Object-Oriented Modeling of Convective Flows Using the Dymola Thermo-Bond-Graph Library

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Abstract
In this paper, a new thermo-bond-graph library is introduced that has been designed for modeling convective flows. It is programmed as part of the Dymola object-oriented graphical modeling environment.

The use of the new library is demonstrated by means of a fairly complex example: a pressure cooker.

This paper is a companion paper to the paper Object-Oriented Modeling of Complex Physical Systems Using the Dymola Bond-Graph Library, which is presented at the same conference.

INTRODUCTION
Thermodynamic systems can be modeled using the standard bond-graph library of Dymola [1], as long as no convective flows are simulated. In fact, the standard bond-graph library contains a section on thermodynamics, which offers models of thermal capacitors, of resistive sources, and of heat exchangers. The problem with convective flows is that they require modeling of mass flows beside from energy flows.

If a mass of some physical quantity is moved macroscopically from point A to point B, it carries its heat along on the trip, and thus, the total effect of the mass transport includes heat transport, which must be reflected in the thermodynamic model of that mass transport. Thus, heat transport without mass transport is possible, by means of either conduction or radiation, but mass transport without heat transport can never occur.

In the new thermo-bond-graph library, thermo-bonds are represented by six variables rather than two, three pairs of adjugate variables describing (i) temperature and entropy flow, (ii) pressure and volume flow, and (iii) enthalpy and mass flow.

Each of the three pairs represents a power flow, i.e., the product of the two variables of each pair has the dimension of power; yet the three power flows are coupled, and only together, they describe the overall effects of a convective flow.

THE THERMO-BOND CONNECTORS
The thermo-bond connectors contain 11 variables:

Of those, the first three are the three efforts relating to heat flow, volume flow, and mass flow: the temperature, \(T\), the pressure, \(p\), and the Gibbs potential, \(g\). The next three variables are the corresponding flow variables, the entropy flow, \(S\dot{\text{ot}}\), the volume flow, \(q\), and the mass flow, \(M\dot{\text{ot}}\). The next three variables are the integrals of the three flows, i.e., the three generalized positions, the entropy, \(S\), the volume, \(V\), and the mass, \(M\).

It is convenient to carry them along in the connector, since these quantities act frequently as modulators. The tenth variable is the directional variable, \(d\), just like in the case of the normal bonds [1]. The eleventh and last variable is an indicator variable telling the system, whether a substance exists or not. In chemical reactions, new products may be created on the fly, and the simulator needs to know whether any of that substance exists, in order to avoid divisions by zero.

Just like in the case of the regular bond-graph library, there exist two variants of this model for causal thermo-bond graphs, which are the \(e\)- and the \(f\)-connectors [1].
We are now ready to model the bonds. Each *Dymola* model is associated with four separate windows: (i) the *diagram window*, used to make composite models out of simpler ones, by dragging them from the library into the diagram window, and interconnecting them there; (ii) the *equation window*, used to provide additional equations for the model that have no graphical representation; (iii) the *icon window*, used to provide a graphical representation for the new model; and (iv) the *documentation window*, used to provide additional information about the model.

I shall only show the causal bonds here. The *e*-bond has the following iconic representation:

![Diagram of the e-bond](image)

We use the notation of a *bus*, as used in electronics, to denote the fact that this bond really transports three different (though inseparable) power flows.

The diagram window demonstrates the flow of information across the causal bond:

![Diagram of the flow](image)

The only semantically relevant information on this diagram is given in the form of the two thermo-bond connectors to the left and to the right of the harpoon. These are connector models that were dragged into the diagram window. Everything else is just text placed here for documentation purposes. *Dymola* allows to place text anywhere and everywhere for documentation purposes.

The equation window contains the information shown below. The thermo-bond is defined analogous to the regular bond. It simply needs to connect more variables across.

![Equation window](image)

Notice that the causal bonds are again modeled as *Dymola blocks* rather than *Dymola models* [1].

**THE THERMO-BOND JUNCTIONS**

Thermo-bonds only have 0-junctions. Since thermal systems do not have an inductive storage, 1-junctions are not essential. Resistive phenomena shall be modeled here as two-ports placed between two neighboring 0-junctions.

The three-bond 0-junction is modeled as follows:

![Equation window showing the three-bond 0-junction](image)

It is programmed analogous to the case of the three-bond 0-junction of the regular bond-graph library [1]. The three flow variables are summed up to zero separately, whereas the three efforts as well as the auxiliary variable are set equal across the 0-junction.

