ECE 304: Two-Mesh Design Project

Problem[1]

In the circuit of Figure 1, a sensing device provides a current I_1 = 10 mA and a receiving device demands a current of I_2 = 5 mA. The matching T-section of resistors is subject to the conditions that it keeps the input voltage V_1 in the range $-20V \le V_1 \le 20V$ and the output voltage V_2 in the same range. Find values for the resistors R_1, R_2 and R_3.

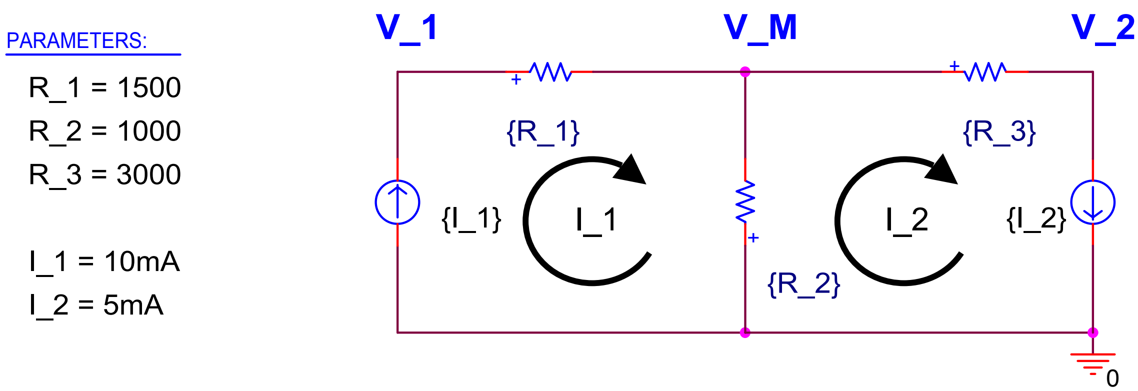

FIGURE 1
Schematic for the problem

Procedure

Our procedure is to analyze the circuit to find the constraints on the resistors imposed by the current limitations and node voltage requirements. We will find formulas expressing the range of resistance values that meet these conditions. Then we will plot these constraints using a spreadsheet. Then we will verify the spreadsheet using PSPICE.

Analysis

Using Kirchhoff's laws we find

EQ. 1

$$V_1 = I_1(R_1 + R_2) - I_2R_2$$

EQ. 2

$$V_2 = I_1R_2 - I_2(R_2 + R_3).$$

Approach to the design problem

We introduce an extra variable into the problem, namely, the voltage on the middle node (top of the center resistor). Let's call this voltage V_M (M = middle node). The reason for introducing this variable is an anticipation that its behavior will be easy to understand and make solutions to the design problem easy to understand. We will see later whether this idea works. From Ohm's law we find

EQ. 3

$$V_M = (I_1 - I_2)R_2.$$

Using V_M, we can rewrite EQ. 1 and EQ. 2 as follows:

EQ. 4

$$V_1 = V_M + I_1R_1$$

EQ. 5

$$V_2 = V_M - I_2R_3.$$

[1] This example is based on a suggestion by Larry Huelsman

Next we solve for the resistor values using EQ. 3, EQ. 4 and EQ. 5. From EQ. 4, R_1 is found as EQ. 6 below.

EQ. 6

$$R_1 = \frac{V_1 - V_M}{I_1}$$

From EQ. 5, R_3 is found as

EQ. 7

$$R_3 = \frac{V_M - V_2}{I_2},$$

and rearranging EQ. 3, R_2 is found as

EQ. 8

$$R_2 = \frac{V_M}{I_1 - I_2}.$$

Putting in the values given for $\mathbf{I}_1$ =10 mA and $\mathbf{I}_2$ = 5 mA, we find

EQ. 9

$$R_1 = \frac{V_1 - V_M}{10}\ \text{k}\Omega$$

EQ. 10

$$R_2 = \frac{V_M}{5}\ \text{k}\Omega$$

EQ. 11

$$R_3 = \frac{V_M - V_2}{5}\ \text{k}\Omega.$$

Intuitive behavior of equations

The expected advantage of V_M as a variable is that we know intuitively how it should behave. For example, because $\mathbf{I}_1$ flows from V_1 to V_M, $V_M \le V_1$. Likewise, because $\mathbf{I}_2$ flows from V_M to V_2, we know $V_M \ge V_2$. Or, putting these two facts together, we know intuitively that V_M lies between V_1 and V_2, that is,

EQ. 12

$$V_1 \ge V_M \ge V_2.$$

Using our understanding of V_M it is easy to understand EQ. 9: $R_1 \to 0$ as $V_M \to V_1$ because the current in R_1 is fixed at $\mathbf{I}_1$ = 10 mA. As $V_M \to V_1$, the voltage drop across R_1 is reduced, so R_1 has to be smaller to allow the same current to flow. A similar understanding applies to EQ. 11.

