


THE UNIVERSITY OF  
**ARIZONA**  
TUCSON, ARIZONA

## CORRECTION AND CALIBRATION

*Reading: Chapter 7, 8.1-8.3*

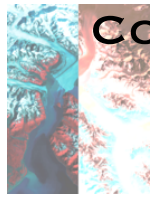
ECE/OPTI 531 – IMAGE PROCESSING LAB FOR REMOTE SENSING FALL 2005



## PREPROCESSING

- *Required for certain sensor characteristics and systematic defects*
- *Includes:*
  - *noise reduction*
  - *radiometric calibration*
  - *distortion correction*
- *Usually performed by the data provider, as requested by the user (see Data Systems)*

CORRECTION AND CALIBRATION 2 FALL 2005



## CORRECTION AND CALIBRATION

- **Noise Reduction**
- *Radiometric Calibration*
- *Distortion Correction*



## NOISE REDUCTION

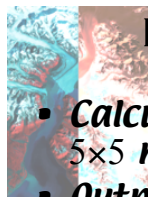
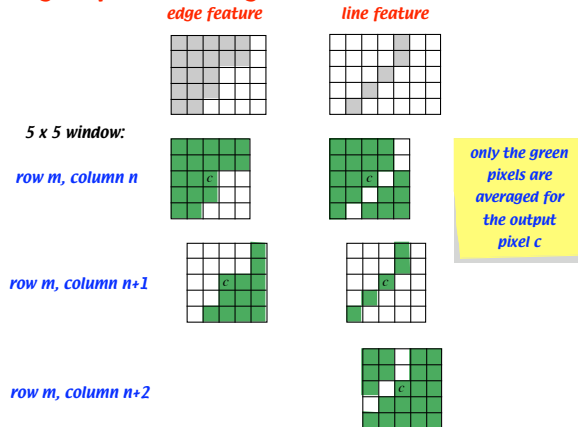
- *Requires knowledge of noise characteristics*
- *Since noise is usually random and unpredictable, we have to estimate its characteristics from noisy images*
- **Global Noise**
  - *Random DN variation at every pixel*
  - *LPFs will reduce this noise, but also smooth image*
  - **Edge-preserving smoothing** algorithms attempt to reduce noise and preserve signal



## SIGMA FILTER

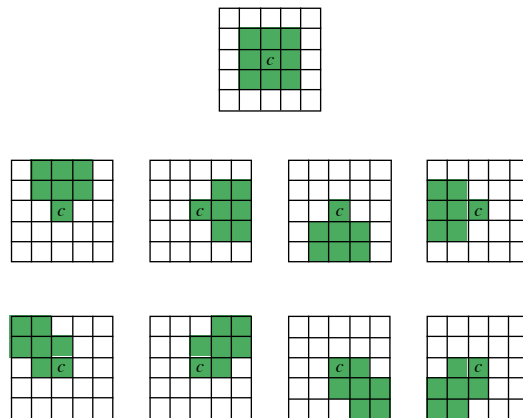
- Average moving window pixels that are within a threshold difference from the DN of the center pixel,  $DN_C \pm \Delta$

### sigma filter near edges and lines



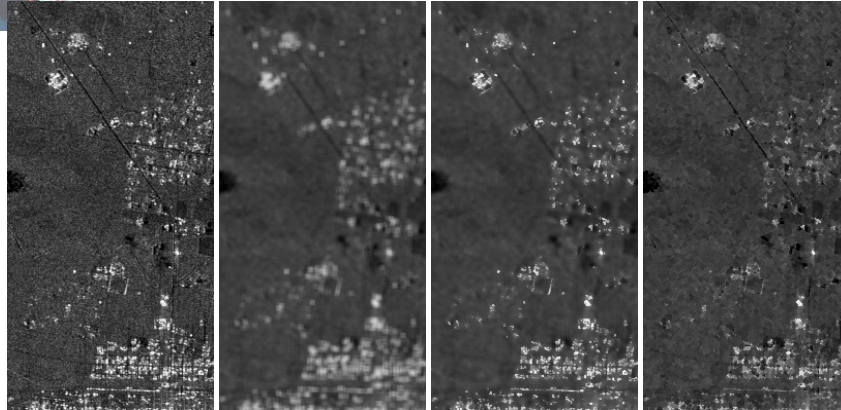
## NAGAO-MATSUYAMA FILTER

- Calculate the **variance** of 9 subwindows within a 5x5 moving window
- Output pixel is the **mean** of the subwindow with the lowest variance





## EXAMPLE: SAR NOISE REDUCTION



*original*

*5 x 5 LPF*

*5 x 5 sigma (k=2)*

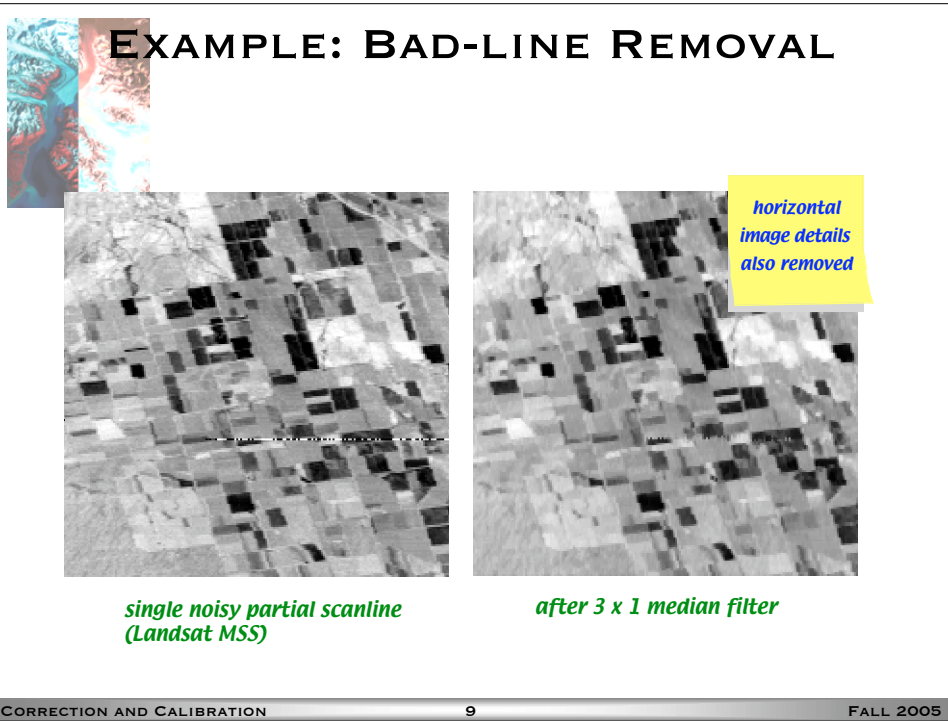
*Nagao-Matsuyama*



## NOISE REDUCTION (CONT.)

- **Local Noise**
  - *Individual bad pixels or lines*
  - *Often DN = 0 ("pepper"), 255 ("salt") or 0 and 255 ("salt and pepper")*
  - **Median filter** *reduces local noise because it is insensitive to outliers (Chapter 6)*
    - *For bad lines, set filter window vertical*

