3rd Midterm - Solution

- 3.1 Transformer
- 3.2 Capacitive Field
- 3.3 Fan
- 3.4 Refrigerator

3.1 Transformer

The standard electrical library of Dymola offers the following electrical transformer model:
Transformer II

- Create a bond-graphic representation of this circuit element, assuming that both positive currents point into the circuit.
- Represent the transformer as an inductive field, i.e., an IF-element.
- Find the induction matrix.

To create a bond-graphic representation, it may be easiest to work with the replacement T-circuit:

![Diagram of the circuit with bond-graphic representation and the induction matrix.](image)
For the IF-element representation, it is more convenient to work directly with the equation window:

\[
\begin{bmatrix}
  v_1 \\
  v_2
\end{bmatrix} =
\begin{bmatrix}
  L_1 & M \\
  M & L_2
\end{bmatrix} 
\begin{bmatrix}
  \frac{di_1}{dt} \\
  \frac{di_2}{dt}
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \frac{di_1}{dt} \\
  \frac{di_2}{dt}
\end{bmatrix} =
\begin{bmatrix}
  L_2 & -M \\
  -M & L_1
\end{bmatrix}^{-1} 
\begin{bmatrix}
  v_1 \\
  v_2
\end{bmatrix}
\]

3.2 Capacitive Field

- Given the circuit segment shown on the left.
- Find a bond graph representing this circuit segment.
- Represent this segment as a capacitive field.
- Find the capacitive matrix for this multi-port element.
• We can draw the bond graph easily:

\[
\begin{align*}
\dot{i}_1 &= i_{10} + i_{12} + i_{13} \\
\dot{i}_2 &= i_{20} - i_{12} + i_{23} \\
\dot{i}_3 &= i_{30} - i_{13} - i_{23}
\end{align*}
\]

\[
\begin{align*}
\dot{i}_{10} &= C_{10} \cdot \frac{dv_1}{dt} \\
\dot{i}_{20} &= C_{20} \cdot \frac{dv_2}{dt} \\
\dot{i}_{30} &= C_{30} \cdot \frac{dv_3}{dt} \\
\dot{i}_{12} &= C_{12} \cdot \frac{dv_1}{dt} - C_{12} \cdot \frac{dv_2}{dt} \\
\dot{i}_{13} &= C_{13} \cdot \frac{dv_1}{dt} - C_{13} \cdot \frac{dv_3}{dt} \\
\dot{i}_{23} &= C_{23} \cdot \frac{dv_2}{dt} - C_{23} \cdot \frac{dv_3}{dt}
\end{align*}
\]

⇒

\[
\begin{align*}
\dot{i}_1 &= (C_{10} + C_{12} + C_{13}) \cdot \frac{dv}{dt} - C_{12} \cdot \frac{dv}{dt} - C_{13} \cdot \frac{dv}{dt} \\
\dot{i}_2 &= -C_{12} \cdot \frac{dv}{dt} + (C_{20} + C_{12} + C_{23}) \cdot \frac{dv}{dt} - C_{13} \cdot \frac{dv}{dt} \\
\dot{i}_3 &= -C_{13} \cdot \frac{dv}{dt} - C_{23} \cdot \frac{dv}{dt} + (C_{30} + C_{13} + C_{23}) \cdot \frac{dv}{dt}
\end{align*}
\]
3.3 Fan

- We wish to model the big fan that sucks humid air out of Biosphere 2, drying it out by cooling it down, reheating it to the desired temperature, and blowing the conditioned air back in at the same speed as it was sucked out.

- We only wish to model the fan. The air contains two separate CF-elements, one representing the air mass, the other representing the water vapor mass contained in the air. The two masses are always in perfect mixture.
Fan II

- The fan moves a given amount of volume out of the Biosphere 2 dome. However, since the same amount of mass is blown back in, you may assume that the pressure remains more or less constant.
- Find a bond-graphic representation of the fan, which sits between 4 CF-elements, representing the air and the water vapor on both sides of the fan.
3.4 Refrigerator

- We wish to model a refrigerator.
- The refrigerator operates similar to thought experiment #3 and #4 in the second presentation on convective flows.
- A compressor placed outside the refrigeration chamber compresses air into a container of fixed volume. The compressed air heats up, but since the compression is slow, the container gives off most of the heat to the environment by means of conduction, convection, and radiation.

Refrigerator II

- Once the air in the container is sufficiently compressed, the air is released to the inside of the refrigerator.
- Because of the quick relaxation of the air, the air cools down rapidly and significantly.
- During the compression, the volume is fixed, and the pressure increases. During the relaxation, the pressure decreases in the container at fixed volume, whereas the pressure is constant in the refrigerator at variable volume.
Refrigerator III

• Make two separate bond graphs for the compression and relaxation phase of the refrigeration process.
• You don’t know yet how to connect the two models, because we haven’t discussed the switch elements yet.

• Usually, a refrigerator uses freon rather than air as the compression medium. The freon circulates in a closed circulation system (just like the air conditioner in the car).
• The system looks similar to the water serpentine. Half of the serpentine is inside the refrigerator, the other is outside. The compressor (pump) is also outside. There is a clamp that closes the entrance to the refrigerator during compression.
• Thus, the compressor over-pressurizes the freon between the compressor and the clamp. It underpressurizes the freon everywhere else.
• Once the desired pressure has been achieved, the clamp is opened, and the freon pressure equalizes again.