GEOMETRICAL OPTICS

Used to find locations and sizes of images formed by optical systems

- **Assume:**
  - optical elements have rotational symmetry about a common optical axis
  - light paths are along rays, which are normal at any point to a wavefront, a surface of constant time phase for an electromagnetic wave
  - paraxial approximation:
    - rays make small angles $\theta$ to optical axis $\sin \theta \equiv \tan \theta \equiv \theta$, $\cos \theta \equiv 1$
    - plane or spherical wavefronts ("perfect" optics with no aberrations)
Reflection and refraction

- **Snell’s Law of Reflection**

\[ \alpha_1 = \alpha_2 \]
**Snell’s Law of Refraction**

\[
\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1}
\]

where *index of refraction*:

\[
n = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} = \frac{c}{v}
\]

- \( n_{\text{air}} = 1 \)
- \( 1.4 < n_{\text{glass}} < 1.7 \)
Dispersion

- different wavelengths of light $\lambda$ refract by different amounts
- typical dispersion from blue to green:
- prism

![Diagram of a prism](image)

- White light
- Red (650nm)
- Green (550nm)
- Blue (450nm)
Thin Lens Image Formation

- thickness << radii of curvature of lens surfaces

- Gauss’ Formula (in air) \( \frac{1}{z_i} = \frac{1}{z_o} + \frac{1}{f} \)

  where

  \( z_i = \) image distance from lens (+ to right)

  \( z_o = \) object distance from lens (- to left)

  focal length \( f = \) image distance for an object distance of infinity; defines the focal point \( F \)

- thin lenses have two focal points, symmetric on either side of the lens:
**Ex: find focal length of a converging (+) lens, given:**

\[
\frac{1}{3f} = \frac{-1}{6} + \frac{1}{f} \quad \Rightarrow \quad f = 4
\]
**Newtonian Formula:** \( s_1 s_2 = f^2 \)

*where* \( s_1 \) *is distance of object from* \( F_1 \) (+ to left), \( s_2 \) *is distance of image from* \( F_2 \) (+ to right)*

**Ex:** object \( 2f \) to left of + lens; find image location

Therefore, image is \( 2f \) to right of lens (symmetric to object)
Magnification

- lateral scale (normal to optical axis) between image space and object space

- definition

\[ m = \frac{h_i}{h_o} = \frac{z_i}{z_o} \]

where

- \( h_i \) = image height
- \( h_o \) = object height

- \textbf{Ex: find location and size of image, given} \( f = +10, h_o = +5, z_o = -50 \)
- Gauss’ Formula \( \rightarrow z_i = +12.5 \)
- \textbf{magnification} \( m = 12.5/-50 = -0.25 \) (image is inverted and 1/4 size of object)
• image size $h_i = (-0.25)(+5) = -1.25$
Ray tracing

- Graphical way to find image location and size

- Any two of three rays needed from off-axis object point to locate and size image:
  - parallel ray - parallel to optical axis in object space, intersects focal point \( F_2 \) in image space
  - chief ray - straight ray that intersects optical axis at lens location (center of lens)
  - focal ray - intersects focal point \( F_1 \) in object space, parallel to optical axis in image space

- Replace thin lens by effective refracting plane and trace rays
- Use graph to check calculations from Gauss’ Formula

- What if the object is within the focal distance?

  “virtual” image is left of the lens, and therefore doesn’t exist
virtual image can only be seen if re-imaged by another optical system, e.g. eye

Multiple, thin lenses

- Treat in serial fashion
- Image formed by first lens is the object for the second lens, and so forth

Ex: 2 lenses, \( f_1 = +3 \), \( f_2 = +4 \), \( d = 2 \) (lens separation), \( z_1 = -4 \)

- lens #1: Gauss’ Formula \( \rightarrow z_2 = +12 \) (10 units to right of lens #2, “virtual” object)
- lens #2: \( z_1 = +10 \); Gauss’ Formula \( \rightarrow z_2 = 2.86 \) (2.86 units to right of lens #2)

Can you draw the ray trace for this system? (hint: find image of lens #1)
ignoring lens #2, and then re-image with lens #2)

- **total magnification** \( m_{tot} = m_1m_2 \)

Show that the total magnification of the above system is \( m_{tot} = -0.858 \)
Spherical Mirrors

