1) Deadbeat Response:

Given the discrete transfer function:

\[ G(z) = \frac{(z+0.5)(z-0.5)}{z^4} \]

a) Simulate this system's response to a step input over five steps:

\[ u(t) = \varepsilon(t) \rightarrow y(t) = ? \]

(This can be done either in the frequency domain by using the
modified partial fraction expansion on \( G(z) \), or in the time-domain, e.g.
in controller-canonical form.)

b) By generalizing the result from (a), prove that the step
response of any discrete-time
system of the deadbeat type
reaches its steady-state value
at the latest after \( n \) steps
where \( n \) is the order of
the system. (The proof is
possible in the frequency or time domain.)
2) **Controllability & Observability**:

Given the discrete-time system:

\[
\begin{align*}
\dot{x}(k+1) &= \begin{bmatrix} \Phi & 1 \\ 0.24 & -0.2 \end{bmatrix} x(k) + \begin{bmatrix} \Phi \\ 1 \end{bmatrix} u \\
y(k) &= \begin{bmatrix} K & 1 \end{bmatrix} x(k)
\end{align*}
\]

a) For which values of \( K \) (if any) does this system lose its full controllability?

b) For which values of \( K \) (if any) does this system lose its full observability?

c) For the values of \( K \) found under (a) and (b), find the transfer function. What do you notice?
3) **Functional Observer Design:**

Given the open-loop system:

\[ G(z) = \frac{1}{(z+1)(z-2)(z-3)(z+4)} \]

which is highly unstable.

(a) Find a functional observer which will place the poles of the closed-loop system all at the origin. The three observer poles are to be placed at \( z = -0.5, \pm 0.5j \frac{3}{2} \).

(b) Write a program segment of a discrete controller that will implement the functional observer as designed under (a).