## EXAMPLE: BAD-LINE REMOVAL



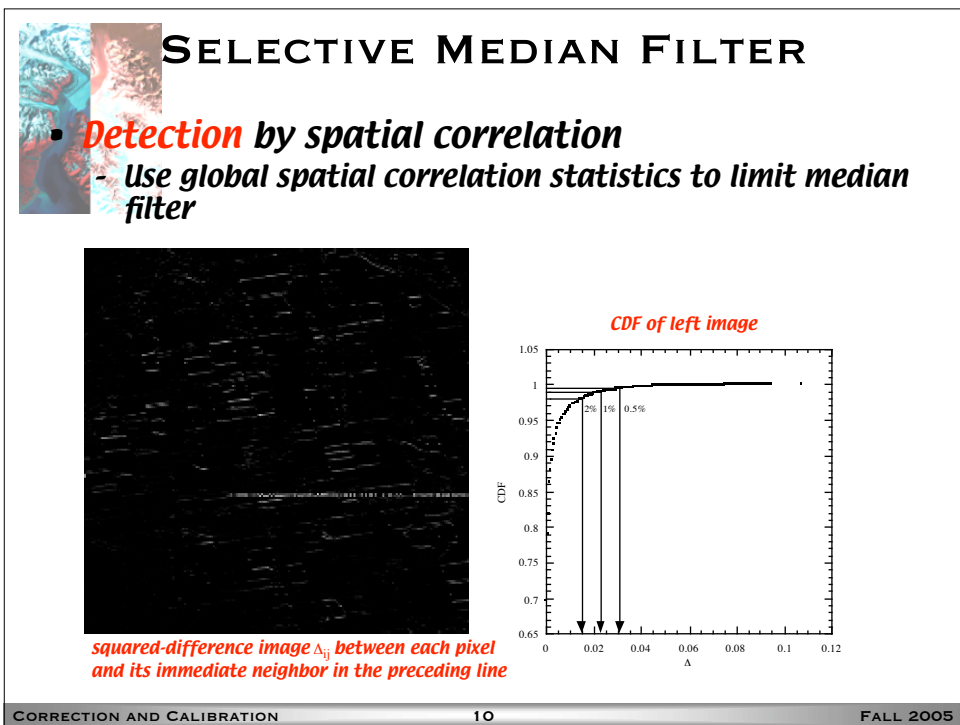
*horizontal image details also removed*

*single noisy partial scanline (Landsat MSS)*      *after 3 x 1 median filter*

CORRECTION AND CALIBRATION      9      FALL 2005

## SELECTIVE MEDIAN FILTER

- **Detection** by spatial correlation
  - Use global spatial correlation statistics to limit median filter

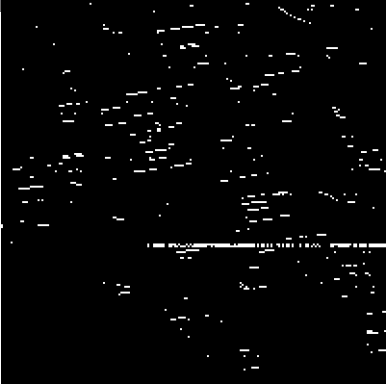


*squared-difference image  $\Delta_{ij}$  between each pixel and its immediate neighbor in the preceding line*

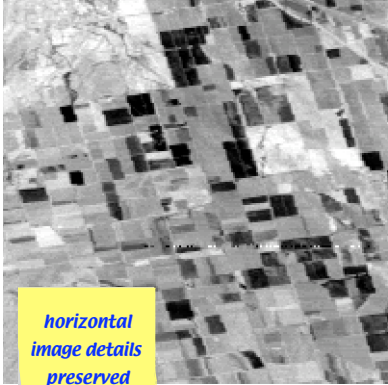
*CDF of left image*

CORRECTION AND CALIBRATION      10      FALL 2005

## SELECTIVE MEDIAN FILTER (CONT.)



98% threshold mask  
for median filter



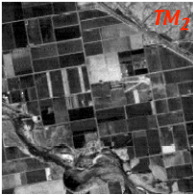
horizontal  
image details  
preserved

after 3 x 1 selective  
median filter

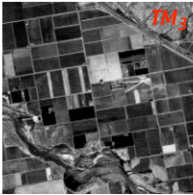
CORRECTION AND CALIBRATION
11
FALL 2005

## PCT COMPONENT FILTER


- **Detection by spectral correlation**
  - Use PCT to isolate spectrally-uncorrelated noise into higher order PCs
  - Remove noise and do inverse PCT



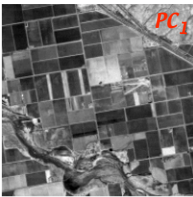
TM<sub>1</sub>



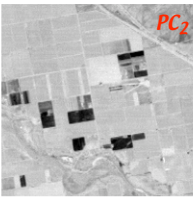
TM<sub>2</sub>



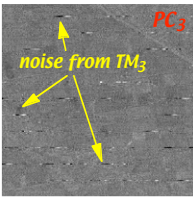
TM<sub>3</sub>



PC<sub>1</sub>



PC<sub>2</sub>



PC<sub>3</sub>

noise from TM<sub>3</sub>

CORRECTION AND CALIBRATION
12
FALL 2005



## PERIODIC (COHERENT) NOISE

- Repetitive pattern, either consistent throughout the image (global) or variable (local)
- Destriping
  - **Striping** is caused by unequal detector gains and offsets in whiskbroom and pushbroom scanners
  - Destripe **before** geometric processing
  - Global, linear detector matching
    - Adjust pixel DNs from each detector  $i$  to yield the same mean and standard deviation over the whole image

$$DN_i^{new} = \frac{\sigma_{ref}}{\sigma_i} (DN_i - \mu_i) + \mu_{ref}$$

- Nonlinear detector matching
  - Adjust each detector to yield the same histogram over the whole image (CDF reference stretch)

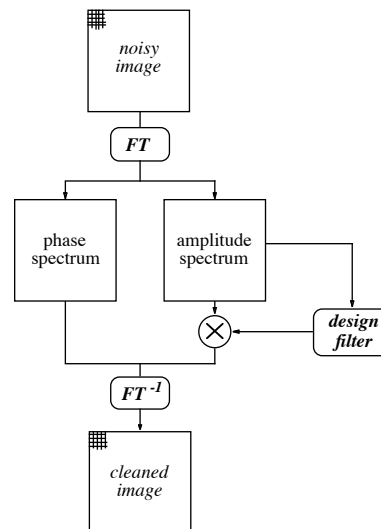
$$DN_i^{new} = CDF_{ref}^{-1} [CDF_i(DN_i)]$$



## PERIODIC NOISE (CONT.)

- Spatial filtering approaches
  - Detector striping causes characteristic “spikes” in the Fourier transform of the image
  - For line striping, the spike is vertical along the  $v$ -axis of the Fourier spectrum
  - A “notch” amplitude filter (removes selected frequency components) can reduce striping without degrading image

data flow for amplitude Fourier filtering



## DESTRIPING EXAMPLES

*noisy image*      *original spectrum*  
*striping notch-filter*      *removed noise*  
*striping and within-line-filter*      *removed noise*

CORRECTION AND CALIBRATION      15      FALL 2005

## AUTOMATIC DESTRIPING

- **“Automatic” periodic noise filter design**
  - Attempt to avoid manual noise identification in spectrum
  - Requires two thresholds