- **why use mirrors instead of lenses?**
  - *folds optical path – makes smaller systems*
  - *metallic reflecting coatings have more uniform spectral reflectance over broad spectral range than glass*

- **concave mirror**
  - *converges parallel rays (f > 0)*
  - *forms a real image*

- **convex mirror**
  - *diverges parallel rays (f < 0)*
  - *forms a virtual image*
• Spherical mirror equation

\[ f = \frac{-R}{2} \]

where \( R \) = radius of curvature

\( (R < 0 \text{ for } + \text{ (concave) mirror, } R > 0 \text{ for } - \text{ (convex) mirror}) \)

• Gauss’ formula for spherical mirrors

\[ \frac{1}{z_i} = \frac{1}{z_o} + \frac{1}{f} = \frac{1}{z_o} - \frac{2}{R} \]

• real image has \( z_i > 0 \) because image distance measured in direction of light propagation
+ mirror \((f > 0)\)

object @ \(z_o = -2f\)

center of curvature \(R\)

real image @ \(z_i = +2f\)

- mirror \((f < 0)\)

object @ \(z_o = +2f\)

virtual image @ \(z_i = +2f/3\)
• Ex: concave (+) mirror, \( h = +2, z_0 = -10, R = -16 \)

  • From mirror equation, \( f = +8 \)
  
  • from Gauss’ formula, \( z_i = +40 \)

  • therefore, image is real (left of mirror)

  • magnification \( m = \frac{40}{-10} = -4 \)

  • image height \( h_i = mh_o = -8 \)
- ray trace

object @ $z_o = -10$

parallel ray

real image @ $z_i = +40$

focal ray
Aberrations

- Expand $\sin \theta$ in Maclaurin Series

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$$

- linear term in $\theta$ is paraxial, first-order approximation
- cubic term in $\theta$ is third-order approximation, etc

- Any deviations of real rays from the rays predicted by paraxial approximation are called **aberrations**

- Third-order aberrations
  - on-axis
    - **spherical aberration**
• focal length varies with ray height
- chromatic aberration (longitudinal)
  - focal length varies with wavelength

![Diagram of chromatic aberration](image.png)
• correctable at any 2 given wavelengths with an achromatic doublet

For example, select $n_1$ and $n_2$ so that red and blue light focus at the same point
- off-axis
- coma
  - off-axis spherical aberration

paraxial rays
marginal rays

paraxial focus
marginal focus

image of a point source

comet-like shape
• astigmatism

• different focal points for **sagittal** (in plane orthogonal to off-axis angle) and **tangential** (in plane of off-axis angle) rays

*optical axis*

*off-axis point*

*S: sagittal focus*

*T: tangential focus*
- curvature-of-field
  - off-axis rays focus on a curved, not plane, surface
- distortion
  - off-axis rays do not intersect a flat focal surface in correct locations

object

"barrel" distortion

"pincushion" distortion
• Stops and pupils

  • stops are physical apertures that limit the rays through a system

    • aperture stop
      • controls the amount of light through a system
      • usually located at the lens (single element) or within a multi-element system

    • field stop
      • limits the Field-Of-View (FOV), i.e. the extent of the image
      • usually located at the focal plane

  • pupils are images of physical stops formed by elements of the system

    • entrance pupil
      • image of the aperture stop formed by all elements before it

    • exit pupil
• image of the aperture stop formed by all elements after it

• entrance and exit pupils are conjugate, just as an object and image

• entrance and exit pupils coincide with the aperture stop at the lens for a single, thin lens

• Defocus

• image is out-of-focus if receiving plane is not at the image focus
By geometry,

\[ \frac{D}{f} = \frac{d}{\Delta/2} \]

or \( \Delta = \frac{2df}{D} = 2dN \)

where \( N \) is the f-number (ratio of focal length to aperture diameter)

- For a given blur circle diameter (as specified in a design, for example), the depth-of-focus is proportional to \( N \)
- Depth-of-field is the object space equivalent to depth-of-focus
• photographers use the f-number N to control the depth-of-field, e.g. when taking a portrait outside, they often use a low N (say 2.8 or 4) to make the background out-of-focus

• The maximum allowable blur circle diameter d (and consequently, the minimum N) is commonly set to the maximum acceptable spot size in the system’s image

• For digital cameras, the maximum d should be no more than the size of a single detector element (pixel), and preferably less