Bounds on resistor values due to voltage limits

Looking at EQ. 9 assuming V_M is given, the value of R_1 increases linearly as V_1 increases, which means the maximum value of R_1 occurs when V_1 is its maximum value of V_1 = 20V. The reason R_1 is a maximum when V_1 = 20 V is that V_1 = 20V will place the largest voltage drop across R_1, so to keep the current at I_1 = 10 mA, R_1 will be its biggest. In other words, the maximum value R_{1MAX} is given by

EQ. 13

$$R_{1MAX} = \frac{20 - V_M}{10}\ \text{k}\Omega.$$

The minimum value of R_1 is zero and occurs when $V_1 = V_M$.
According to EQ. 10, for a non-negative value of R_2 we must have

EQ. 14

$$V_M \ge 0$$

From EQ. 11 for a given value of V_M, the value of R_3 decreases linearly as V_2 increases, reaching zero when $V_2 = V_M$. Again, the reason is that when V_M is fixed and V_2 is increased, the voltage drop across R_2 is decreased. Therefore, R_2 must be reduced to keep the current at I_2 = 5 mA. Correspondingly, the minimum value of R_3 occurs when V_2 has its greatest value of V_2 = 20 V, so R_{3MIN} is given by

EQ. 15

$$R_{3MIN} = \frac{V_M - 20}{5} k\Omega ,$$

so long as this value is not negative. We can see that EQ. 15 will not affect the design because the largest value of V_M allowed is V_M = 20 V. Therefore, for all allowed values of V_M, EQ. 15 is negative or zero, which places no constraint on R_3 because R_3 is always ≥ 0 Ω anyway.

Finally, looking at EQ. 11 the other way around, for decreasing V_2, the value of R_3 increases linearly as V_2 decreases, so the largest value of R_3 occurs when V_2 is its most negative value V_2 = –20 V and R_{3MAX} is

EQ. 16

$$R_{3MAX} = \frac{V_M + 20}{5} k\Omega .$$

The bounds on $R_{1=}$and R_3 are now put into a spreadsheet, and the value of R_2 is calculated from V_M using EQ. 10.

Spreadsheet

The spreadsheet contains several worksheets. The ORGANIZATION worksheet shows the arrangement.

Worksheet ORGANIZATION

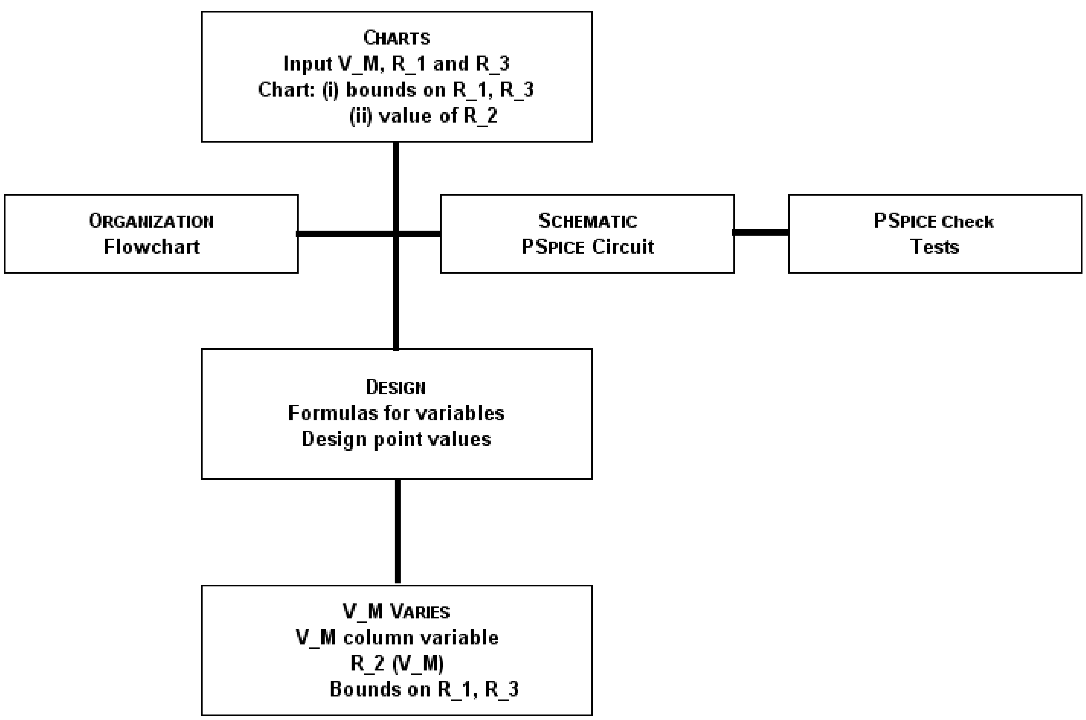

FIGURE 2

Organization of the spreadsheet as seen on the ORGANIZATION worksheet

Figure 2 introduces the spreadsheet organization. Data input is made on CHARTS, formulas are defined on DESIGN, and functional dependence is calculated on VM_VARIES. In more complex spreadsheets there will be several worksheets for calculations and it is best to keep all data input in one place, namely CHARTS, to insure that all the worksheets are working with the same design point. CHARTS also has graphs showing the design tradeoffs. These graphs, or charts, automatically update when the data input on CHARTS is changed, allowing immediate visual appraisal of the benefit of the change.

