1) Given the continuous system:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

which can be discretized to:

\[
\begin{align*}
x(k+1) &= F \cdot x(k) + G \cdot u(k) \\
y(k) &= H \cdot x(k) + I \cdot u(k)
\end{align*}
\]

where:

\[
\begin{align*}
F &= e^{AT} \\
G &= \int_0^T e^{AT} B \, dt \\
H &= C \\
I &= D
\end{align*}
\]

and \( T \) is the sampling rate.
a) Show that with the abbreviation: 
\[ \Psi = \int_0^t e^{At} \, dt \]
i.e., \( \Psi = \Psi \cdot B \)
we can compute \( \Psi \) as:
\[ \Psi = I + A \cdot \Psi \, . \]
Hint: Remember that:
\[ e^{At} = I + A t + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \ldots \]

b) Develop an algorithm to compute \( \Psi \) using the idea of a squaring-down procedure (similar to the case where \( e^{AT} \) is computed directly).
2) We wish to prove that the eigenvalues of the Hamiltonian matrix are indeed symmetric to the imaginary axis. The proof proceeds along the following line:

a) Prove that the inverse of the matrix:

\[ M = \begin{bmatrix} A & \mathcal{O} \\ C & B \end{bmatrix} \]

is:

\[ M^{-1} = \begin{bmatrix} A^{-1} & \mathcal{O} \\ -B^{-1}CA^{-1} & B^{-1} \end{bmatrix} \]

b) Apply a similarity transformation to the Hamiltonian matrix:
\[ H = \begin{bmatrix} A & -BR^{-1}B' \\ -Q & -A' \end{bmatrix} \]

with:
\[ T = \begin{bmatrix} I^{(n)} & \Phi \\ -P & I^{(n)} \end{bmatrix} \]

where \( P \) is the solution to the algebraic matrix Riccati equation:
\[ PA + A'P + Q - PBR^{-1}B'P = 0 \]

It turns out that:
\[ \hat{H} = T \cdot H \cdot T^{-1} \]
can be written as a function of:
\[ \hat{H} = f(A, B, R, K) \]

where \( K \) is the state feedback:
\[ K = R^{-1}B'P \]

What can you conclude about \( \text{eig}(\hat{H}) \)?
3) Given:

\[ \Delta = \begin{bmatrix} \Delta_1 & \phi & \phi \\ \Delta_2 & \Delta_3 & \phi \\ \phi & \phi & \Delta_4 \end{bmatrix} \]

Find matrices $M$ and $N$, such that:

\[ \Delta = M \cdot \Delta \cdot N \]

where $\Delta$ is block-diagonal:

\[ \Delta = \begin{bmatrix} \Delta_1 & \Delta_2 & \times \\ \times & \Delta_3 & \times \\ \times & \times & \Delta_4 \end{bmatrix} \]
4) Given the following non-linear plant:

\[
\begin{align*}
\dot{x}_1 &= -ax_1 + bx_1x_2 + eu \\
\dot{x}_2 &= cx_1 - dx_1x_2 + fu \\
y &= x_1 + x_2
\end{align*}
\]

We wish to design a controller such that the controlled system behaves as much as possible like the reference model:

\[
\begin{align*}
\dot{e} &= -e + r \\
e &= y - y_r
\end{align*}
\]
Recipe:

We connect the reference model in cascade with the inverse plant:

\[ R \rightarrow [M_{\text{ref}}] \xrightarrow{u_y} \left[ \text{Plant}^{-1} \right] \rightarrow M_{\text{opt}} \]

Find a state-space model for this cascade system!

Hint: \[ \dot{y} = \xi = \dot{x}_1 + x_2 \]
5) Given the following programmable-logic controller:

\[ e_1, e_2 \in \{NL, NS, PS, PL\} \]

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\[ NL = \text{negative large} \]
\[ NS = \text{negative small} \]
\[ PS = \text{positive small} \]
\[ PL = \text{positive large} \]

\[ u \in \{N, Z, P\} \]
\[ N = \text{negative} \]
\[ Z = \text{zero} \]
\[ P = \text{positive} \]
We wish to implement this controller using a counterpropagation neural network.

a) Draw a schematic of all boxes (functions) needed.

b) Design the neural network.

6) Given the non-linear function:

\[
x \rightarrow \square \rightarrow y
\]

characterized by the following measurements:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>82.8</td>
</tr>
<tr>
<td>2</td>
<td>68.0</td>
</tr>
<tr>
<td>3</td>
<td>54.8</td>
</tr>
<tr>
<td>4</td>
<td>45.8</td>
</tr>
<tr>
<td>5</td>
<td>54.8</td>
</tr>
<tr>
<td>6</td>
<td>72.8</td>
</tr>
</tbody>
</table>
We fuzzify input and output using the following triangular fuzzy membership functions:

\[ \mu_x(1) \quad \mu_x(2) \quad \mu_x(3) \quad \mu_x(4) \quad \mu_x(5) \quad \mu_x(6) \]

\[ \mu_y(600) \quad \mu_y(700) \quad \mu_y(800) \quad \mu_y(900) \]

a) Find the qualitative triples for \( x \) and \( y \).
b) Predict class, side, and membership value of the output for $x = 2.7$.

c) Defuzzify the predicted output.

d) Compute the confidence in that prediction using the proximity measure.

e) Compute the confidence in that prediction using the similarity measure.

f) The real data were taken from the function:

$$y = 1000 \cdot x + 720 / x$$

Find the real value for $y(x = 2.7)$. What can you say about the quality of the prediction made?