*HPF spectrum*      *noise filter*  
*cleaned image*      *removed noise*

Automatic Periodic Noise Filter

1. Apply “soft” (Gaussian) high-pass filter to noisy image to remove image components
2. Threshold HPF-filtered spectrum to isolate noise frequency components
3. Convert thresholded spectrum to 0 (noise) and 1 (non-noise) to create noise amplitude notch filter
4. Apply filter to noisy image

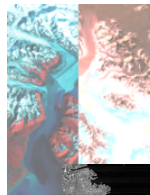
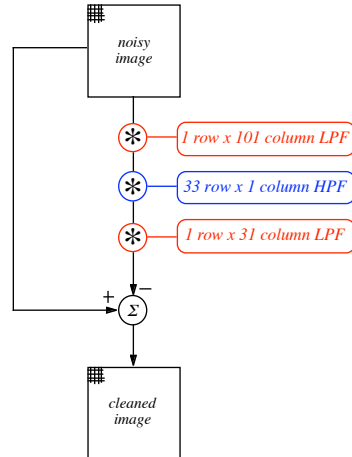
CORRECTION AND CALIBRATION      16      FALL 2005



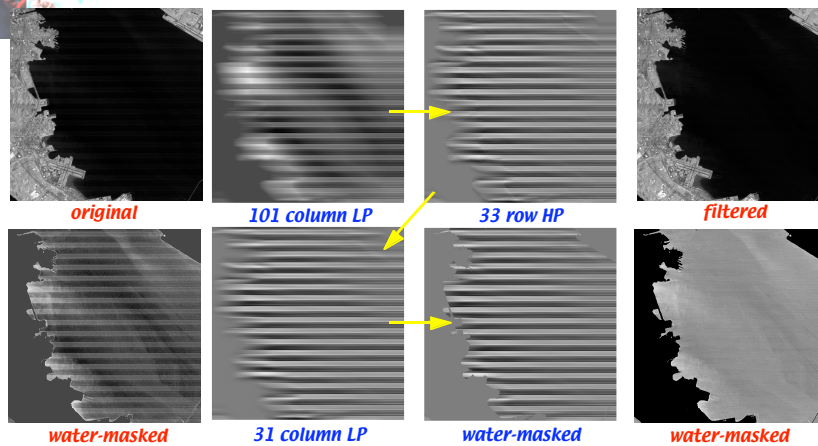


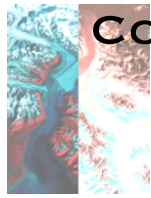
## DEBANDING

- Use combination of LPFs and HPFs (Crippen, 1989)



## EXAMPLE: TM DEBANDING





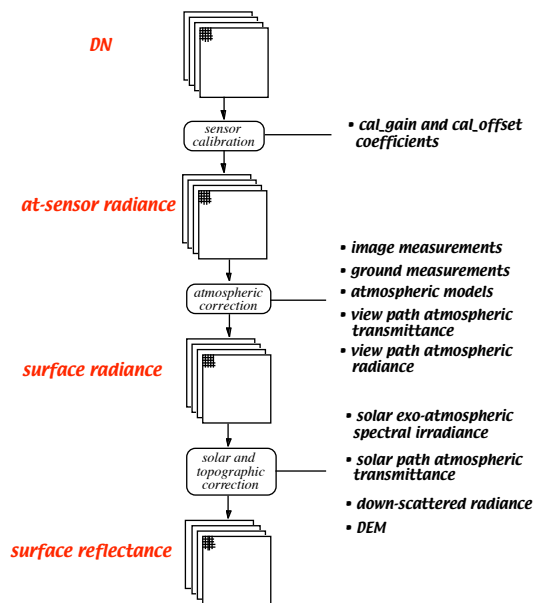
## CORRECTION AND CALIBRATION

- **Noise Reduction**
- **Radiometric Calibration**
- **Distortion Correction**



## RADIOMETRIC CALIBRATION

- **Many remote sensing applications require calibrated data**
  - comparison of geophysical variables over time
  - comparison of geophysical variables derived from different sensors
  - generation of geophysical variables for input to climate models
- **Extent of calibration depends on the desired geophysical parameter and available resources for calibration**





## SENSOR CALIBRATION

- **Calibrate sensor gain and offset in each band (and possibly for each detector) to get at-sensor radiance**
- **Use pre-launch or post-launch measured gains and offsets**

$$L_b^s = \text{cal\_gain}_b \cdot \text{DN}_b + \text{cal\_offset}_b$$

*pre-flight TM calibration coefficients*

Band	Landsat-4		Landsat-5	
	cal_gain	cal_offset	cal_gain	cal_offset
1	0.672	-3.361	0.642	-2.568
2	1.217	-6.085	1.274	-5.098
3	0.819	-4.917	0.979	-3.914
4	0.994	-9.936	0.925	-4.629
5	0.120	-0.7208	0.127	-0.763
6	0.0568	1.252	0.0552	1.238
7	0.0734	-0.367	0.0677	-0.0338



## ATMOSPHERIC CORRECTION

- **Estimate atmospheric path radiance and view-path transmittance to obtain *at-surface radiance* (sometimes called *surface-leaving radiance*)**

$$\text{At-sensor: } L_b^s(x, y) = \tau_{vb} L_b(x, y) + L_b^{sp}$$

- **Solve for at-surface radiance**

$$\text{Earth-surface: } L_b(x, y) = \frac{L_b^s(x, y) - L_b^{sp}}{\tau_{vb}}$$

- **In terms of calibrated at-sensor satellite data**

$$\text{Earth-surface: } L_b(x, y) = \frac{\text{cal\_gain}_b \cdot \text{DN}_b + \text{cal\_offset}_b - L_b^{sp}}{\tau_{vb}}$$



## IN-SCENE METHODS

- **Path radiance can be estimated with the *Dark Object Subtraction (DOS)* technique**
  - *In-scene method assumes dark objects have zero reflectance, and any measured radiance is attributed to atmospheric path radiance only*
  - *Subject to error if object has even very low reflectance*
  - *View-path atmospheric transmittance is **not** corrected by DOS*

### Dark-Object Subtraction

1. Identify “dark object” in the scene
2. Estimate lowest DN of object,  $DN_{0b}$
3. Assume  $DN_{0b} \equiv L_b^{sp}$
4. DN values (calibrated to at-sensor radiance) within the dark object assumed to be due only to atmospheric path radiance
5. Subtract  $DN_{0b}$  from all pixels in band b



## ATMOSPHERIC MODELING

- **DOS can be improved by incorporating atmospheric models**
  - *Reduces sensitivity to dark object characteristics*
  - *For example, coarse atmospheric characterization (Chavez, 1989)*

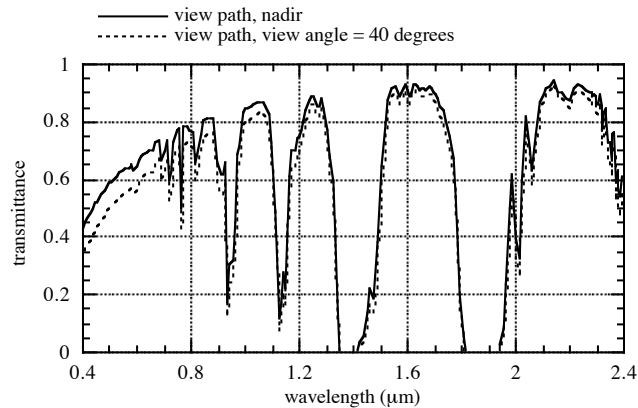
visibility	scattering model
very clear	$\lambda^{-4.0}$
clear	$\lambda^{-2.0}$
moderate	$\lambda^{-1.0}$
hazy	$\lambda^{-0.7}$
very hazy	$\lambda^{-0.5}$