The worksheets shown in the row under CHARTS are supplementary, serving to remind the user how the spreadsheet works, what problem it has solved, what the notation means, and what PSPICE verifications have been done to insure everything is working. The worksheets are separately introduced next.

Worksheet CHARTS

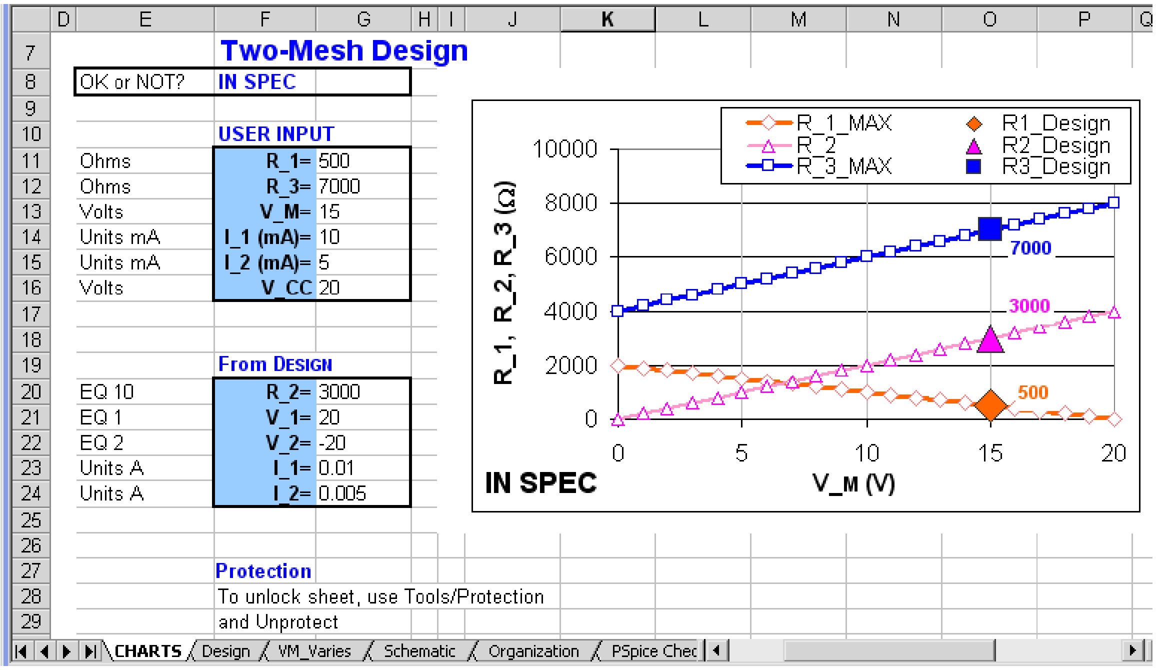

FIGURE 3

Worksheet CHARTS showing USER INPUT field for data and design point values from DESIGN; the maximum voltage has been labeled as V_{CC} = 20 V

Figure 3 shows the CHARTS worksheet where data is input and charts of the resistor values are plotted. To avoid accidental overwriting of the FROM DESIGN area, this area of the worksheet is protected. Instructions to unprotect it are provided in case we forget how later on.

Worksheet DESIGN

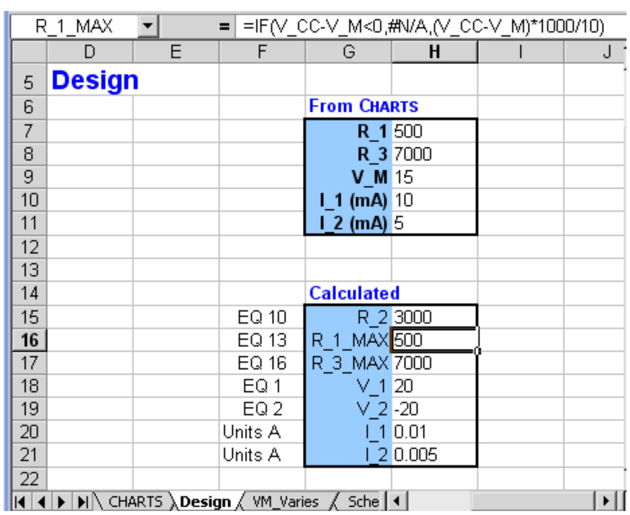

FIGURE 4

DESIGN worksheet that contains formulas for the values of the bounds and of R_2

The DESIGN worksheet contains the formulas used in the design. It is useful to have all the formulas in one place. Then the other worksheets are made by copying these formulas, so all use the same formulas. If a formula is changed, it is changed here, and then copied to the other worksheets.

