2-D LINEAR SYSTEM EXAMPLES

Example 1: desktop scanner

- **input signal (photo):** \( f(x, y) = \frac{1}{2} + \cos(2\pi x^2) \)

- “chirp” function

- frequency of pattern increases as \( x \) increases (\( u = x \))
  - this pattern therefore “maps” the spatial frequency dimension onto a spatial dimension for visualization
• **Scan with 2 different-sized square spots (detectors)**

  • **scan spot 1:** \( h(x, y) = \frac{1}{W^2} \text{rect}\left(\frac{x}{W}, \frac{y}{W}\right) \)

  • **scan spot 2:** \( h(x, y) = \frac{1}{4W^2} \text{rect}\left(\frac{x}{2W}, \frac{y}{2W}\right) \)

• **output signal (scanned photo):** \( g(x,y) = f(x,y) \ast \ast h(x,y) \)
**Square scan spot 1**

**Spatial frequency, \( u = \lambda \)**

**Input pattern**

**Output pattern**

1st zero (\( u = 1/W \))

2nd zero (\( u = 2/W \))

"Spurious" resolution (\( \pi \) phase shift)
square scan spot 2

input pattern

output pattern

spatial frequency, $u = x$

1st zero ($u = \frac{1}{2W}$) 2nd zero ($\frac{2}{2W}$) 3rd zero ($\frac{3}{2W}$)

"spurious" resolution ($\pi$ phase shift)
• **Scan with 2 different-sized Gaussian spots**

  - **scan spot 1:** \[ h(x, y) = \frac{1}{W^2} \text{gaus} \left( \frac{x}{W}, \frac{y}{W} \right) \]

  - **scan spot 2:** \[ h(x, y) = \frac{1}{4W^2} \text{gaus} \left( \frac{x}{2W'}, \frac{y}{2W} \right) \]
Gaussian scan spot 1

input pattern

spatial frequency, \( u = x \)

output pattern

"effective" cutoff frequency
Gaussian scan spot 2

input pattern

output pattern

spatial frequency, $u = \lambda$

"effective" cutoff frequency
Example 2: digital image processing

- **input signal (digital image):**

- **scan with various impulse responses** $h(x,y)$ **representing an (unspecified) LSI system**
  - represent $h(x,y)$ by small, discrete array of “weights”
  - approximate continuous convolution by discrete convolution between digital image and weight array
• Low-pass box-filter

• continuous impulse response: \( h_{LP}(x, y) = \frac{1}{W^2} \text{rect} \left( \frac{x}{W}, \frac{y}{W} \right) \)

• discrete approximation \((W = 3)\): \( h_{LP}(x, y) = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \)

  • sum of weights = 1
  • preserves signal mean
**Low-pass box-filter**

- **Input**
- **Output**
  - $3 \times 3$
  - $5 \times 5$
  - $7 \times 7$

The filter removes high-frequency content (micro-contrast: edges, lines, details) and keeps low-frequency content (macro-contrast: shading).
• **High-pass box-filter**

  • *continuous impulse response:*

    \[
    h_{HP}(x, y) = \delta(x, y) - \frac{1}{W^2} \text{rect}\left(\frac{x}{W}, \frac{y}{W}\right)
    \]

    \[
    = \delta(x, y) - h_{LP}(x, y)
    \]

  • *discrete approximation (W = 3):*

    \[
    h_{HP}(x, y) = \frac{1}{9} \cdot \begin{bmatrix}
    -1 & -1 & -1 \\
    -1 & 8 & -1 \\
    -1 & -1 & -1
    \end{bmatrix}
    \]

    • *sum of weights = 0*

    • *zeros signal mean*
**High-pass box-filter**

- **3 x 3**
- **5 x 5**
- **7 x 7**

removes low-frequency content, keeps high-frequency content.
**High-boost box-filter**

- **continuous impulse response:**
  \[ h_{HB}(x, y) = 2\delta(x, y) - \frac{1}{W^2} \text{rect}\left(\frac{x}{W}, \frac{y}{W}\right) \]

  \[ = \delta(x, y) + h_{HP}(x, y) \]

- **discrete approximation \((W = 3)\):**
  \[ h_{HP}(x, y) = \frac{1}{9} \cdot \begin{bmatrix} -1 & -1 & -1 \\ -1 & 17 & -1 \\ -1 & -1 & -1 \end{bmatrix} \]

- **sum of weights = 1**
- **preserves signal mean**
preserves low-frequency content, boosts high-frequency content

“sharpens” image
Example 3: optical image formation (incoherent light)

- An imaging system is a **LPF of spatial frequency components in the input object**
- the image is a blurred copy of the object
- \( h(x, y) \) depends on the parameters and quality of the optical system