IMAGE WARPING

- Correct system distortions
- Register multiple images
- Register image to map (rectification)
- Register multiple images (rectification)
- Correct system distortions

Applications

Rectification of Landsat TM Image
WHAT IS REGISTRATION?

- Alignment of two images such that pixels in each correspond to the same area

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Examples of registered remote sensing images

SAME SENSOR

Different sensors

Date 1, date 2 with off-nadir pointing

Or

Date 1 and date 1 + revisit period

Orbit

Orbit K

Orbit J
THREE COMPONENTS TO WARPING

• Resampling (interpolation)
• Coordinate transformation
• Appropriate mathematical distortion model(s)
For example, a quadratic polynomial is written as:

\[
\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \left( a_{ij} x_{\text{ref}}^i y_{\text{ref}}^j + b_{ij} x_{\text{ref}}^i y_{\text{ref}}^j \right) = x,\ y
\]

Coordinate system \((x', y')\) and Reference

- Relates distorted coordinate system \((x, y)\) and Reference
- Known as "rubber sheet stretch"

Generic model useful for registration, rectification and geocoding

**POLYNOMIAL DISTORTION MODELS**

**IMAGE WARPING**
The coefficients in the polynomial can be associated with particular types of distortion.

The coefficients in the polynomial can be associated with particular types of distortion.

### Image Warping

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Warp Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{00} )</td>
<td>Nonlinear scale in ( x )</td>
</tr>
<tr>
<td>( b_{00} )</td>
<td>Nonlinear scale in ( y )</td>
</tr>
<tr>
<td>( a_{10} )</td>
<td>( x )-dependent scale in ( y )</td>
</tr>
<tr>
<td>( b_{01} )</td>
<td>( y )-dependent scale in ( x )</td>
</tr>
<tr>
<td>( a_{01} )</td>
<td>( x )-dependent shear in ( y )</td>
</tr>
<tr>
<td>( b_{10} )</td>
<td>( y )-dependent shear in ( x )</td>
</tr>
<tr>
<td>( a_{11} )</td>
<td>Nonlinear scale in ( x )</td>
</tr>
<tr>
<td>( b_{11} )</td>
<td>Nonlinear scale in ( y )</td>
</tr>
<tr>
<td>( a_{20} )</td>
<td>( x )-dependent shift in ( y )</td>
</tr>
<tr>
<td>( b_{02} )</td>
<td>( y )-dependent shift in ( x )</td>
</tr>
</tbody>
</table>

These coefficients represent different types of distortions that can be applied to an image.
different types of distortion
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**IMAGE WARPING**

• A linear (affine) polynomial (number of terms $K = 3$)

Special case that can include:

• shift
• scale
• rotation
• shear

In vector-matrix notation

$$
\begin{bmatrix}
00 q \\
00 p
\end{bmatrix} + \begin{bmatrix}
\hat{f}_{\alpha \lambda} & \hat{f}_{\alpha \xi} \\
\hat{f}_{\xi \lambda} & \hat{f}_{\xi \xi}
\end{bmatrix} \begin{bmatrix}
10 q & 01 q \\
10 p & 01 p
\end{bmatrix} = \begin{bmatrix}
\hat{x} \\
\hat{y}
\end{bmatrix}
$$

rotation

shear

scale

shift

Special case that can include:

• A linear (affine) polynomial (number of terms $K = 3$)
Varying degree, depending on the problem
often located by visual examination, but can be automated to
- Often located by visual examination, but can be automated to
  - all are at the same elevation (unless topographic relief is being specifically addressed)
  - unchanging over time
  - small feature size
  - high contrast in all images of interest

Characteristics:
- Use to control the polynomial, i.e., to determine its coefficients

GROUND CONTROL POINTS (GCPS)

IMAGE WARPING
Automated GCP Location

- Use image "chips"
- Small segments that contain one easily identified and well-located GCP
- Normalized cross-correlation between template chip T (Reference)
- Normalization adjusts for changes in mean DN within area

\[
\begin{align*}
K & = R_{ij}T_{mn}S_{im}\sum_{n=1}^{N} + 1 = u, 1 = w \\
K & = R_{ij}S_{im}\sum_{n=1}^{N} + 1 = u, 1 = w
\end{align*}
\]

where

\[
R_{ij} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{T_{mn} - \mu_T}{\sigma_T} \cdot \frac{S_{imjn} - \mu_S}{\sigma_S}
\]
Schematic for correlation