Worksheet VM_VARIES

R_1_MAX = =IF(V_CC-V_M<0,#N/A,(V_CC-V_M)*1000/10)

	D	E	F	G	H	I	J
5	V_M Varies				EQ 10	EQ 13	EQ 16
6				V_M	R_2	R_1_MAX	R_3_MAX
7				0	0	2000	4000
8				1	200	1900	4200
9				2	400	1800	4400
10				3	600	1700	4600
11				4	800	1600	4800
12				5	1000	1500	5000
13				6	1200	1400	5200
14				7	1400	1300	5400

CHARTS / Design / VM_Varies / Schema

FIGURE 5

Worksheet VM_VARIES showing the formula used to calculate R_{1MAX}

Figure 5 shows the worksheet VM_VARIES. This worksheet is easily formed using the DESIGN worksheet: we simply highlight the cells F15:G18 in Figure 4, copy them, and go to a blank worksheet where we use EDIT/PASTE SPECIAL with the TRANSPOSE box checked. That pastes the columns as rows on the new worksheet. The implementation of EQ. 13 using an IF-statement as shown in Figure 5 and Figure 4 serves to check for negative values of R_1, and inserts #N/A (not available) if a negative value would result. Negative values should not happen if we keep $V_M \leq 20$ V, a requirement of EQ. 12 already discussed.

Worksheet VM_VARIES does all the graphical calculations. V_M is a column variable, and for each value in the V_M column the resistor value R_2 is found, and the limiting values on R_1 and R_3.

Worksheet SCHEMATIC

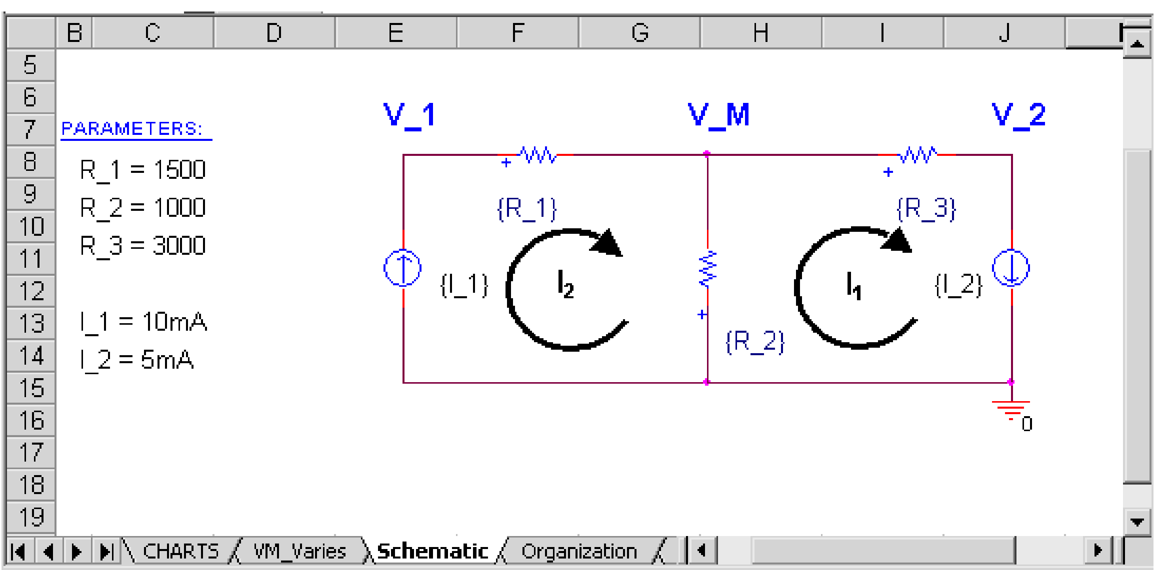

FIGURE 6

The SCHEMATIC worksheet

Figure 6 shows the SCHEMATIC worksheet, which is useful to remind you what the variables in the spreadsheet mean.

Worksheet PSPICE CHECK

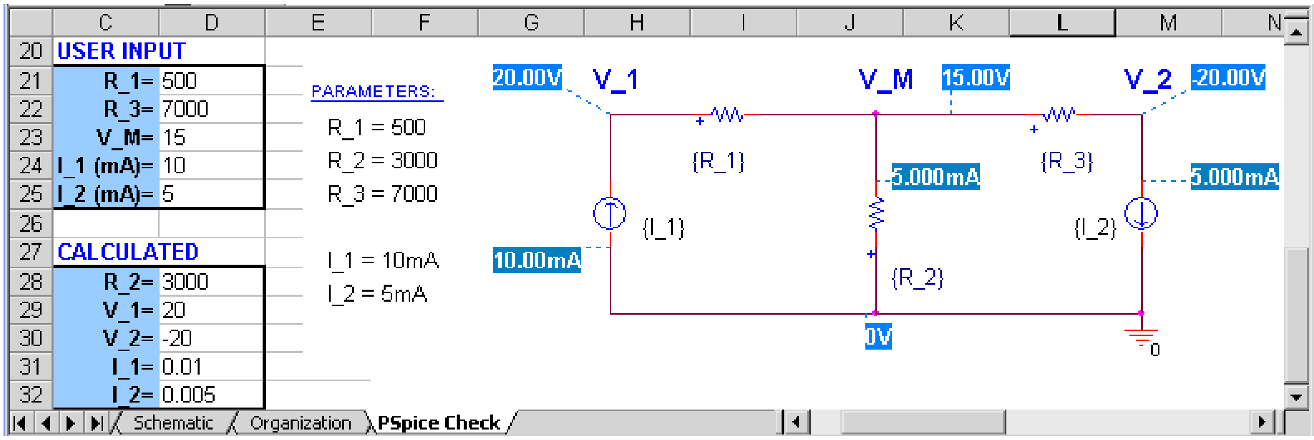

FIGURE 7

Worksheet PSPICE CHECK showing a particular spot check of a design point where V_1 and V_2 are at their limiting values