- *Can also fit  $DN_{0b}$  with  $\lambda^{-K}$  function to determine K, and use fitted model instead of  $DN_{0b}$*
- **View-path atmospheric transmittance can be estimated using atmospheric models, such as MODTRAN**



## ATMOSPHERIC MODELING (CONT.)

- **Atmospheric modeling requires knowledge of many parameters**
  - **Imaging geometry**
  - **Atmospheric profile and aerosol model**



## SOLAR GEOMETRY AND TOPOGRAPHY

- **Further calibration to reflectance requires 3 more parameters**

$$\rho_b(x, y) = \frac{\pi L_b(x, y)}{\tau_{sb} E_b^0 \cos[\theta(x, y)]}$$

- **solar path atmospheric transmittance (from model or measurements)**
- **exo-atmospheric solar spectral irradiance (known)**
- **incident angle (from DEM)**

## EXAMPLE: MSI CALIBRATION

- Partial calibration with correction for sensor gains and offsets and DOS

band 1 (blue)				note the strong correction for Rayleigh scattering
2 (green)				
3 (red)				
	DN	at-sensor radiance	scene radiance	

CORRECTION AND CALIBRATION 27 FALL 2005

## EXAMPLE: PARTIAL CALIBRATION

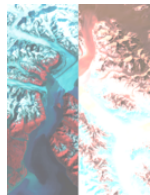
band 1 (blue)				Rayleigh scattering correction
2 (green)				
3 (red)				
4 (NIR)				low solar irradiance
5 (SWIR)				
7 (SWIR)				
	DN	at-sensor radiance	scene radiance	

CORRECTION AND CALIBRATION 28 FALL 2005



## HYPERSENSPECTRAL NORMALIZATION

- **Calibration of hyperspectral imagery is particularly important because:**
  - *High sensitivity to narrow atmospheric absorption features or the edges of broader spectral features*
  - *Spectral band shift in operating sensor*
  - *Need for precise absorption band-depth measurements*
  - *Computational issues*
- **A number of “normalization” techniques that use scene models have been developed to partially calibrate hyperspectral data**

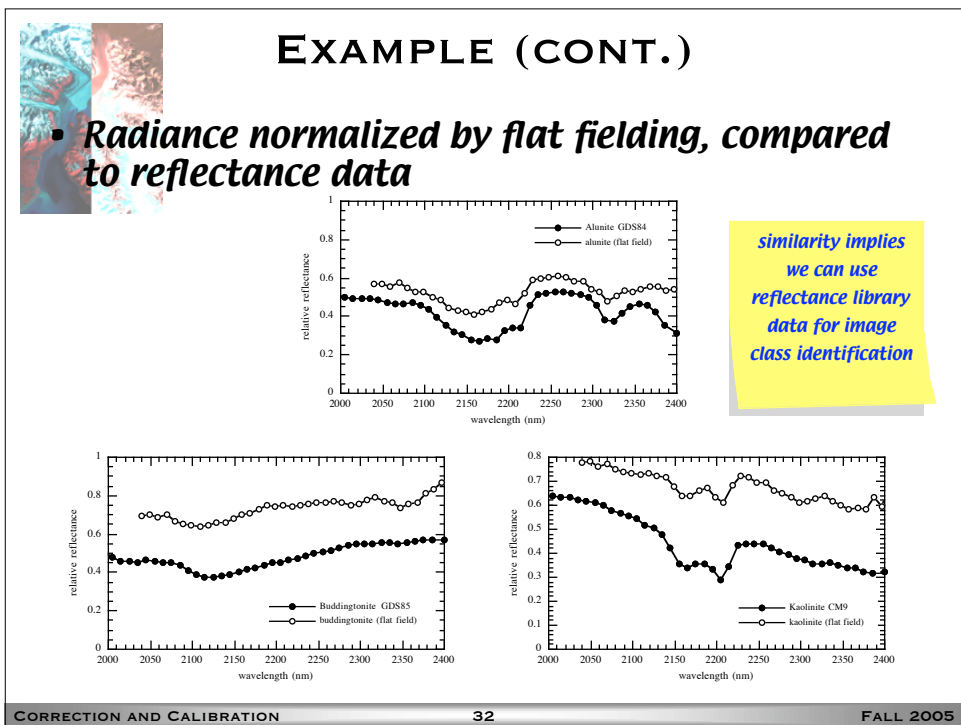
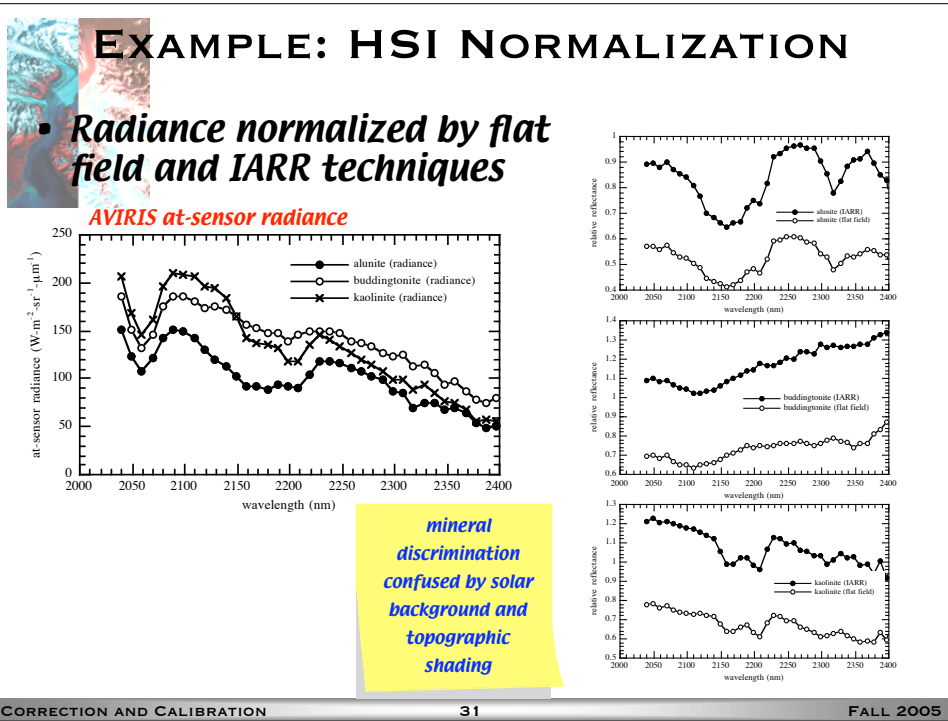


## NORMALIZATION (CONT.)

<i>technique</i>	<i>description</i>	<i>reference</i>
residual image	scale each pixel's spectrum by a constant such that the value in a selected band equals the maximum value in that band for the entire scene  subtract the average normalized radiance in each band over the entire scene from the normalized radiance in each band	(Marsh and McKeon, 1983)
continuum removal	generate a piecewise-linear or polynomial continuum across "peaks" of image spectrum and divide each pixel's spectrum by the continuum	(Clark and Roush, 1984)
internal average relative reflectance (IARR)	divide each pixel's spectrum by the average spectrum of the entire scene	(Kruse, 1988)
empirical line	band-by-band linear regression of pixel samples to field reflectance spectra for dark and bright targets	(Kruse <i>et al.</i> , 1990)
flat-field	divide each pixel's spectrum by the average spectrum of a spectrally-uniform, high reflectance area in the scene	(Rast <i>et al.</i> , 1991)

