1) Given the stiffly-stable back-interpolation algorithm $BI_{1,2}(\delta=0.4)$.

a) Find $f(q)$ for this method.

b) Determine the location of the roots and zeros in the complex $\omega$-plane.

c) Sketch the damping plot.

d) Sketch the stability domain.

e) Sketch the order stars.
2) Given the implicit extrapolation technique IEX2.
   
a) Write down the equations of this algorithm for non-linear state-space models.

b) Write the Butcher tableau for this method.

3) Given two n-th order accurate RK algorithms in at least \((n+1)\) stages:

   \[ f_1(q) = 1 + q + \frac{q^2}{2!} + \ldots + \frac{q^n}{n!} + c_1 \cdot q^{n+1} \]
   
   \[ f_2(q) = 1 + q + \frac{q^2}{2!} + \ldots + \frac{q^n}{n!} + c_2 \cdot q^{n+1} \]

   with \(c_1 \neq c_2\)
a) Prove that it is always possible to use the blending technique:

\[ x^{bl}(k+1) = \varphi \cdot x'(k+1) + (1-\varphi) \cdot x^e(k+1) \]

such that \( x^{bl}(k+1) \) is accurate up to order \( n+1 \).

b) Find a formula for \( \varphi \) that will make \( x^{bl}(k+1) \) accurate to order \( n+1 \).

c) Prove that the stability domains of Heun's method and the explicit midpoint rule are identical.