Solutions to Homework Set 1

February 11, 2000

1 Problem 1a

We write $f_1(x)$ as:

$$f_1(x) = \text{rect}(\frac{x}{w}) * [\delta(x) + \frac{1}{|2w|} \delta(\frac{x}{2w})] \quad (1)$$

The Fourier Transform of $f_1(x)$ is:

$$F_1(u) = w.sinc(w.u).[1 + 2.\frac{2w}{2w}.\cos(2\pi.2w.u)]$$

Therefore,

$$F_1(u) = w.sinc(w.u).[1 + 2.\cos(4\pi.w.u)] \quad (2)$$

The amplitude is shown on Figure 2. The phase $\Phia(F_1(u))$ is 0 or $\pi$ depending on the sign of $Re(F_1(u))$.

2 Problem 1b

The are several ways to write $f_2(x)$. One simple way to do is:

$$f_2(x) = \frac{1}{2} - \text{rect}(\frac{x}{w}) * [\frac{1}{|2w|}.\text{comb}(\frac{x}{2w})] \quad (3)$$

The Fourier Transform of $f_2(x)$ is:

$$F_2(u) = \frac{1}{2}.\delta u - w.sinc(w.u).\text{comb}(2w.u) \quad (4)$$
See Figure 3.
Once again the phase $\text{Pha}(F_2(u))$ is 0 or $\pi$ depending on the sign of $\text{Re}(F_2(u))$.

3 Problem 2a

Simple case of lowpass filtering on space domain. The resulting function is the $\text{tri}(\frac{x}{b})$ with the sides chopped at a width ’$b$’(see Figure 1).

![Figure 1: Plots of $\text{rect}(\frac{x}{b}), \text{tri}(\frac{x}{b})$ for $b = 500$](image-url)
4 Problem 2b

\[ g_2(x) = \cos\left(\frac{2\pi x}{b}\right) * \delta\left(\frac{2x}{b}\right) \]  

(5)

We can rewrite the function as:

\[ g_2(x) = \cos\left[\frac{2\pi}{b}(x - \frac{b}{2})\right] + \cos\left[\frac{2\pi}{b}(x + \frac{b}{2})\right] \]  

(6)

or

\[ g(x) = -b\cos\left(\frac{2\pi x}{b}\right) \]  

(7)

or, we can find the Fourier Transform of \( g_2(x) \) in equation 5:

\[ FT[g_2(x)] = \frac{|b|}{2}\delta(b,u).2\frac{|b|}{2}\cos(2\pi x \frac{b}{2}.u) = \frac{|b|^2}{2}\delta(b,u).\cos(2\pi x \frac{b}{2}.u) \]

\[ = -\frac{|b|^2}{2}\delta(b,u) = F_2(u) \]

And find the Inverse Fourier Transform of \( F_2(u) \):

\[ FT^{-1}[F_2(u)] = -\frac{|b|^2}{2} \frac{2\pi x}{b}.\cos(2\pi x \frac{b}{2} = -|b|.\cos(2\pi x \frac{b}{2}) \]

See Figure 4.
Figure 2: Plots of $F_1(u)$ for 3 different values of $w$
Problem 1b

Figure 3: Plots of $F_2(u)$ for 2 different values of w
Figure 4: Plots of $G_2(u)$ for 3 different values of $b$