Treatment of Discontinuities II

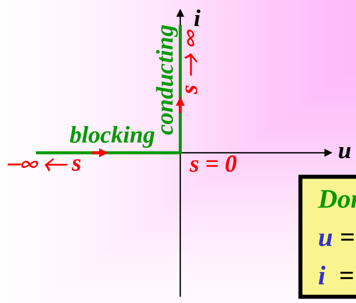
- We shall today once more look at the ***modeling of discontinuous systems***.
- First, an additional method to their mathematical description shall be discussed. This method makes use of a ***parameterized description of curves***.
- Subsequently, we shall deal with the problem of variable causality.
- Finally, a method shall be discussed that permits to solve causality problems elegantly.

Table of Contents

- Parameterized curve descriptions
- Causality of the switch equation
- Leaking diodes
- Singularity of switch equation
- Inline integration
- Causality of inline integration

Parameterized Curve Descriptions

- It is always possible to describe discontinuous functions by mean of parameterized curves. This technique shall be illustrated by means of the diode characteristic.



Domain:	Condition:	Equations:
blocking:	$s < 0$	$u = s;\ i = 0$
conducting:	$s > 0$	$u = 0;\ i = s$

Domain = **if** *s* < *0* **then** *blocking* **else** *conducting*;
u = **if** *Domain* == *blocking* **then** *s* **else** *0* ;
i = **if** *Domain* == *blocking* **then** *0* **else** *s* ;

ECE 449/549: Continuous System Modeling

The Causality of the Switch Equation I

- Let us consider once more the switch equation in its algebraic form:

$$0 = s \cdot i + (1 - s) \cdot u$$

Switch open: $s = 1$
Switch closed: $s = 0$

- We can solve this equation either for u or for i :

	$u = \dfrac{s}{s-1} \cdot i$	$i = \dfrac{s-1}{s} \cdot u$
Switch open:	*Division by 0!*	$i = 0$
Switch closed:	$u = 0$	*Division by 0!*

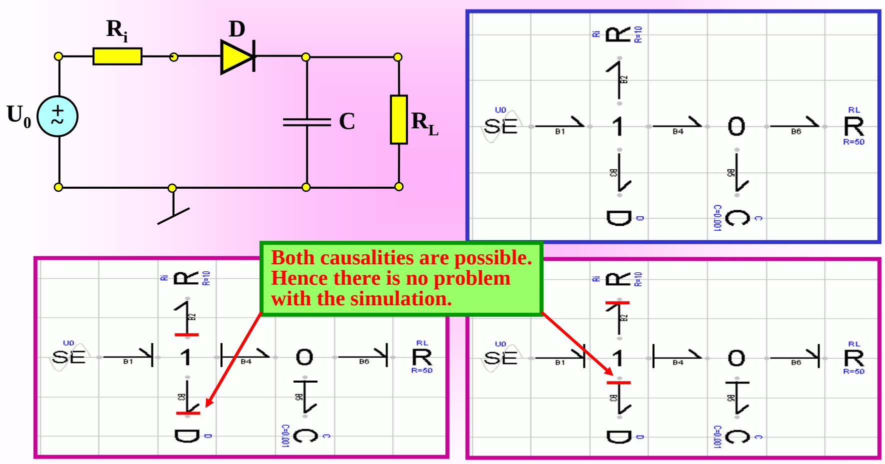
The Causality of the Switch Equation II

- Neither of the two causal equations can be used in both switch positions. Either one or the other switch position leads to a ***division by 0***.
- This is exactly what happens in the simulation, when the causality of the switch equation is fixed.