### *effectiveness of various normalization techniques for calibration*

<i>technique</i>	<i>view-path radiance</i>	<i>topography</i>	<i>solar irradiance</i>	<i>solar path atmospheric transmittance</i>
residual images	✓	✓	✓	✓
continuum removal	✗	✗	✓	✗
IARR	✗	✗	✓	✓
empirical line	✓	✗	✓	✓
flat field	✗	✗	✓	✓

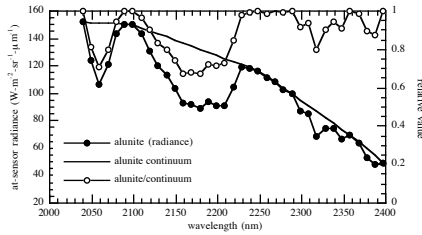




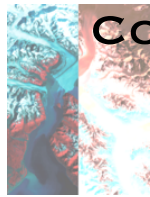
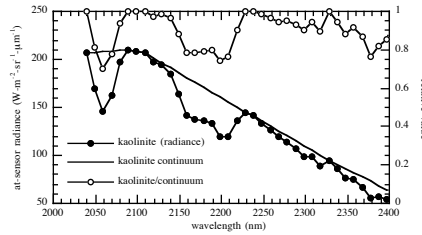
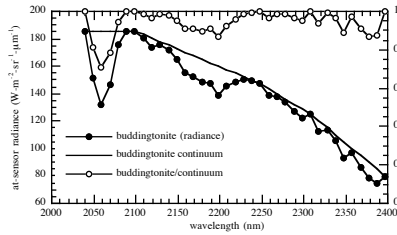


## EXAMPLE (CONT.)

- **Radiance normalized by continuum removal**



removes solar background



## CORRECTION AND CALIBRATION

- **Noise Reduction**
- **Radiometric Calibration**
- **Distortion Correction**



## DISTORTION CORRECTION

- **Applications**
  - correct system distortions
  - register multiple images
  - register image to map
- **Three components to warping process**
  - selection of suitable mathematical distortion model(s)
  - coordinate transformation
  - resampling (interpolation)

- **Distortion Correction Implementation**
- 1. Create an empty output image (which is in the reference coordinate system)
- 2. Step through the integer reference coordinates, one at a time, and calculate the coordinates in the distorted image  $(x,y)$  by Eq. 7-27
- 3. Estimate the pixel value to insert in the output image at  $(x_{ref},y_{ref})$  from the original image at  $(x,y)$  (resampling)

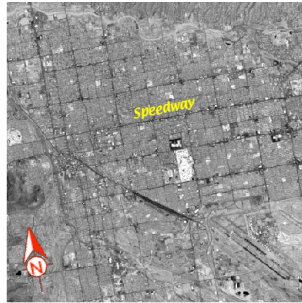


## DEFINITIONS

- **Registration:** The alignment of one image to another image of the same area.
- **Rectification:** The alignment of an image to a map so that the image is planimetric, just like the map. Also known as **georeferencing**.
- **Geocoding:** A special case of rectification that includes scaling to a uniform, standard pixel GSI.
- **Orthorectification:** Correction of the image, pixel-by-pixel, for topographic distortion.

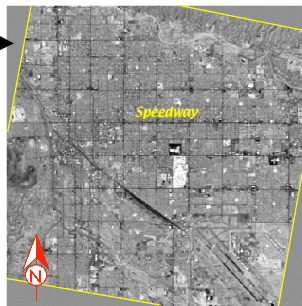


## EXAMPLE: TM RECTIFICATION



Original (Tucson, AZ)

fill →



Rectified



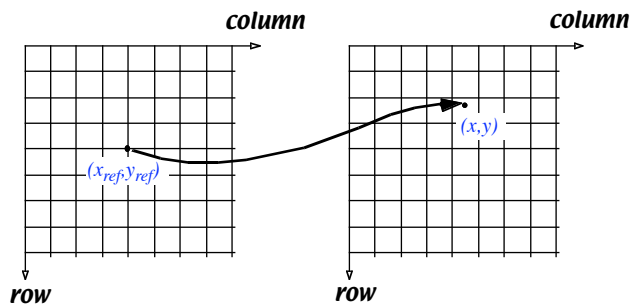
## COORDINATE TRANSFORMATION

- *Coordinates are mapped from the reference frame to the distorted image frame*

$$(x, y) = f(x_{ref}, y_{ref})$$

reference frame

image frame



- *“Backwards” mapping  $(x_{ref}, y_{ref}) \rightarrow (x, y)$  avoids “holes” or overlaying of multiple pixels in the processed image*



## POLYNOMIAL DISTORTION MODEL

- *Generic model useful for registration, rectification and geocoding*
- *Known as a “rubber sheet stretch”*
- *Relates distorted coordinate system (x,y) to the reference coordinate system (x<sub>ref</sub>,y<sub>ref</sub>)*

$$x = \sum_{i=0}^N \sum_{j=0}^{N-i} a_{ij} x_{ref}^i y_{ref}^j$$

$$y = \sum_{i=0}^N \sum_{j=0}^{N-i} b_{ij} x_{ref}^i y_{ref}^j$$

- *For example, a **quadratic polynomial** is written as:*

$$x = a_{00} + a_{10}x_{ref} + a_{01}y_{ref} + a_{11}x_{ref}y_{ref} + a_{20}x_{ref}^2 + a_{02}y_{ref}^2$$

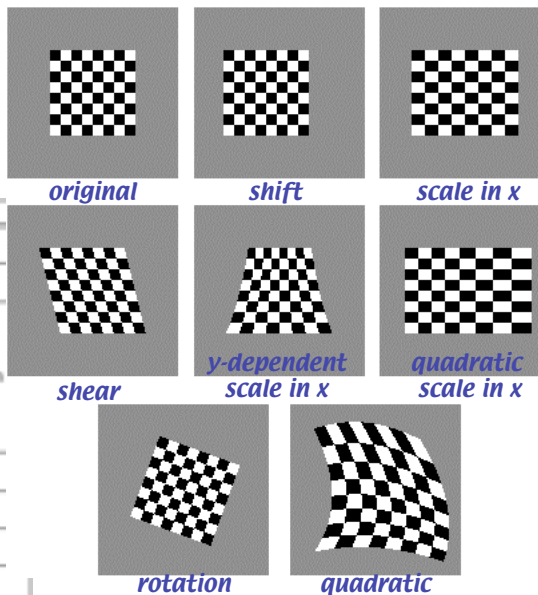
$$y = b_{00} + b_{10}x_{ref} + b_{01}y_{ref} + b_{11}x_{ref}y_{ref} + b_{20}x_{ref}^2 + b_{02}y_{ref}^2$$

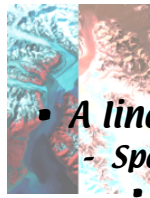


## DISTORTION TYPES

- *The coefficients in the polynomial can be associated with particular types of distortion*

coefficient	warp component	
$a_{00}$	shift in x	rotation
$b_{00}$	shift in y	
$a_{10}$	scale in x	
$b_{01}$	scale in y	
$a_{01}$	shear in x	
$b_{10}$	shear in y	
$a_{11}$	y-dependent scale in x	rotation
$b_{11}$	x-dependent scale in y	
$a_{20}$	nonlinear scale in x	
$b_{02}$	nonlinear scale in y	