Special 0-junctions with only one single thermo-bond connector were created as interfaces between the thermo-
bond-graph library and the regular bond-graph library: For example, the model:

makes heat and volume flows separately available for use in a regular bond graph, whereas the mass flow is set to zero. In the upper right corner, the Boolean variable Exist is made available to the regular bond graph. The equation window of this model is programmed as follows:

These special junctions will be used inside the heat exchange models shown later in this paper.

THERMAL MODELS OF THE REGULAR BOND-GRAPH LIBRARY

Let us now look at some of the thermal models that are provided as part of the regular bond-graph library. The regular bond-graph library currently contains seven thermal models.

The RS-element is the standard resistive source element, as introduced by Thoma [2]. It is a directed element with a non-thermal primary side, and a thermal secondary side. The causality of the primary side is free (as for all resistors), whereas the causality of the secondary side is fixed. The secondary side acts as a source of entropy, never as a source of temperature.

The RSth-element is a modified resistive source, used whenever the primary side is thermal as well. As shown in [3], the thermal resistor needs to be modulated by the absolute temperature (it is multiplied by the absolute temperature) in the case of true bond graphs. The thermal resistor is constant in the case of pseudo-bond graphs, i.e., when heat flow is used as a flow variable, instead of entropy flow. However, the Dymola bond graph library is built around true bond graphs, not pseudo-bond graphs.

The Cth-element is the thermal capacitor. As shown in [3], thermal capacitors need to be divided by the absolute temperature in a true thermal bond graph.

The HEs- and HEas-elements represent symmetric and asymmetric heat exchange elements without simultaneous exchange of mass. The symmetric heat exchanger is modeled graphically as follows:

As shown in [3], the R-element of the heat exchanger must be replaced by a resistive source with a primary thermal side, i.e., an RSth-element. In the symmetric model, the RSth-element is split in two, each having twice the resistance that would be present in an asymmetric heat exchanger.

The mGS- and mGSth-elements are identical to the RS- and RSth-elements, except that they can be turned off
by a Boolean control signal. Since the resistance in the off state would be infinite, it makes more sense to model these elements as conductive sources rather than resistive sources. The mGSth-element furthermore models the conductance as a modulating real-valued input signal, rather than as a parameter.

THE THERMO-BOND-GRAPH LIBRARY

We shall now look at some models of the thermo-bond-graph library. This library also provides a heat exchange element.

The thermo-bond HE-element models the exchange of heat between two media without simultaneous exchange of mass. It is similar to the HE-element of the regular bond-graph library. However, the heat exchanging media themselves are modeled using thermo-bonds. This heat exchanger is modeled as follows:

The model has two thermo-bond connectors. These are interfaced to the heat exchanger using the special 0-junctions introduced earlier. Heat exchange only takes place if both media exist. Furthermore, the specific thermal conductance, \( \lambda \), is computed elsewhere and imported as a real-valued input signal. One half of the conductance goes to the left 0-junction, the other half goes to the right 0-junction.

The iconic representation of this model is provided below. It has two thermo-bond connectors to the left and to the right of the thermo-bond HE-symbol, and it has a (triangular) signal input connector at the bottom to import the value of \( \lambda \).

Notice that, from the outside, this model looks like any other thermo-bond-graph element, i.e., it is not visible from the outside that the element internally is modeled primarily by regular bond-graph elements.

The PVE-element models the pressure and volume exchange between neighboring media with a movable membrane between them.

In the process, additional entropy is being generated. Notice that this model uses the mGS-element, rather than the mGSth-element, because the primary side is hydraulic, not thermal. It picks up the two pressure values at the top of the special 0-junctions using bonds \( B_1 \) and \( B_2 \), and delivers entropy flows back to the special 0-junctions from the bottom using bonds \( B_3 \) and \( B_7 \).

The RFq-element models the forced convective flow of mass. This is a general resistive field element. It contains a modulated thermo-bond flow source, the mSF-element, which models forced volume flow. Mass, volume, and entropy enter and leave the flow source, but depending on the differences in temperature, pressure, and Gibbs potential, additional entropy may be generated that is being re-introduced in the forward direction of the flow. Switch elements (Sw-elements) are used to determine, in which direction the additional entropy is being introduced. The amount of forced volume flow, \( q \), is entered to the model as an external modulating input signal.
Notice that the switch element has variable causality. Hence a-causal bonds are used in the vicinity of the switch elements.

In thermal modeling, it is often numerically meaningful to introduce fictitious boundary layers. It is important that these boundary layers are in hydrodynamic equilibrium with the bulk, i.e., that the two pressure values are being equilibrated between the boundary layer and the bulk. Yet, the modeler may prefer to keep the total volume of the boundary layer constant.