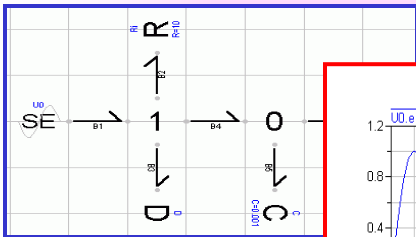
⇒ ***The causality of the switch equation must always be free.***

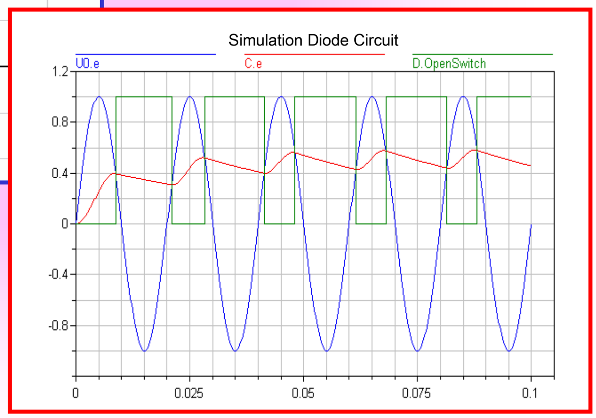
⇒ ***The switch equation must always be placed in an algebraic loop.***

An Example I

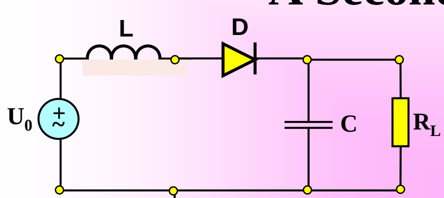


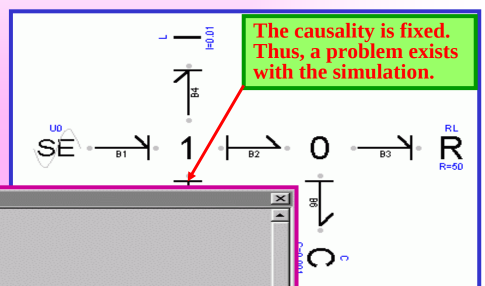
An Example II

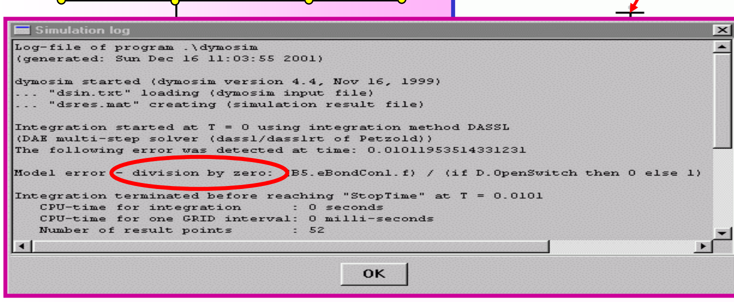




A Second Example



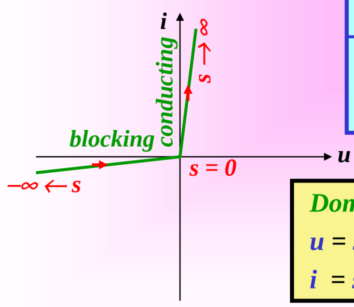





```
Log-file of program .\dymosim
(generated: Sun Dec 16 11:03:55 2001)

dymosim started (dymosim version 4.4, Nov 16, 1999)
... "dsin.txt" loading (dymosim input file)
... "dsres.mat" creating (simulation result file)

Integration started at T = 0 using integration method DASSL
(DAE multi-step solver (dassl/dasslrt of Petzold))
The following error was detected at time: 0.01011953514331231

Model error - division by zero: (B5.eBondCon1.f) / (if D.OpenSwitch then 0 else 1)

Integration terminated before reaching "StopTime" at T = 0.0101
   CPU-time for integration      : 0 seconds
   CPU-time for one GRID interval: 0 milli-seconds
   Number of result points       : 52
```

Not So Ideal Diode I

- One possibility for circumventing the causality problem consists in defining a ***leakage resistance*** R_{on} for the closed switch, as well as a ***leakage conductance*** G_{off} for the open switch.

