) \, dx = (\alpha)^x \int_{-\infty}^{\infty} \text{LSF}(\alpha) \, d\alpha
\]

- Integrate LSF up to location of ESF.
- Measure irradiance (watts/m²) vs. distance (arbitrary units).

Dr. Robert A. Schowengerdt
OPTICAL IMAGE FORMATION

Relations among PSF, OTF, LSF and ESF (x-direction)

PSF(x,y)  \rightarrow  OTF(u,v)  \rightarrow  LSF(x)

OTF(u,0)  \rightarrow  Profile

One-sided 1-D integration

OTF(u,v)  \rightarrow  2-D Fourier Transform

PSF(x,y)  \rightarrow  1-D Fourier Transform

PSF(x,y)  \rightarrow  One-sided 1-D Integration

ESF(x)  \rightarrow  Derivative

One-sided 1-D Integration

LSTF(x)

relations among PSF, OTF, LSF and ESF (x-direction)
DEFOCUS

By geometry, diameter of blur circle, \( d \), is:

\[
\frac{\Delta}{2}, \quad \text{where } \Delta = f \text{-number}
\]

\[
\frac{\Delta}{D} = \frac{f}{D}
\]

OPTICAL IMAGE FORMATION
Filter Model
Spatial Phase Shift

- π spatial phase shift at spatial frequencies where OTF is negative
- First-zero in OTF defocus
- $\nabla = 2\Delta$ defocus

Spatial Phase Shift

Optical Image Formation