1) Given the matrix:

\[ A = \begin{bmatrix}
-9 & 11 & -21 & 63 & -252 \\
70 & -69 & 141 & -427 & 1684 \\
-575 & 575 & -1149 & 3451 & -13801 \\
3891 & -3891 & 7782 & -23345 & 93365 \\
1024 & -1024 & 2048 & -6144 & 24572
\end{bmatrix} \]

a) Calculate the eigenvalues and eigenvectors (possibly generalized) of this matrix.

Hint: It may be possible to exploit the fact that all matrix elements are integer. Any technique that operates on \( +, *, - \) for as long as possible won't pick up numerical errors during those parts of the algorithm.
b) Calculate $e^A$.

c) Perturb $A$ as follows:

$$
\hat{A} = A + 0.001 \hat{R} \\
\hat{R} = R / \| R \|_2 \\
R = 2 \cdot \text{rand}(5) - \text{ones}(5)
$$

Repeat problem (a) for $\hat{A}$.

d) Repeat problem (b) for $\hat{A}$.

2) Prove that $\text{eig}(A^k) = [\text{eig}(A)]^k$.

3) Prove that the eigenvalues of a skew-Hermitian matrix are purely imaginary.

4) Prove that the singular values of a square matrix are invariant to unitary transformations.
5) Prove that any square matrix can be unitarily transformed to another matrix whose QR decomposition has a diagonal R-matrix. Show that $R = \Sigma$ in that case.