Chip layout over full scene

two relative shift positions

Cross-correlation of one area

IMAGE WARPING
Example with TM "chips"
IMAGE WARPING

Using GCPs to find polynomial coefficients

• Set up system of simultaneous equations using GCPs and solve for polynomial coefficients

Given M pairs of GCPs

For each GCP pair, m, create two equations

Example with quadratic polynomial (number of terms K = 6)

Using GCPs to find polynomial coefficients
So, for each set of GCP x- and y-coordinate pairs, we can write

**Exact solution which passes through GCPs, i.e., they are mapped exactly**

\[ X \mathbf{M} = \mathbf{B} \]
\[ X \mathbf{M} = \mathbf{V} \]

- **Determined case** ($M = K$, just enough GCPs) solution:

\[ \mathbf{B} \mathbf{W} = \mathbf{X} \]
\[ \mathbf{V} \mathbf{W} = \mathbf{X} \]

So, for each set of GCP x- and y-coordinate pairs, we can write
315

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315

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IMAGE WARPING

• Overdetermined case (M > K, more than enough GCPs):

\[
\begin{align*}
(\mathbf{WM} - \mathbf{X}) (\mathbf{WM} - \mathbf{X}) & = [X_3 X_3] \min \\
(\mathbf{YM} - \mathbf{X}) (\mathbf{YM} - \mathbf{X}) & = [X_3 X_3] \min
\end{align*}
\]

This solution results in least-squares minimum error at GCPs:

This solution is called the pseudoinverse of \( \mathbf{W} \) is called the pseudoinverse of \( \mathbf{W} \)

\[
\begin{align*}
\mathbf{XM} & = \mathbf{B} \\
\mathbf{XM} & = \mathbf{V}
\end{align*}
\]

Solution:

\[
\begin{align*}
X_3 + BM & = X \\
X_3 + VM & = X
\end{align*}
\]
1-D Example of Polynomial Curve Fitting

$y = 13.228 + 0.82085x \quad R^2 = 0.93425$
Second order

\[ y = M_0 + M_1 x + \ldots + M_8 x^8 + M_9 x^9 \]
IMAGE WARPING

third order

\[ Y = M_0 + M_1 x + \ldots + M_8 x^8 + M_9 x^9 \]

\begin{tabular}{|c|c|}
\hline
\textbf{Y} & \textbf{x} \\
\hline
0.98745 & 0 \\
0.011585 & 30 \\
-0.14393 & 60 \\
5.9866 & 90 \\
-24.455 & 120 \\
\hline
\end{tabular}
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**Fourth order**

\[ Y = M_0 + M_1 x + \ldots + M_9 x^8 + M_{9,x} x^9 \]

<table>
<thead>
<tr>
<th>( M_0 )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99999</td>
<td></td>
</tr>
<tr>
<td>3.4942e-05</td>
<td></td>
</tr>
<tr>
<td>-0.0049359</td>
<td></td>
</tr>
<tr>
<td>3.4942e-05</td>
<td></td>
</tr>
<tr>
<td>-2.5272</td>
<td></td>
</tr>
<tr>
<td>31.173</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1: Coefficients**
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**IMAGE WARPING**

- Fifth order

\[ y = M_0 + M_1 x + \ldots + M_8 x^8 + M_9 x^9 \]
Example Application

• Register aerial photo to map in an urban area

  (red circles) points (GPs) in each
  each (black crosses)
  locate 6 GCPs in
  locate 4 Ground
  not used for control, only

  for testing

  GCP 1
  GCP 2
  GCP 3
  GCP 4
  GCP 5
  GCP 6

  aerial photo (x,y)

  map reference (x_{ref}, y_{ref})
Mapping of GCPs

**Image Warping**

Mapping of GCPs
Error at GCPs and GPS versus polynomial order

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Refinement of GCPs by error analysis

Original six GCPs

Five GCPs, with outlier deleted
Final Result

IMAGE WARPING
IMAGE WARPING

Piecewise Polynomial Distortion Model

- For severely distorted images that can't be modeled with a single, global polynomial of reasonable order.