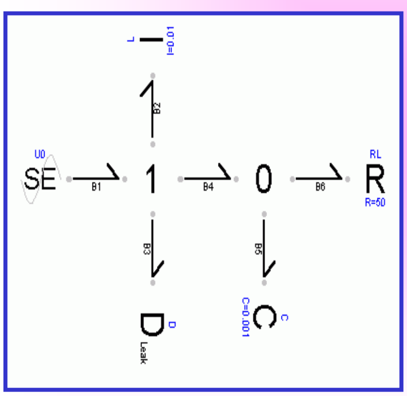


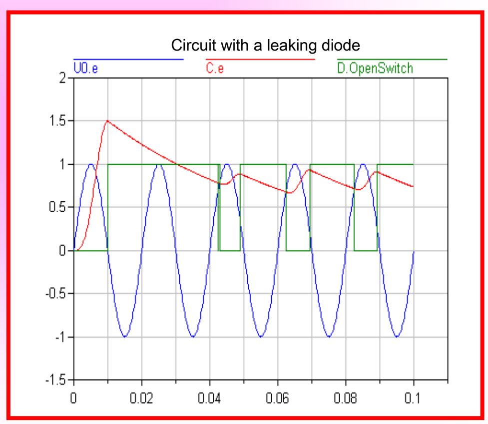
Domain:	Condition:	Equations:
blocking:	$s < 0$	$u = s;\ i = G_{off} \cdot s$
conducting:	$s > 0$	$u = R_{on} \cdot s;\ i = s$

Domain = **if** $s < 0$ **then** *blocking* **else** *conducting*;
$u = s$*(**if** *Domain* == *blocking* **then** *1* **else** R_{on});
$i = s$*(**if** *Domain* == *blocking* **then** G_{off} **else** *1*);

Not So Ideal Diode II

- This is the solution that was chosen in ***Modelica***.





Problems I

- For ***electrical applications***, the solution with the leaking diode is frequently acceptable.
- One problem has to do with the numerics. When circuit using the ideal diode is plagued by division problems, the circuit with the leaking diode leads invariably to a ***stiff system***.
- Stiff systems can be integrated in ***Modelica*** by means of the (standard) ***DASSL integration algorithm***.
- However, this is time consuming and may not be suitable, at least for real-time applications.

ECE 449/549: Continuous System Modeling

Problems II

- In the case of ***mechanical applications***, the method is less suitable, since for example friction characteristics must frequently be computed rather accurately, and since in mechanical applications, the causalities are almost invariably fixed.
- The masses (and inertias) determine all velocities, and the friction as well as spring forces (and torques) must therefore be determined by the ***R***- and. ***C***-elements in a pre-set causality.
- Consequently, another solution approach should be sought for these applications.

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"Inline" Integration Algorithm

- When using ***Inline Integration***, the integration algorithm is directly substituted into the model equations (or inversely: the model equations are being substituted into the integration algorithm).
- Let us consider an inductor integrated by means of the ***implicit Euler algorithm***.

$$u_L = L \cdot di_L/dt$$
$$i_L(t) = i_L(t-h) + h \cdot di_L(t)/dt$$

$$\Rightarrow \quad i_L(t) = i_L(t-h) + (h/L) \cdot u_L(t)$$

ECE 449/549: Continuous System Modeling

The Causality of Inline Integration

$$i_L(t) = i_L(t-h) + (h/L) \cdot u_L(t)$$

Known, since computed in the past.

This constitutes an algebraic relation between *i* and *u*. This now looks like a resistor. Hence the causality is now free.

When using the inline integration algorithm, the causalities of the so integrated storage elements are being freed up. Consequently, the division problem disappears.

References I

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- Otter, M., H. Elmqvist, and S.E. Mattsson (1999), "Hybrid modeling in Modelica based on the synchronous data flow principle," *Proc. CACSD'99, Computer-Aided Control System Design*, Hawaii.

ECE 449/549: Continuous System Modeling

References II

- Krebs, M. (1997), *Modeling of Conditional Index Changes*, MS Thesis, Dept. of Electr. & Comp. Engr., University of Arizona, Tucson, AZ.

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