Figure 7 shows the worksheet PSPICE Check where spot checks of the spreadsheet using PSPICE are pasted. The value of this sheet is to remind the user just what has been checked, so a new user knows whether their application of the spreadsheet is new territory or just routine. Of course, a verbal listing of checks performed might be useful too.

Verification of the spreadsheet

A spot check of the spreadsheet is accomplished by selecting a value of V_M and putting the spreadsheet values for R_1, R_2 and R_3 into PSPICE. An example is shown in the worksheet of Figure 8 below and its PSPICE verification in Figure 9.

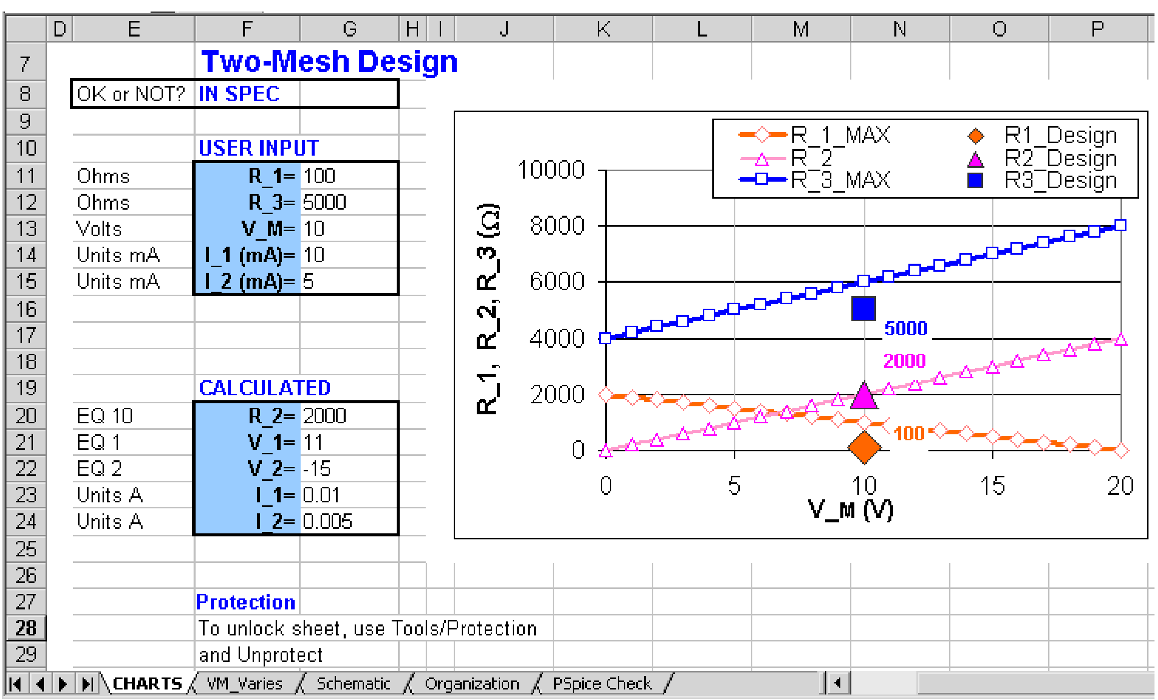

FIGURE 8

A random example in which R_1 lies below its upper bound (R_1_MAX) and R_3 lies below its upper bound (R_3_MAX); R_2 is calculated, and lies on the calculated curve R_2

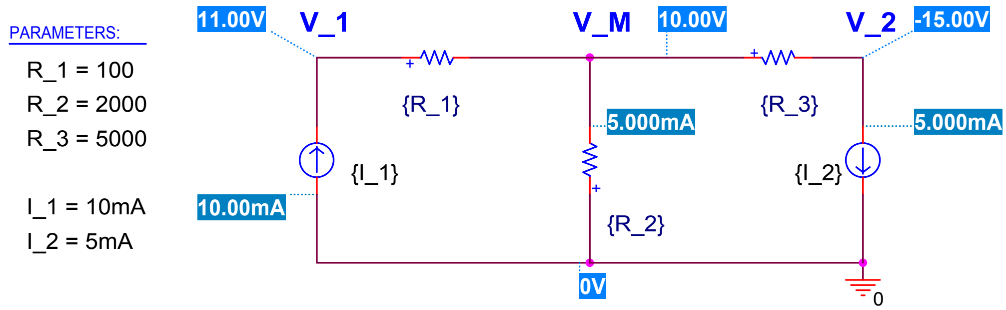

FIGURE 9

PSPICE verification of spreadsheet results in Figure 8

Verification of design boundaries

A useful test of the spreadsheet (and also the analysis behind the spreadsheet) is to check points on the boundary. On the boundary V_1 or V_2 or both should be at their limiting values because their limits are what set the boundaries in the first place. An example of a design point on the boundary is shown in Figure 10 below.

