RADIOMETRY OF LAMBERTIAN SOURCES

Relate $M_\lambda$ and $L_\lambda$

- Valuable exercise in spherical coordinate integration

- Area element of sphere, radius $r$
  $dA_2 = rd\phi r\sin\theta d\theta$
  $= r^2\sin\theta d\theta d\phi$

- Start with radiance to get flux within area $dA_2$
Total flux in hemisphere above source

Now, in general \( L = L(\theta, \phi) \), but for a Lambertian source
\( L = \text{constant} \)

- Therefore

\[
\Phi = dA_1 2\pi L \left( \frac{\sin \theta}{2} \right)^2 \left. \frac{\pi/2}{2} \right|_0 \\
= \pi L dA_1
\]

**Radiant exitance definition**

\[
M = \frac{\Phi}{dA_1} \\
= \pi L
\]
Include spectral variation

\[ M_\lambda = \pi L_\lambda \]

or \[ L_\lambda = M_\lambda / \pi \]

- Can think of \( \pi \) as having units of \( sr \), which cancels the \( sr^{-1} \) units of \( L \)

- Note: \( M \neq 2\pi L \), as one might guess since hemisphere is \( 2\pi \) \( sr \)
  
  *Why?*
TRANSMITTANCE

Defined as ratio of transmitted output flux to input flux

\[ t(\lambda) = \frac{\Phi^\text{out}_\lambda}{\Phi^\text{in}_\lambda} \]
REFLECTANCE

Three types:

- Specular (mirror): \( \theta \) \( \theta \)
- Directional
- Diffuse (Lambertian)

Most natural surfaces are approximately Lambertian
for $\theta < 40^\circ$, snow and sand for $\theta < 60^\circ$

- At larger $\theta$, natural surfaces tend to become directional

Reflectance of Lambertian surface

- Reflectance defined similarly to transmittance

\[ \rho(\lambda) = \frac{\Phi_{\lambda}^{out}}{\Phi_{\lambda}^{in}} \]

- From earlier derivation
Given reflectance of surface and incident irradiance, can get radiance

\[ L_\lambda = \rho(\lambda) E_\lambda / \pi \]
BAND-AVERAGED IRRADIANCE

Cascade spectral quantities from source-to-surface-to-
surface . . .

\[ E_\lambda = (\text{geometric factors}) \cdot L_\lambda \cdot t_1(\lambda) \cdot t_2(\lambda) \cdots \rho_1(\lambda) \cdots \]

- until arriving at a spectrally-integrating element, i.e. a detector,

\[ E_{total} = \int E_\lambda \cdot S(\lambda) d\lambda \]

where \( S(\lambda) \) is the detector’s spectral sensitivity

Example of a specific detector \( S(\lambda) \) is human vision system sensitivity \( V(\lambda) \)

\( E_{total} \) is the effective irradiance, because it is what is
detected and measured

Band-averaged spectral irradiance

\[ E_b = \frac{E_{total}}{\int_{\infty}^{0} S(\lambda) d\lambda} \]
RADIOMETRY OF OPTICAL SYSTEMS

telescope

- collects light from point source
- “light bucket”

• assume:
  - source and detector on optical axis
  - object-to-sensor distance much greater than aperture
**diameter** \( z_0 \gg d \)

- no transmission losses

- **irradiance at lens aperture** \( E = I/z_0^2 \) (inverse square law)

- **flux collected by aperture**

\[
\Phi_c = E \cdot A_{aperture} \\
= E \cdot \frac{\pi d^2}{4} \\
= \frac{I}{z_0} \cdot \frac{\pi d^2}{4}
\]

- **flux at detector (no losses)** \( \Phi_i = \Phi_c \)

- proportional to radiant intensity of source

- proportional to square of aperture diameter
inversely proportional to square of object-to-sensor distance

What is the only way to increase the amount of light collected by a telescope?

Camera

assume:
- Lambertian source and detector plane normal to optical axis
- magnification \[ m = \frac{h_i}{h_o} = \frac{z_i}{z_o} \]
• **flux collected by aperture**

\[
\Phi_c = L_o A_o \Omega \\
= L_o A_o \frac{A}{z_o^2} \\
= \frac{L_o A_o}{z_o^2} \cdot \frac{\pi d^2}{4}
\]

• **NOTE: similarity to “light bucket” equation**

• **flux at detector (no losses)** \( \Phi_i = \Phi_c \)
irradiance at detector

\[
E_i = \frac{\Phi_c}{A_i} = \frac{\Phi_c}{m^2 A_o} = L_o \frac{\pi d^2}{m^2 4 z_o^2}
\]

by definition of magnification (see Geometrical Optics)

\[
E_i = L_o \frac{\pi d^2}{4 z_i^2}
\]

define effective f-number \( N \) of sensor, \( N = z_i/d \)

then,

\[
E_i = L_o \frac{\pi}{4N^2} \quad \text{Camera Equation}
\]

NOTE:

proportional to radiance of source
- inversely proportional to square of sensor f-number
- does not depend on $z_0$, the source-to-sensor distance

- f-number $N$ typically preset on camera to 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22

What is rationale for the above preset values of $N$?
Camera imaging reflecting object

- Irradiance on object $E$

- Reflectance of object $\rho_o$

From earlier derivation:

$$E_i = \frac{E_o \rho}{4N^2}$$
Camera imaging extended reflecting object

- \( \cos^4 \theta \) Law applies:

\[
E_i(\theta) = \frac{E_o \rho}{4N^2} (\cos \theta)^4
\]

With object irradiance and reflectance fixed, what is the only way to increase the
amount of light collected by a camera?