## AFFINE TRANSFORMATION

- A linear polynomial (number of terms  $K = 3$ )

- Special case that can include:

- shift
- scale
- shear
- rotation

$$x = a_{00} + a_{10}x_{ref} + a_{01}y_{ref}$$

$$y = b_{00} + b_{10}x_{ref} + b_{01}y_{ref}$$

- In vector-matrix notation

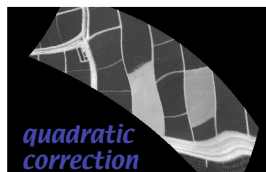
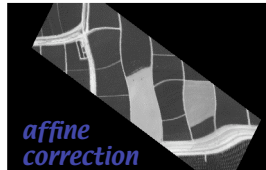
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{bmatrix} \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix} + \begin{bmatrix} a_{00} \\ b_{00} \end{bmatrix}$$



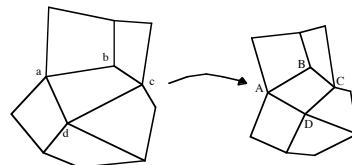
## PIECEWISE POLYNOMIAL DISTORTION

- For severely-distorted images that can't be modeled with a single, global polynomial of reasonable order

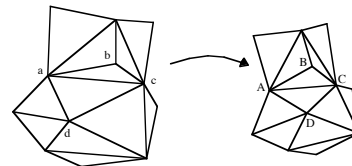
Example airborne scanner image



piecewise quadrilateral warping



piecewise triangle warping

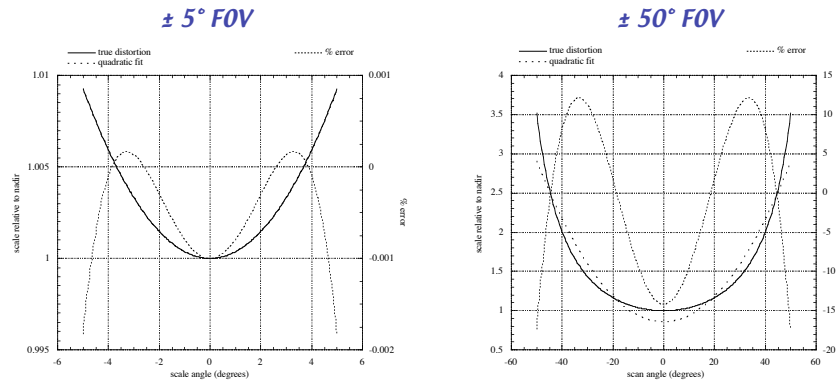


reference frame

distorted frame

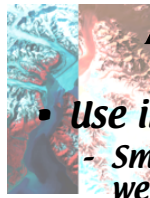
## APPLICATION TO WIDE FOV SENSORS

- **Comparison of actual distortion to global polynomial model**
  - accurate for small FOV sensors such as Landsat, but not for large FOV scanners such as AVHRR



## GROUND CONTROL POINTS (GCPs)

- Use to **control** the polynomial, i.e. to determine its coefficients
  - Characteristics:
    - high contrast in all images of interest
    - small feature size
    - unchanging over time
    - all are at the same elevation (unless topographic relief is being specifically addressed)
  - Often located by visual examination, but can be automated to varying degree, depending on the problem



## AUTOMATIC GCP LOCATION

- **Use image “chips”**
  - Small segments that contain one easily identified and well-located GCP
- **Normalized cross-correlation between template chip  $T$  (reference) and search area  $S$  in image to be registered**

$$R_{ij} = \frac{\sum_{m=1}^N \sum_{n=1}^N T_{mn} S_{i+m, j+n}}{\sqrt{\sum_{m=1}^N \sum_{n=1}^N T_{mn}^2} \sqrt{\sum_{m=1}^N \sum_{n=1}^N S_{i+m, j+n}^2}}$$

- normalization adjusts for changes in mean DN within area



## AREA CORRELATION

**search chip**

**target chip**

**search chip**

**target chip**

**Cross-correlation of one area**

target area  $T$

search area  $S$

two relative shift positions

$ij = 0,0$

$ij = 0,1$

**Chip layout over full scene**

reference image

distorted image

## FINDING POLYNOMIAL COEFFICIENTS

- Set up system of simultaneous equations using GCPs and solve for polynomial coefficients
- Example with quadratic polynomial (number of terms  $K = 6$ )
  - Given  $M$  pairs of GCPs
  - For each GCP pair,  $m$ , create two equations

$$\begin{aligned} x_m &= a_{00} + a_{10}x_{refm} + a_{01}y_{refm} + a_{11}x_{refm}y_{refm} + a_{20}x_{refm}^2 + a_{02}y_{refm}^2 \\ y_m &= b_{00} + b_{10}x_{refm} + b_{01}y_{refm} + b_{11}x_{refm}y_{refm} + b_{20}x_{refm}^2 + b_{02}y_{refm}^2 \end{aligned}$$

- Then, for all  $M$  GCP pairs, in vector-matrix notation

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} 1 & x_{ref1} & y_{ref1} & x_{ref1}y_{ref1} & x_{ref1}^2 & y_{ref1}^2 \\ 1 & x_{ref2} & y_{ref2} & x_{ref2}y_{ref2} & x_{ref2}^2 & y_{ref2}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{refM} & y_{refM} & x_{refM}y_{refM} & x_{refM}^2 & y_{refM}^2 \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \\ a_{20} \\ a_{02} \end{bmatrix}$$

## SOLVING FOR COEFFICIENTS

- So, for each set of GCP  $x$ - and  $y$ -coordinate pairs, we can write a linear system

- **Determined case** ( $M = K$ , just enough GCPs) solution:

$$M = K: \begin{matrix} \mathbf{X} = \mathbf{W}\mathbf{A} \\ \mathbf{Y} = \mathbf{W}\mathbf{B} \end{matrix} \quad \text{solution: } \begin{matrix} \mathbf{A} = \mathbf{W}^{-1}\mathbf{X} \\ \mathbf{B} = \mathbf{W}^{-1}\mathbf{Y} \end{matrix}$$

- Exact solution which passes through GCPs, i.e. they are mapped exactly

- **Overdetermined case** ( $M > K$ , more than enough GCPs):