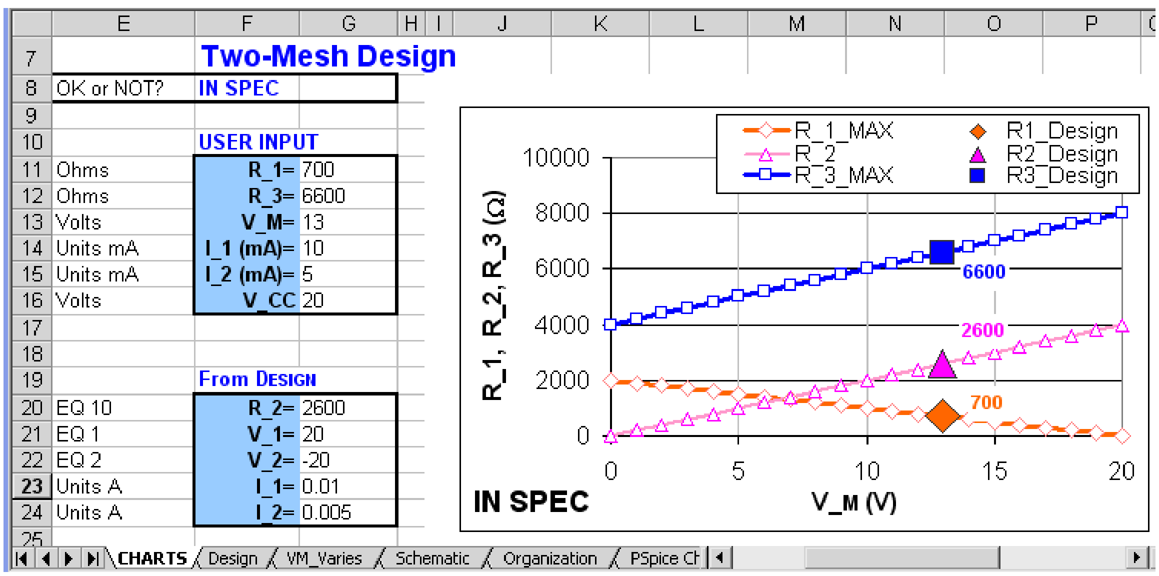

FIGURE 10

An example design point where R_1 and R_3 both are at their limiting boundaries, and the spreadsheet predicts V_1 = 20 V and V_2 = –20 V, which are limiting values for the voltages

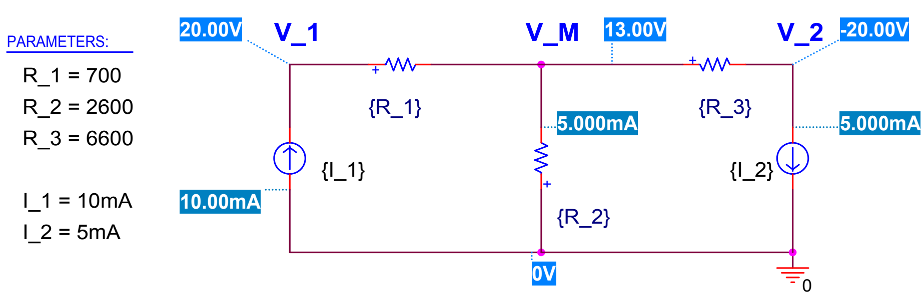

FIGURE 11

PSPICE verification of the design point of Figure 10

Figure 11 shows that PSPICE agrees with the spreadsheet predictions for V_1 and V_2. Of course, PSPICE input was R_2, not V_M, so another check is that V_M from PSPICE is the value V_M = 13V used in the spreadsheet.

Bells and whistles

EXCEL allows us to implement bells and whistles. These conveniences make sense only if the spreadsheet is used a lot, and they aren't worthwhile for a single design. Here's an example, which introduces features more valuable in complex designs.

The spreadsheet has a check feature to warn the user whether the design is in spec. This warning is in cell F8 on CHARTS, and is implemented using an IF-statement as shown below:

=IF(OR(ABS(V_1)>20,ABS(V_2)>20),"OUT OF SPEC","IN SPEC")

The IF statement evaluates the OR condition to see whether the voltages are out of spec. If they are, it prints the value **OUT OF SPEC**. Otherwise it prints the value **IN SPEC**. To emphasize the violation of specifications, CONDITIONAL FORMATTING is used to change the font color depending on whether the design is in or out of specification. CONDITIONAL FORMATTING is implemented by highlighting the cell F8 and selecting the menu FORMAT/CONDITIONAL FORMATTING. The menu is filled out as shown in Figure 12.

