1)

\[ \text{A} \quad \text{B} \quad \text{C} \quad \text{D} \quad \text{E} \quad \text{F} \quad \text{X} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

b) 

\[ \begin{array}{cc}
A & B \\
\hline
1 & 0 \\
1 & 1 \\
\end{array} \]

c) \( X = \overline{A} + B \)

d) 

\[ \text{A} \quad \text{B} \quad \text{X} \]
2) a) \[ X = A \cdot (B+C) + \overline{A} \cdot \overline{B} \]
\[ \overline{X} = \overline{A} \cdot B + A \cdot \overline{B} \cdot \overline{C} \]

b) It may be easiest to do it using a Karnaugh map.
\[ X = AB + AC + \overline{A} \overline{B} \]
\[ \overline{X} = \overline{AB} + A \overline{BC} \]

\[ \overline{X} \] is indeed the true negation of \( X \).
\[ q.e.d. \]
3) \( \overline{A}((A \oplus B) + (C \oplus D)) \overline{D} \)
\[= \overline{A}((\overline{A}B + \overline{A}B) + (C\overline{D} + \overline{C}D)) \overline{D} \]
\[= \overline{A}\overline{A}B\overline{D} + \overline{A}\overline{A}B\overline{D} + \overline{A}C\overline{D}\overline{D} + \overline{A}\overline{C}D\overline{D} \]
\[= \emptyset + \overline{A}BD + \overline{A}CD + \emptyset \]
\[= \overline{A} (B + C) \overline{D} \]

q.e.d.

4) 

[Diagram showing a circuit with inputs AB, CD, E, F and outputs shown in the diagram.]

[Diagram showing a circuit with inputs AB, CD, E, F and outputs shown in the diagram.]
There are two (actually four) equally good answers:

(1) \( X = BD + \overline{BD} + DEF + BEF \)
(2) \( X = BD + \overline{BD} + \overline{BEF} + \overline{DEF} \)

* Each optimal solution will contain the first two (16-field) terms, then one of the 3rd terms and one of the 4th terms.

* Obviously, it would also be possible to combine the first two terms into one, since:

\[
BD + \overline{BD} = \overline{B \oplus D}.
\]

\[ \text{or: } BD + \overline{BD} = \overline{B \oplus D}. \]

\[ \text{or: } BD + \overline{BD} = B \oplus \overline{D}. \]
Let us next eliminate the XOR.

\[ B \oplus D \oplus E = (B \oplus D) \oplus E \]
\[ = (B \overline{D} + \overline{B}D) \oplus E \]
\[ = (B \overline{D} + \overline{B}D) \cdot E + (B \overline{D} + \overline{B}D) \cdot \overline{E} \]
\[ = (B \overline{D} + \overline{B}D) \cdot E + ((B \overline{D}) \cdot (\overline{B}D)) \cdot \overline{E} \]
\[(B\overline{D} + \overline{B}\overline{D}) \cdot \overline{E} + (\overline{B} + D) \cdot (B + D) \cdot E\]
\[= (B\overline{D} + \overline{B}\overline{D}) \cdot \overline{E} + (BD + \overline{BD}) \cdot E\]
\[= BD\overline{E} + B\overline{DE} + BDE + \overline{BD}E\]

Next, we eliminate the ORs.

Now, we move some more bubbles.
6) \( x = (A+B)(B+C)(A+C) + \overline{AC} + \overline{BC} \)

\[
= AAB + ABC + A\overline{AC} + A\overline{CC} + A\overline{BB} + B\overline{BC} \\
+ A\overline{BC} + B\overline{CC} + \overline{AC} + \overline{BC}
\]

\[
= AB + ABC + A\overline{C} + \phi + AB + BC \\
+ A\overline{BC} + \phi + \overline{AC} + \overline{BC}
\]

\[
= AB + BC + AB(C+\overline{C}) + \overline{AC} + \overline{BC} + AC
\]

\[
= AB + BC + AB + AC + BC + AC
\]

\[
= AB + BC + AC + BC + AC
\]

\[
= AB + (B+\overline{B})C + \overline{AC} + AC
\]
\[
= A\overline{B} + C + (A\overline{A}) \overline{C}
\]
\[
= A\overline{B} + C + \overline{C} = A\overline{B} + 1 = 1
\]
\[
\Rightarrow \quad X = 1
\]

7) \hspace{1cm} \begin{array}{c}
\text{A} \\
\text{B} \\
C
\end{array} \hspace{1cm} \begin{array}{c}
\text{HA} \\
\text{S}_1 \\
C_1
\end{array} \hspace{1cm} \begin{array}{c}
\text{D} \\
\text{E}
\end{array} \hspace{1cm} \begin{array}{c}
\text{HA} \\
\text{S}_2 \\
C_2
\end{array} \hspace{1cm} \begin{array}{c}
X
\end{array}

9) \hspace{1cm} \begin{array}{c|c|c|c|c|}
\text{A} & \text{B} & \text{S}_1 & \text{C}_1 & \text{X}
\hline
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 \\
\end{array}

\text{HA} \quad \text{XOR} \quad \text{XNOR}

\begin{array}{c|c|c|c|c|c|}
\text{A} & \text{B} & \text{C} & \text{S}_1 & \text{C}_1 & \text{D} & \text{E} & \text{S}_2 & \text{C}_2 & \text{X}
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0
\end{array}
b) 

\[ A \quad B \quad C \]

\[ \begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array} \]

c) \[ X = \overline{A}BC + ABC\overline{C} + A\overline{B}C \]

d) 

We eliminate the ANDs :

\[ A \quad B \quad C \]

\[ \begin{array}{ccc}
\overline{A} & \overline{B} & \overline{C} \\
\overline{A} & \overline{B} & \overline{C} \\
\overline{A} & \overline{B} & \overline{C} \\
\end{array} \]
Now we move bubbles:

\[ \text{Done!} \]

**Corollary:**

\[
X = \overline{A}BC + \overline{A}BC + ABC + \overline{A}BC \\
= (\overline{A}C + AC)B + (\overline{A}B + A\overline{B})C \\
= (A \oplus C)B + (A \oplus B)C
\]

would be a nice realization also.
a)  

<table>
<thead>
<tr>
<th>$A_3$</th>
<th>$A_2$</th>
<th>$A_1$</th>
<th>$A_0$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

b) 

\[ X = A_2 \overline{A}_1 + A_2 \overline{A}_0 + A_3 \overline{A}_1 + A_3 \overline{A}_0 + \overline{A}_3 \overline{A}_2 A_1 \]