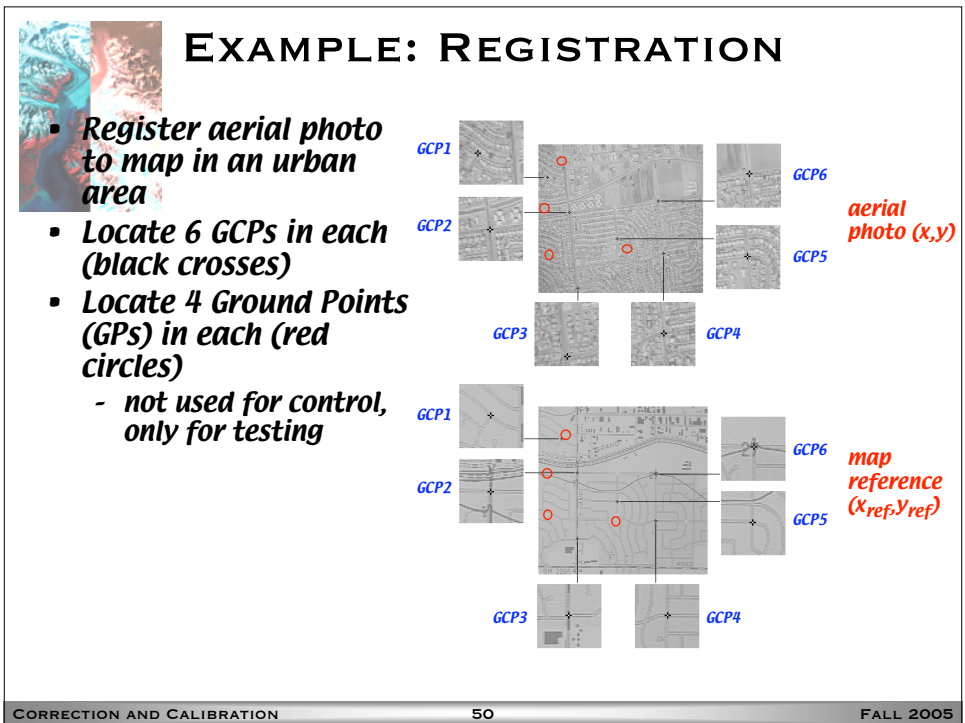
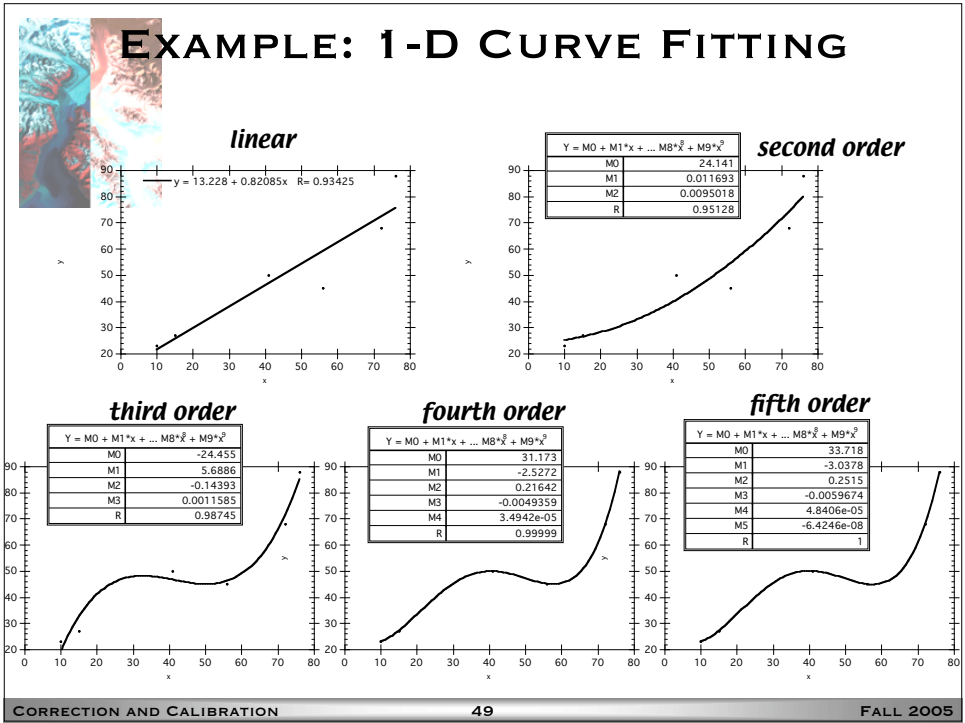
$$M \geq K: \begin{matrix} \mathbf{X} = \mathbf{W}\mathbf{A} + \varepsilon_X \\ \mathbf{Y} = \mathbf{W}\mathbf{B} + \varepsilon_Y \end{matrix} \quad \text{solution: } \begin{matrix} \hat{\mathbf{A}} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\mathbf{X} \\ \hat{\mathbf{B}} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\mathbf{Y} \end{matrix}$$

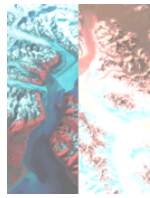
- $(\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T$  is called the **pseudoinverse of  $\mathbf{W}$**
- solution results in least-squares minimum error at GCPs:

$$\min [\varepsilon_X^T \varepsilon_X] = (\mathbf{X} - \mathbf{W}\hat{\mathbf{A}})^T (\mathbf{X} - \mathbf{W}\hat{\mathbf{A}})$$

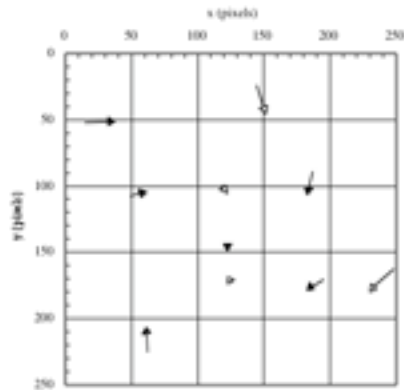
$$\min [\varepsilon_Y^T \varepsilon_Y] = (\mathbf{Y} - \mathbf{W}\hat{\mathbf{B}})^T (\mathbf{Y} - \mathbf{W}\hat{\mathbf{B}})$$





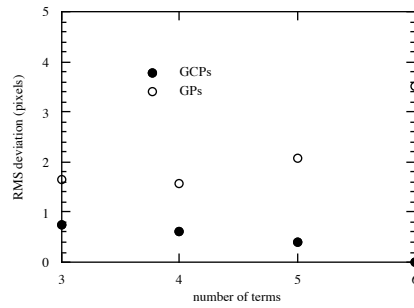


## GCP ERROR



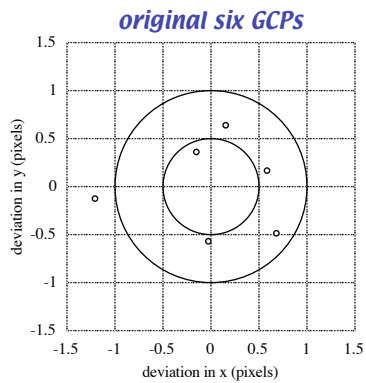
*Mapping of GCPs*

*Error at GCPs and Ground Points (GPs) as function of polynomial order*

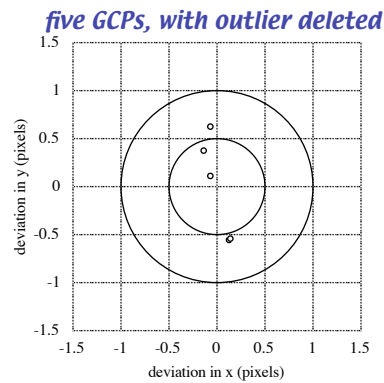


## GCP REFINEMENT

- Analyze GCP error and remove outliers



*original six GCPs*



*five GCPs, with outlier deleted*

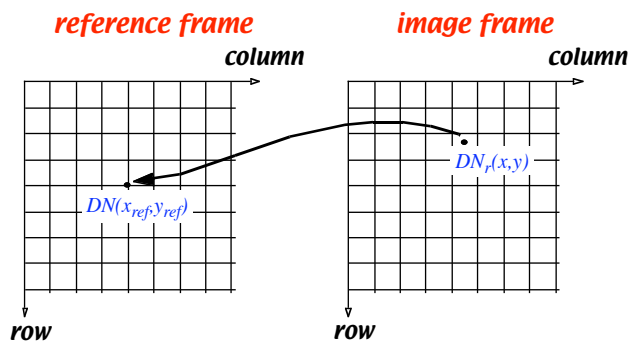


## FINAL REGISTRATION RESULT



## RESAMPLING

- The  $(x,y)$  coordinates calculated by  $(x,y) = f(x_{ref},y_{ref})$  are generally **between** the integer pixel coordinates of the array
- Therefore, must estimate (interpolate or **resample**) a new pixel at the  $(x,y)$  location