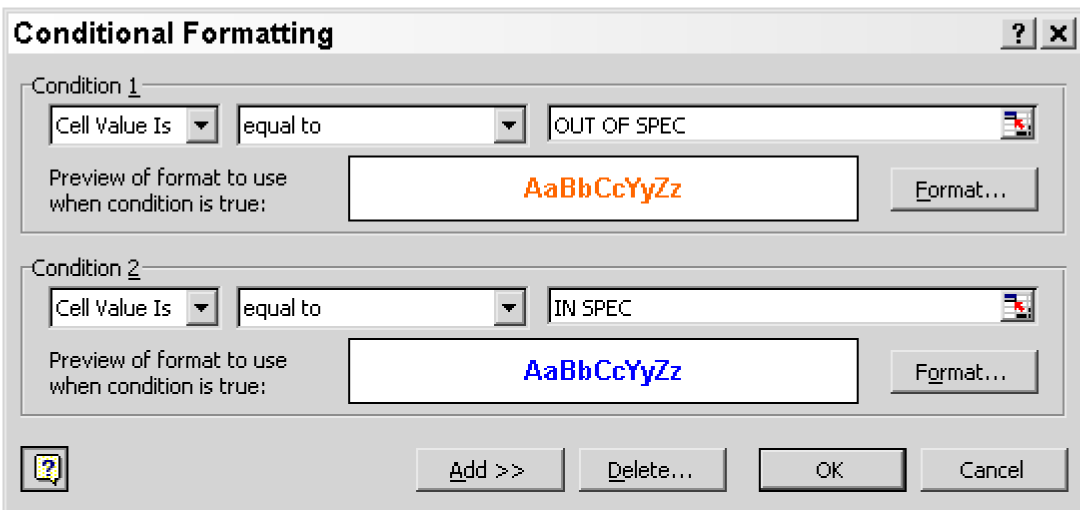

FIGURE 12

Conditional formatting menu for making the font red for a violation of specification, and blue for within specification; this is how the menu is filled out: once OK is clicked, EXCEL will implement the entries as text strings with quotation marks

To add to the warning, a label is added to the chart when a violation occurs. An example is shown in Figure 13 below.

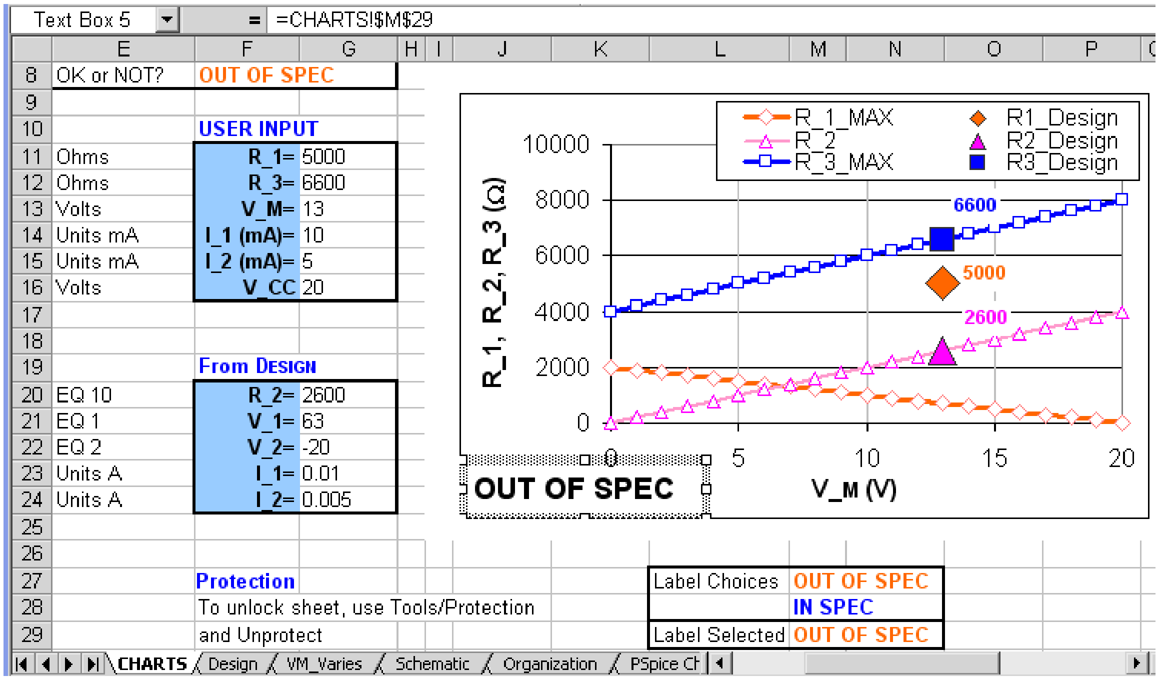

FIGURE 13

Warning labels when design is out of specification; V_1 = 63V, which exceeds the 20 V specification; the label at lower left of the chart is the content of cell M29

Highlighting the chart and hitting the equals sign adds the chart label. This procedure introduces a text box inside the chart, and right after hitting the equals sign, you can make the text box contain whatever you enter in the formula box, for example the content of cell F8.