Example airborne scanner image

Reasonable order single, global polynomial of that can't be modeled with a

Distortion Model

Piecewise Polynomial
Coordinate Transformation

Image Warping

Overlaying of multiple pixels in the processed image

“Backwards” mapping from \((x_{re}^f, y_{re}^f)\) to \((x, y)\) avoids “holes” or

\(\begin{align*}
(x_{re}^f, y_{re}^f) & \text{ from the original image at } (x, y) \\
& \text{resampling} \\
& \text{Implemntation} \\
\end{align*}\)

\(\begin{align*}
(x, y) & \text{ by Eq. 7-27} \\
& \text{disortion correction} \\
& \text{coordinate system} \\
& \text{distorted Image Frame} \\
& \text{coordinates are mapped from the reference frame to the}
\end{align*}\)

\(\begin{align*}
(x_{re}^f, y_{re}^f)_{implied} & = (x, y) \\
\end{align*}\)
Image warping is the transformation between two frames.

Coordinate transformation between two frames:

\[(x_{\text{ref}}, y_{\text{ref}})\]

\[(x', y')\]
The (x,y) coordinates calculated by are generally between the integer pixel coordinates of the array. Therefore, must estimate (interpolate or resample) a new pixel. The (x,y) coordinates calculated by the array are generally between the integer pixel coordinates of the array 

\[ f(x',y') = (x',y') \]

**Resampling**

**Image Warping**
Image Warping

- Pixels are resampled using a weighted-average of the neighboring pixels.

- Three common weighting functions:
  - Nearest-neighbor: Fast, but discontinuous
  - Bilinear: Slower, but continuous
  - Nearest-neighbor: Fast, but discontinuous

Implement 2-D bilinear resampling as two successive 1-D resamplings:

- Resample E between A and B
- Resample F between C and D
- Resample DN(x,y) between E and F

Mathematically:

\[ \Delta y \left[ \frac{\partial DN \left( x - 1 \right) + \partial \triangle x \Delta y}{\partial y} \right] + \left( \Delta x - 1 \right) \left[ \frac{\partial DN \left( y - 1 \right) + \partial \triangle y \Delta x}{\partial x} \right] = \frac{\partial DN}{\partial x} \]

Three common weighting functions:

- Nearest-neighbor: Fast, but discontinuous
- Bilinear: Slower, but continuous
- Nearest-neighbor: Fast, but discontinuous
 IMAGE WARPING

bicubic: slowest, but results in sharpest image

- piecewise polynomial; special case of Parametric Cubic Convolution (PCC)
- approximates image as a set of parabolas centered on the grid points
- formula involves cubic polynomials for each pixel
- suitable for small displacements

bicubic

where $\nabla$ is the distance from $(x', y')$ to the grid points in 2D

- $\nabla = |x' - x|$
- $\nabla = |y' - y|$
- $\alpha$ is a parameter
- $\Delta = 1 - \alpha^2$
- $\Delta = -0.5$
- $\Delta = -1$
- $\Delta = -1.5$
- $\Delta = -2$
- $\Delta = -2.5$
- $\Delta = -3$
- $\Delta = -3.5$
- $\Delta = -4$

High-pass filter characteristics: amount of boost depends on amplitude of side-lobes, which is proportional to $\alpha$.

- "Standard" bicubic is $\alpha = -1$; superior bicubic is $\alpha = -0.5$.

$DN_{\alpha}(\nabla, \alpha) = (\nabla^2 x + \alpha)(\nabla^2 y + \alpha)$
IMAGE WARPING

\[ \Delta x_1 - \Delta x \cdot \alpha; \delta \Delta y_1 - \Delta y \cdot \alpha; = \Delta x \cdot \alpha; \delta \Delta y = \Delta \alpha; \delta \]

PCC resampling procedure

- resample along each row, A-D, E-H, I-L, M-P
- resample along new column Q-T
Comparison of nearest-neighbor and bilinear resampling for image magnification:

- **1x** (Original image)
- **2x**
- **3x**

**NEAREST-NEIGHBOR:**

- **1x**
- **2x**
- **3x**

**BI线ERLINEAR:**

- **1x**
- **2x**
- **3x**

**IMAGE WARPING**
Surface plots from previous figure

**bilinear**

**nearest-neighbor**
Comparison of 4 resampling functions for image magnification:

- **Nearest Neighbor**
- **Bilinear**
- **PCC ($\alpha = -0.5$)**
- **PCC ($\alpha = -1.0$)**
Note, resampling function affects local radiometric accuracy. While polynomial distortion model affects global geometric accuracy.

- nearest-neighbor - bilinear
- bilinear - PCC

Image difference maps resulting from different resampling functions.