## RESAMPLING (CONT.)

- **Pixels are resampled using a weighted-average of the neighboring pixels**
- **Common weighting functions:**
  - **nearest-neighbor:** fast, but discontinuous

$$DN_r = DN_C$$

- **bilinear:** slower, but continuous

$$DN_r = [\Delta x DN_B + (1 - \Delta x) DN_A] (1 - \Delta y) + [\Delta x DN_D + (1 - \Delta x) DN_C] \Delta y$$

*resampling distances*

- **Implement 2-D bilinear resampling as two successive 1-D resamplings**
  - resample E between A and B
  - resample F between C and D
  - resample  $DN_r(x,y)$  between E and F

## RESAMPLING (CONT.)

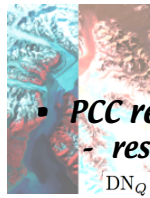
- **bicubic:** slowest, but results in sharpest image

- **piecewise polynomial; special case of Parametric Cubic Convolution (PCC)**

$$DN_r(\Delta; \alpha) = PCC(\Delta, \alpha) = \begin{cases} (\alpha + 2)|\Delta|^3 - (\alpha + 3)|\Delta|^2 + 1, & |\Delta| \leq 1 \\ \alpha(|\Delta|^3 - 5|\Delta|^2 + 8|\Delta| - 4), & 1 \leq |\Delta| \leq 2 \\ 0, & |\Delta| \geq 2 \end{cases}$$

where  $\Delta$  is the distance from  $(x,y)$  to the grid points in 1-D

- “standard” bicubic is  $\alpha = -1$ ; superior bicubic is  $\alpha = -0.5$
- High-boost filter characteristics; amount of boost depends on amplitude of side-lobe, which is proportional to  $\alpha$



## PCC RESAMPLING

- **PCC resampling procedure**

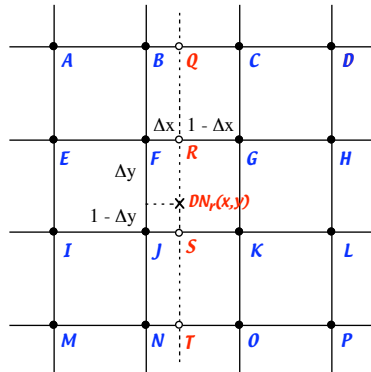
- **resample along each row, A-D, E-H, I-L, M-P**

$$DN_Q = DN_A \cdot PCC(\Delta x + 1; \alpha) + DN_B \cdot PCC(\Delta x; \alpha) + DN_C \cdot PCC(1 - \Delta x; \alpha) + DN_D \cdot PCC(2 - \Delta x; \alpha)$$

*etc.*

- **resample along new column Q-T**

$$DN_r = DN_Q \cdot PCC(\Delta y + 1; \alpha) + DN_R \cdot PCC(\Delta y; \alpha) + DN_S \cdot PCC(1 - \Delta y; \alpha) + DN_T \cdot PCC(2 - \Delta y; \alpha)$$



## EXAMPLE: MAGNIFICATION (“ZOOM”)



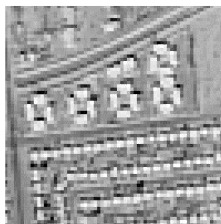
1x

**nearest-neighbor**

**bilinear**

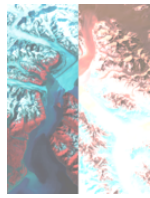


2x

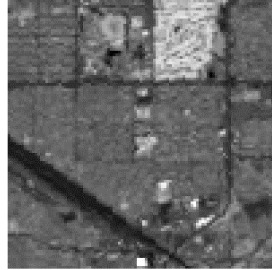


3x

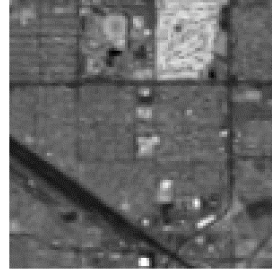




## EXAMPLE: RECTIFICATION



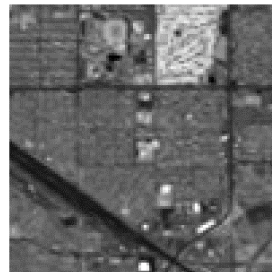
*nearest-neighbor*



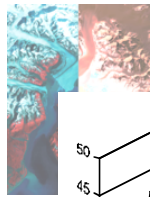
*bilinear*



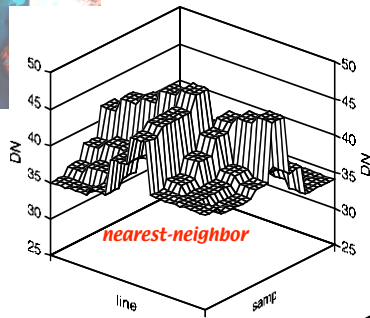
*PCC ( $\alpha = -0.5$ )*



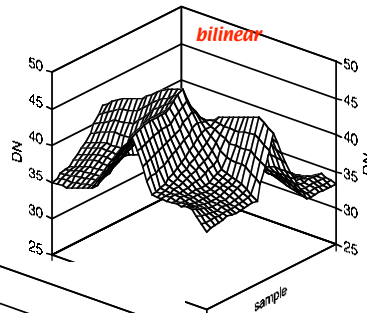
*PCC ( $\alpha = -1.0$ )*



## INTERPOLATION QUALITY

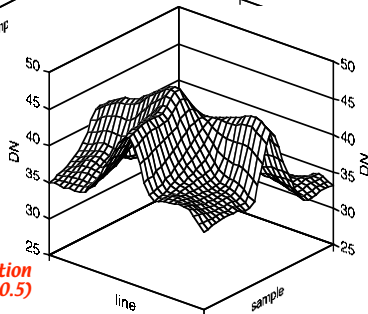


*nearest-neighbor*



*bilinear*

*resampled  
image surface  
plots (4x zoom)*

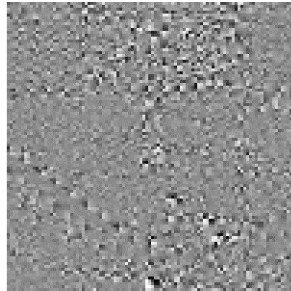


*parametric cubic convolution  
( $\alpha = -0.5$ )*

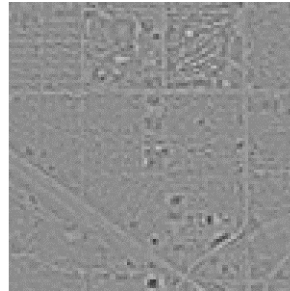


## RESAMPLING DIFFERENCES

- *Difference images between different resampling functions*



*nearest-neighbor - bilinear*



*bilinear - PCC*

- *polynomial distortion model affects global geometric accuracy*
- *resampling function affects local radiometric accuracy*