Exam: Open-Notes, books, no partial credit

Because there is no partial credit, it is especially important that you clearly define, specify or describe everything that you do.

1. (10%, 10%, 10%) Given the following noisy image segment $g(m,n)$:

$$
\begin{array}{cccc}
  m &=& \\
   1 & 2 & 3 & 4 \\
   4 & 123 & 114 & 188 & 173 \\
   n &=& 3 & 120 & 126 & 193 & 199 \\
   2 & 135 & 127 & 197 & 201 \\
   1 & 182 & 179 & 189 & 193 \\
\end{array}
$$

The global image noise variance is 100, and the noise is additive to and independent of the noiseless image $f(m,n)$. Calculate the adaptive Wiener filter output at pixel (2,2) using a 3x3 pixel neighborhood and uniform box filters.

The adaptive WF output at pixel $(m,n)$ is (notes17,p387):

$$\hat{f}(m, n) = LP(m, n) + \left[ \frac{\sigma^2_f(m, n)}{\sigma^2_f(m, n) + \sigma^2_v} \right] HP(m, n)$$

Using a 3x3 uniform box filter,

$LP(2,2) = 160.89$

$HP(2,2) = 127 - 160.89 = -33.89$

$\sigma^2_f(2, 2) + \sigma^2_v = 1075.86$ (956.3 if variance calculated with N instead of N-1)

therefore, $\sigma^2_f(2, 2) = 975.86$ and $\frac{\sigma^2_f(2, 2)}{\sigma^2_f(2, 2) + \sigma^2_v} = 0.907$ and $\hat{f}(2, 2) = 130.15$ (130.55)

Note that, because this 3x3 region overlaps a high contrast feature and therefore has relatively high SNR, the change due to the WF is small.

2. (5%, 5%) Given the following 1-D image:

\[
\begin{align*}
\hat{f}_{mn}(t) &= \frac{\sigma^2_f(2, 2)}{\sigma^2_f(2, 2) + \sigma^2_v} \cdot HP(2,2) \\
\end{align*}
\]
173 160 156 171 167 189 185 194 173 164 188 173 181

Calculate the other 2 levels in the resolution pyramid using the 1-D Burt kernel,

0.05 0.25 0.4 0.25 0.05

Average 5 pixel segments and downsample by 2 (no padding):

second level:  162.15  173.85  186.75  177.35  177.15
third level:  179.47

Various padding schemes OK, as long as weighting and downsampling are correct
3. You take two portraits of a woman, but accidentally move the digital camera between the two pictures (A and B below). The offset is 33 pixels in the x-direction (horizontal); there is no other distortion. Determine the two polynomials required to register image B to image A (reference). Show your coordinate systems. (20%)

\[x = a_{00} + a_{10}x_{ref} + a_{01}y_{ref} + \ldots\]
\[y = b_{00} + b_{10}x_{ref} + b_{01}y_{ref} + \ldots\]

The only distortion is a horizontal shift; therefore, all other coefficients are either zero or one, including higher-order terms

\[x = a_{00} + x_{ref}\]
\[y = y_{ref}\]

By inspection, \(a_{00} = -33\) and \(x = -33 + x_{ref}\)
4. Given the following DN values for a digital image:

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<tr>
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<td>201</td>
<td>183</td>
<td>190</td>
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The image is resampled for geometric correction. What is the resampled pixel value at \((x,y) = (2.4, 3.7)\), if bilinear resampling is used? **Note the coordinate system.** (10%)

From notes17, p330:
\[
\text{DN}(2.4,3.7) = (0.4*182 + 0.6*187)*0.3 + (0.4*189 + 0.6*200)*0.7 = 192.42
\]
5. Given the following 1-band image,

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<td>200</td>
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Cluster the image into 2 clusters using the K-means iterative nearest-mean clustering algorithm, with seed mean values 188 (cluster a) and 210 (cluster b).

Calculate the cluster map after the first cluster assignment (10%) and after the second cluster assignment (10%).

after first cluster assignment:

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new cluster mean estimates:

mean a = 178.78
mean b = 211.43

after second cluster assignment:

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No change.

Calculate the signature map after the second cluster assignment (10%).

211.43 211.43 211.43 211.43
178.78 178.78 178.78 178.78
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178.78 211.43 211.43 211.43