Discussion

In looking at this problem one sees right away that there are many possible solutions. This latitude allows opportunity to satisfy other conditions that haven't been specified yet, for example, cost requirements or further performance goals.

Besides a variety of design solutions, there are a variety of solution *approaches*. The approach above uses an extraneous variable V_M. We call such a variable a *design* variable, to distinguish it from a circuit component value, or *circuit* variable. This variable is not necessary: one could work directly with the resistor values. In particular, one could use R_2 as a running variable instead of V_M and do the entire design using R_2. According to EQ. 10 these two variables are entirely equivalent mathematically.

The reason for introducing V_M is *transparency*. We hope that using V_M makes the design more understandable, which will help us think about the design choices, be more creative, and avoid errors. We have seen that easy interpretation of V_M makes it clear why the boundary on R_3 increases with V_M: bigger V_M → bigger voltage drop across R_3, which requires a larger R_3 to keep I_2 = 10 mA. Likewise, we know why the bound on R_1 drops with increased V_M: larger V_M → smaller drop across R_1, which requires a smaller R_1 to keep I_1 = 5 mA. And we also understand why R_2 increases with V_M: larger V_M → bigger drop across R_2, which means R_2 must increase to keep its current (I_1–I_2) at 5 mA.

It is helpful to check the design curves of the spreadsheet this way to see that they do make sense and no gross error has crept into our analysis. This understanding also shows the necessity of why things behave as they do, for example, why R_3 and R_2 must go upward when R_1 goes downward.

If a formulation seems perplexing or opaque, not *intuitive*, rethinking the choice of variables can lead to transparency. The best variables are not always the values of circuit components, but variables that convey our concept of how the circuit works. That is, *design* variables that guide our thought may be related only indirectly to the *circuit* variables that specify what circuit to build.

Design variables are inventions related to concept. There is no cut-and-dried approach to concept, but the spreadsheet can help. Spreadsheet organization is tied to concept. For example, spreadsheet layout depends on what variables are selected, and which are elected independent, which dependent. Also, the spreadsheet displays trade-off curves. Explaining these curves forces us to think, learn more, and perhaps change our approach. The spreadsheet is a partner in our thinking, and the design can take new turns as our minds and our spreadsheet evolves.

Summary of solution to design problem

Finally, we summarize the solution to the design problem:

> The specifications that $\mathbf{I_1}$ = 10mA and $\mathbf{I_2}$ = 5mA for input and output voltages less than 20V in magnitude are satisfied for values of R_1 and R_3, that satisfy inequalities EQ. 13 for R_1 and EQ. 16 for R_3 at the selected value of V_M. That is, there is a range of allowed values for R_1 found below the lowest downward sloping curve in Figure 10, and for R_3 found below the highest upward sloping curve in Figure 10. For a solution, R_2 must satisfy EQ. 10, also shown in Figure 10. For any selected R_1, R_2 and R_3, the voltages are reported in the CALCULATED box, as determined by EQ. 1 and EQ. 2.

Exercises

Below are other spreadsheets related to this problem. You'll see spreadsheets affect your thinking and vice versa. In these problems, the "presented" spreadsheet means the spreadsheet just discussed.

1. Redesign the presented spreadsheet so it uses R_2 as a variable and R2_VARIES as calculation worksheet, and does not use the variable V_M or worksheet VM_VARIES at all. CHARTS should show a chart for the allowed values for R_1 and R_3 similar to the presented spreadsheet, but using R_2 as *x*-axis.
2. Redesign the presented spreadsheet so it uses R_1 and R_3 as variables and R1_VARIES and R3_VARIES as calculation worksheets, and does not use the variable V_M or worksheet VM_VARIES at all. CHARTS should show a chart for the allowed values for R_2 for any choice of R_1 and R_3.
3. Redesign the presented spreadsheet to guide selection of R_1, R_2 and R_3 to meet a specification for the power consumed by the resistors. The voltages V_1 and V_2 and the values of I_1 and I_2 still must satisfy the original problem requirements.
4. Redesign the presented spreadsheet so it will solve the design problem for any positive values for I_1 and I_2 input on CHARTS, regardless of which of I_1 and I_2 is larger. Assume the currents are in the range from 0 to 10 mA.
5. Redesign the presented spreadsheet so it will solve the design problem for any values for I_1 and I_2 input on CHARTS, positive or negative. Assume the currents are in the range from –10 mA to 10 mA.
6. Redesign the presented spreadsheet so it will explore the design problem as a <u>function</u> of positive values for I_1 and I_2, regardless of which of I_1 and I_2 is larger. The new spreadsheet will contain additional worksheets I1_VARIES and I2_VARIES, and place charts on CHARTS to show how R_1, R_2, and R_3 behave as these currents are changed. Assume the currents are in the range from 0 to 10 mA.