To this end, the RFv-element was introduced. It replaces the PVE-element between boundary layer and bulk of the same substance. When volume is shifted from one side to the other by the internal PVE-element, a forced volume flow shifts the volume right back to where it came from.

The top-left thermo-bond, $F_1$, is not a normal thermo-bond. It is the thermo-bond counterpart to the $f$-element of the regular bond-graph library [1]. It provides the same function as a normal thermo-bond, yet the flow variables are made available as additional signal outputs. The volume flow, $q$, flowing through the $F_1$ thermo-bond is sent as a modulating input signal to the RFq-element.

Finally, there are the CF-elements, capacitive field elements that describe the material properties of the substances involved. Each CF-element describes one chemical substance.

**THE PRESSURE COOKER**

The pressure cooker model had already been described in [4,5]. However at the time those papers were written, the thermo-bond-graph library had not yet been created, i.e., the bond graphs shown in those papers were manually drawn representations of a simulation program that had been encoded as an ASCII text file.

By now, the new thermo-bond-graph library has been used to re-implement the previous pressure cooker model graphically. The resulting simulation model is present below.

There are five different substances: water, water vapor in the bulk, water vapor in the boundary layer, air in the bulk, and air in the boundary layer. Each substance
is represented by a 0-junction, which is sometimes split into two or even three separate 0-junctions to keep the number of connections to each junction limited. Each 0-junction has a \( CF \)-element attached to it representing the storage of that substance.

\( HE \)- and \( PVE \)-elements connect the different substances, since temperature and pressure are always equilibrated. In the equilibration of the bulk with the boundary layer, the \( PVE \)-elements are replaced by corresponding \( RFv \)-elements.

At the top left of the graph, there is an additional 0-junction representing the hot surface of the stove. It could have been represented using a regular 0-junction, since no mass is exchanged with the pressure cooker. Yet, it was represented using thermo-bonds as well, with only \( HE \)-elements connecting it to the pressure cooker. The temperature of the hot surface is represented by the \( mSE \)-element Heat is conducted from the hot surface through the metal of the pressure cooker. This effect is not modeled. The water and the two boundary layers are heated by the pressure cooker, but not the bulk. At the center right of the graph, there is another 0-junction outside the boxes representing the five substances. It represents cold water being applied to the pressure cooker to cool it down. Another thermo-bond \( SE \)-element is attached to it that represents the temperature of the cold water. The mechanism of cooling the metal of the pressure cooker down is not modeled accurately. The 0-junction simply interacts with the two boundary layers by means of \( HE \)-elements.

Additional \( RF \)-elements of a different type are placed between the water and the water vapor, and between the water vapor in the boundary layer and the water. They are modeled by use of steam tables computing the saturation pressure and volume as well as the enthalpy of boiling as functions of temperature.

**SIMULATION RESULTS**

The above model is a model that can be simulated. The model can be compiled:

Extracting all the equations from the hierarchical bond graph leads initially to a differential algebraic equation system containing 10853 equations in 10853 unknowns. 2221 of these equations compute constants, and 138 equations compute parameter values. 6413 equations are trivial equations of the type \( a=b \). They can be eliminated as well. After the automatic simplification process, there are still 2081 significant equations in 2081 variables to be simulated.

The simulation experiment has been encoded in the equation diagram of the main model, which provides the temperature values of the hot surface of the stove and the cold water as functions of time. In the experiment, the pressure cooker is first heated for 10,000 seconds, and subsequently cooled down for another 10,000 seconds. Notice that the parameter values of the pressure cooker were set such that the pressure cooker is of huge dimensions (1 m\(^3\)).

After setting the final time to 20,000 and the tolerance to 1.0E-10, the model can be simulated. The simulation takes only seconds. The water temperature and the air temperature are plotted below as functions of time.

The graph shows the water and air temperature as functions of time.

**CONCLUSIONS**

The paper demonstrates the power of a modeling environment that restricts the preset modeling concepts to mathematical information only. Everything else is soft-coded.

*Dymola* does not encode any physical knowledge. Yet, the programming environment is rich enough to enable the user to set up a modeling environment that is as generic or as specific as the user desires. This can be done easily and conveniently.

Creating a powerful thermo-bond-graph environment for modeling and simulation of convective flows is not conceptually more difficult than creating a regular bond-graph environment.

Once created, this environment is as easy to use as any specialized bond-graph environment of the past, yet it
is much more flexible than anything that had been available before.

REFERENCES


