1. Given the following 2-D image array \( f(m,n) \)

\[
\begin{array}{ccc}
2 & 5 & 3 \\
1 & 4 & 1 \\
\end{array}
\]

find the output of convolution with each of the following filter arrays \( h(m,n) \). (20%)

Rotate \( h(m,n) \) to \( h(-m,-n) \):

\[
\begin{array}{ccc}
1 & -1 & 1 \\
1 & -1 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
-1 & 4 & -1 \\
-1 & 4 & -1 \\
\end{array}
\]

\[
\begin{array}{ccc}
-1 & 1 & -1 \\
1 & 1 & -1 \\
\end{array}
\]

\[
\begin{array}{ccc}
-1 & 1 & -1 \\
1 & 1 & -1 \\
\end{array}
\]

\[
\begin{array}{ccc}
-1 & 4 & -1 \\
-1 & 4 & -1 \\
\end{array}
\]

\[
\begin{array}{ccc}
-1 & 1 & -1 \\
1 & 1 & -1 \\
\end{array}
\]

\[
\begin{array}{ccc}
-1 & 4 & -1 \\
-1 & 4 & -1 \\
\end{array}
\]
Note, that $h2(-m,-n) = h2(m,n)$ because of symmetry. Now, shift these inverted $h$ arrays over the input $f(m,n)$, multiply the overlapping $h$ and $f$ values, and add to obtain the output at each shift location:

2.a. Prove for $g(m,n) = f(m,n) \ast h(m,n)$, that, (20%)

$$
\sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} g(m, n) = \left[ \sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} f(m, n) \right] \left[ \sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} h(m, n) \right]
$$

This property is called “conservation of volume”

$$
\sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} g(m, n) = \sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} \sum_{k = -\infty}^{\infty} \sum_{l = -\infty}^{\infty} h(m-k, n-l)u(k, l)
$$

$$
= \left( \sum_{k = -\infty}^{\infty} \sum_{l = -\infty}^{\infty} \left[ \sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} h(m-k, n-l) \right] \cdot u(k, l) \right)
$$

$$
= \left( \sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} h(m, n) \right) \cdot \left( \sum_{k = -\infty}^{\infty} \sum_{l = -\infty}^{\infty} u(k, l) \right)
$$

b. Verify the above equation for the examples in Prob. 1. (10%)

(volume of $f = 16$) * (volume of $h1 = 2$) = (volume of $g1 = 32$)

(volume of $f = 16$) * (volume of $h2 = 0$) = (volume of $g2 = 0$)

3. The checkerboard pattern $f(x,y)$ below has the following specifications,

width of each square = 15,
minimum = 0 and
maximum = 255.
The pattern in \( f(x,y) \) repeats indefinitely and is infinite in extent in x and y. (20%) 

a. Write the equation for \( f(x,y) \) using delta and rectangle functions. There are several ways to do this, including:

\[
f(x, y) = 255 \text{rect}
\left(\frac{x}{15}, \frac{y}{15}\right) * \sum_m \sum_n [\delta(x - m30, y - n30) + \delta(x - m30 - 15, y - n30 - 15)]
\]

\[
= 255 \left[ \text{rect}
\left(\frac{x}{15}, \frac{y}{15}\right) + \text{rect}
\left(\frac{x-15}{15}, \frac{y-15}{15}\right) \right] * \sum_m \sum_n \delta(x - m30, y - n30)
\]

b. Write the equation for \( f(x,y) \) using the 2-D comb and rectangle functions. There are several ways to do this, including:

\[
f(x, y) = 255 \text{rect}
\left(\frac{x}{15}, \frac{y}{15}\right) * \frac{1}{30^2} \left[ \text{comb}
\left(\frac{x}{30}, \frac{y}{30}\right) + \text{comb}
\left(\frac{x-15}{30}, \frac{y-15}{30}\right) \right]
\]

\[
= 255 \left[ \text{rect}
\left(\frac{x}{15}, \frac{y}{15}\right) + \text{rect}
\left(\frac{x-15}{15}, \frac{y-15}{15}\right) \right] * \frac{1}{30^2} \text{comb}
\left(\frac{x}{30}, \frac{y}{30}\right)
\]

4. Plot the following functions: (30%)

a. plot the x-profile, y-profile and 45°-profile (y = x) of \( f(x,y) = \text{tri}(x/10,y/10) \) on the same graph
b. plot the x-profile and y-profile of $f(x,y) = \text{sinc}(x/10, y/5)$ on the same graph

c. make a 2-D map in $(x,y)$ of the positive and negative regions of $f(x,y) = \text{sinc}(x/10, y/5)$ (do a plan (vertical) view, not a perspective (oblique) view)