1) Given two state-space representations

\[ S_1 = \{ A, B, C, D \} \]
\[ S_2 = \{ \hat{A}, \hat{B}, \hat{C}, \hat{D} \} \]

which are similar to each other.

We make two polynomial matrix representations out of them:

\[ S_1(s) = \Xi((sI-A), B, C, D) \]
\[ S_2(s) = \Xi((sI-\hat{A}), \hat{B}, \hat{C}, \hat{D}) \]

Prove that they are equivalent to each other, i.e., that:

\[
\begin{bmatrix}
\hat{P}(s) &=& U_L(s) \cdot P(s) \cdot U_R(s) \\
\hat{Q}(s) &=& U_L(s) \cdot Q(s) \\
\hat{R}(s) &=& R(s) \cdot U_R(s) \\
\hat{W}(s) &=& W(s)
\end{bmatrix}
\]
b) Given two polynomial matrix representations of the type:

\[ S_1(s) = \sum (sI - A), B, C, D ] \]
\[ S_2(s) = \sum (sI - \hat{A}), \hat{B}, \hat{C}, \hat{D} ] \]

which are equivalent to each other.

Prove that the corresponding state-space representations:

\[ S_1 = \sum A, B, C, D ] \]
\[ S_2 = \sum \hat{A}, \hat{B}, \hat{C}, \hat{D} ] \]

are similar to each other.
2) The polynomial matrix representation:
\[ P(s) \cdot x(t) = Q(s) \cdot y(t) \]
\[ y(t) = R(s) \cdot x(t) + W(s) \cdot u(t) \]
is sometimes summarized by the "system polynomial matrix":
\[ S(s) = \begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix} \]
Given another system with:
\[ S_2(s) = S^*(s) = \begin{bmatrix} P^*(s) & -R^*(s) \\ Q^*(s) & W^*(s) \end{bmatrix} \]
where \(^*\) denotes the Hermitian transpose. Prove that \( S_2(s) \) is dual to \( S(s) \).

Hint: It suffices to show that the two systems have dual transfer function matrices.
3) Given the system:
\[
\begin{align*}
P(s) \cdot x(t) &= Q(s) \cdot u(t) \\
y(t) &= R(s) \cdot x(t) + W(s) \cdot y(t)
\end{align*}
\]
with:
\[
S_{0L}(s) = \begin{bmatrix}
P(s) & Q(s) \\
-R(s) & W(s)
\end{bmatrix}
\]

We introduce the output feedback:
\[
y(t) = i(t) + F(s) \cdot y(t)
\]
a) Prove that the closed-loop system:
\[
S_{0C}(s) = \begin{bmatrix}
P_{CL}(s) & Q_{CL}(s) \\
-R_{CL}(s) & W_{CL}(s)
\end{bmatrix}
\]
can be represented as:
\[
P_{CL}(s) = \begin{bmatrix}
P(s) & Q(s) & -F(s) \\
-R(s) & W(s) & I \\
\Phi & I & F(s)
\end{bmatrix} ;
Q_{CL}(s) = \begin{bmatrix}
\Phi \\
\Phi \\
I
\end{bmatrix}
\]
\[
R_{CL}(s) = \begin{bmatrix}
\Phi & \Phi & -I
\end{bmatrix} ;
W_{CL}(s) = \begin{bmatrix}
\Phi
\end{bmatrix}
\]
where:
\( \Phi \): zero matrices of appropriate dimensions
\( I \): identity matrices of appropriate dimensions

\[ S_{CL}(s) = \begin{bmatrix}
P(s) & Q(s) & \Phi & \Phi \\
-R(s) & W(s) & I & \Phi \\
\Phi & \Phi & I & I \\
\Phi & \Phi & \Phi & I \\
\end{bmatrix} \]

i.e.: 

6) The partial state vector of the closed-loop system can be decomposed into three subvectors:

\[ X_{CL}(t) = \begin{bmatrix}
X_1(t) \\
X_2(t) \\
X_3(t)
\end{bmatrix} \]

Find the relations between these subvectors and the partial state vector, input vector, and output vector of the open-loop system.
4) For the system with:

\[ P(s) = \begin{bmatrix} s^2 & -1 \\ -s & s^2 \end{bmatrix}; \quad Q(s) = \begin{bmatrix} s & -s \\ \phi & 1 \end{bmatrix} \]

(p. 453)

find the LCLD by triangularization

show the intermediate computations!