1) We wish to analyze our water pump a little further.

→ We assume friction between the piston and the cylinder.

→ We assume real characteristics on the two flaps:

While a flap is closed, there occurs a certain laminar leakage flow:
\[ q_e = k_1 \cdot \Delta p \]

where \( \Delta p \) is the pressure difference between the two sides.

- While the flap is open, the constraint generates a turbulent flow:

\[ q_t = k_2 \cdot \sqrt{\Delta p} \]

a) Draw the pressure/flow characteristic of any of the two flaps:

\[ \uparrow q \]

\[ \rightarrow \Delta p \]
in the ideal case, and in the real case.

b) Identify the type of bond graph element that the real flap represents
    
    \[ \text{(SE, SF, R, C, I, TF, GY)} \]

\[ SF \xrightarrow{\frac{f}{V}} TF \xrightarrow{p} 0 \]

\[ \frac{f}{V} \quad R \quad C \quad \text{P} \quad \text{P}_{\text{loss}} \quad \text{Sw}_2 \]

\[ \text{P}_{\text{city}} \quad \text{Sw}_0 \]

\[ \text{Sw}_1 \]

\[ \text{1} \]

\[ \text{1} \]

\[ \text{1} \]

\[ \text{1} \]

\[ \text{1} \]

c) In MT3, we found the following BG representing the pump:
Modify this BG to represent the real water-pump with mechanical friction and real flap characteristics.

d) Write a Dymola flap model for the BG library.

e) We wish to analyze how the water-pump heats up. Modify the BG twice more to include the thermodynamics.

f) Readout the equations from the BG and show that there is an algebraic loop.
2) In the gravimetric determination of phosphorus, an aqueous solution of dihydrogen phosphate ion, $H_2PO_4^-$, is treated with a mixture of ammonium and magnesium ions to precipitate magnesium ammonium phosphate, $\text{MgNH}_4\text{PO}_4\cdot6\text{H}_2\text{O}$. This is heated and decomposed to magnesium pyrophosphate, $\text{Mg}_2\text{P}_2\text{O}_7$, which is weighed. The reactions are:

$$H_2PO_4^- + Mg^{+} + NH_4^+ + H_2O \rightarrow$$

$$\text{MgNH}_4\text{PO}_4\cdot6\text{H}_2\text{O} + NH^+$$

$$\text{MgNH}_4\text{PO}_4\cdot6\text{H}_2\text{O} \rightarrow \text{Mg}_2\text{P}_2\text{O}_7$$

$$+ NH_3 + H_2O$$

Find the stoichiometric coefficients for these two reactions.
3) For the hydrogen iodide reaction:

\[ H_2 + I_2 \xrightleftharpoons[k_1]{k_2} 2HI \]

a) draw a bondgraph under isothermal and isobaric conditions. Include the thermal and pneumatic sides of the BG (trivial!).

b) Read out the mass balance equations from the O-junctions.

c) Let: \( \mathbf{y}_S = \begin{bmatrix} y_{H_2} \\ y_{I_2} \\ y_{HI} \end{bmatrix} \); \( \mathbf{y}_K = \begin{bmatrix} y_{k_1} \\ y_{k_2} \end{bmatrix} \)

Write the mass balance equations in matrix form:

\[ \mathbf{y}_S = \mathbf{N} \cdot \mathbf{y}_K \]

Find \( \mathbf{N} \).
d) Read out the energy balance equations from the 1-junctions.

e) Let: 
\[ M_s = \begin{bmatrix} M_{H_2} \\ M_{I_2} \\ M_{HI} \end{bmatrix}, \quad M_K = \begin{bmatrix} M_{K_1} \\ M_{K_2} \end{bmatrix} \]

Write the energy balance equations in matrix form:

\[ M_K = M \cdot M_s \]

Find \( M \).