ECE 304 Spring '06 Exam 3 Solutions

For all problems take the thermal voltage as V_{TH} = 25.864 mV.

Problem 1: Resistor-biased class AB amplifier

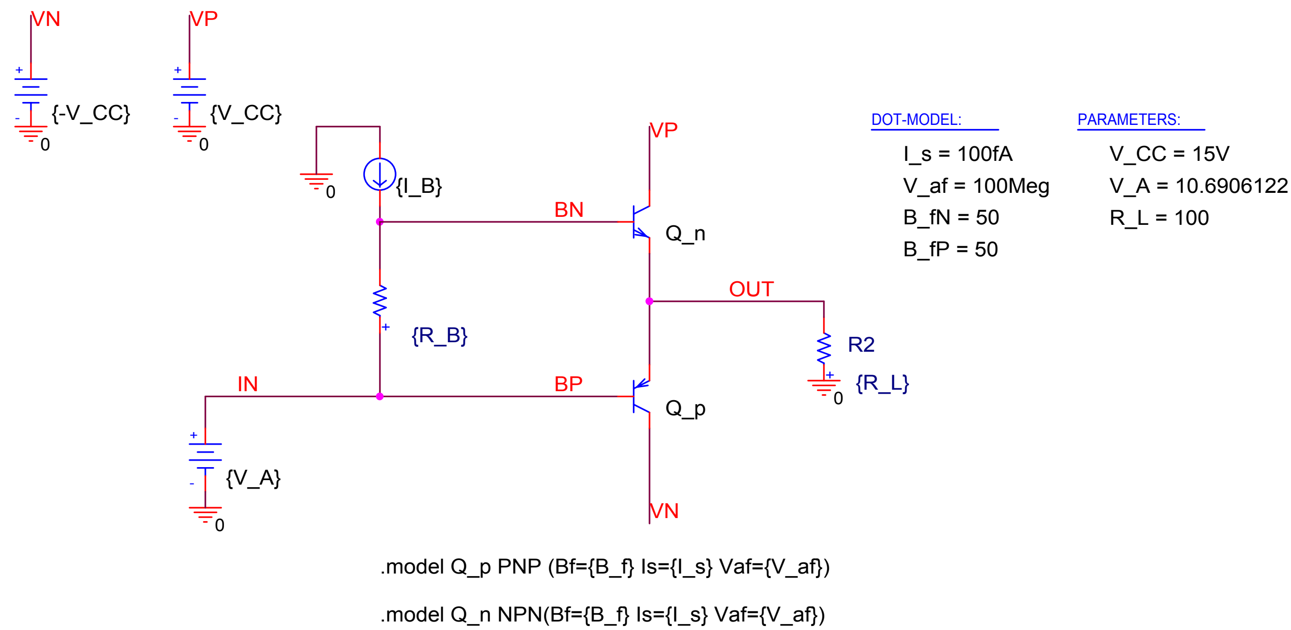

FIGURE 1

Class AB amplifier for Problem 1; notice that Early voltage is very high

1. Select the bias current I_B and the bias resistor R_B so the amplifier meets these specifications:
   - The amplifier can drive the load to an output voltage of V_O = 10 V
   - The amplifier has an emitter current of I_Q = 971.5 μA at V_O = 0 V

Assume V_A always is below V_{BN}, and a minimum current in R_B of I_{RB}(min) =40 μA.

Answer: I_B = 2 mA, R_B = 600 Ω.

Outline: From the high voltage output case with the PNP in cutoff, we find $I_B = I_{RB} + V_O/[(\beta_N+1)R_L]$ = 2 mA. From the zero output voltage case we find $V_{BB} = 2\ V_{BE}(NPN)$ = 1.188 V, and $R_B = V_{BB}/[I_B - I_Q/(\beta_N+1)]$ = 600 Ω.

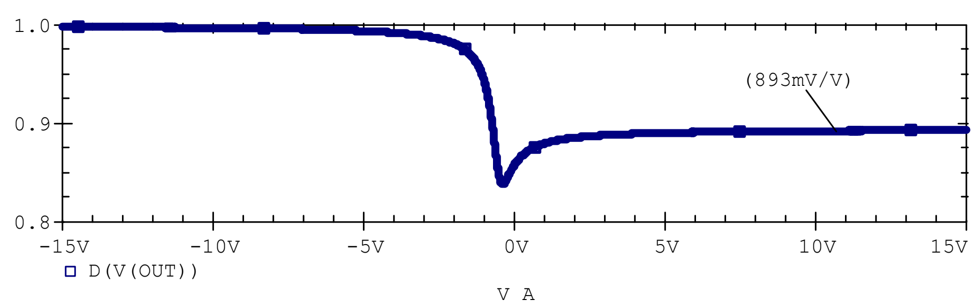

FIGURE 2

Gain plot of amplifier

2. Discuss the minimum and the lower gain at large voltages in the gain plot of Figure 2: what are the origins of these features? What aspects of the circuit control these features?

Answer: The minimum in the gain is due to output transistors being near cutoff when V_O is near zero volts. When transistors are near cutoff, their resistance is high, and gain is therefore low. This effect is controlled by adjustment of I_Q with larger I_Q meaning transistors further away from cutoff and so a better gain. The lower gain at V_O near its maximum is due to the voltage drop across R_B, which affects the gain at large positive voltages, but not at large negative voltages. The gain at high voltages can be improved by reducing R_B.

3. Determine the small-signal gain of the amplifier at V_O = 10 V and compare to Figure 2.

Answer: Gain = 0.893 V/V in complete agreement with Figure 2.

Outline: At V_O = 10 V, the PNP is cutoff, and the small-signal circuit is simply that of a voltage follower with R_L in the emitter branch. The gain of this circuit is

$$\frac{V_O}{V_S} = \frac{1}{1 + \frac{R_B + r_{\pi N}}{\beta_N + 1} \cdot \frac{1}{R_L}}.$$

We evaluate I_C(NPN) as

$$I_C(PNP) = \frac{V_{O\,max} / R_L}{(1 + 1/\beta_N)} = 98 \text{ mA},$$

which leads to $r_{\pi N}$ = 13.19 Ω. Substituting in the gain expression, A_υ = 0.893 V/V.

4. If R_B is increased, discuss the effect on efficiency η (useful power/input power)

Answer: Decrease of R_B increases the width of the dead zone, bringing the amplifier closer to Class B operation and increasing the efficiency.

5. If R_B is increased, discuss the effects on distortion

Answer: Increase in R_B decreases the width of the dead zone, causing less crossover distortion near zero output voltage. However, increase in R_B decreases the gain at large V_O, so the upswing of the output is decreased compared to the downswing, increasing distortion.

Problem 2: Class AB amplifier

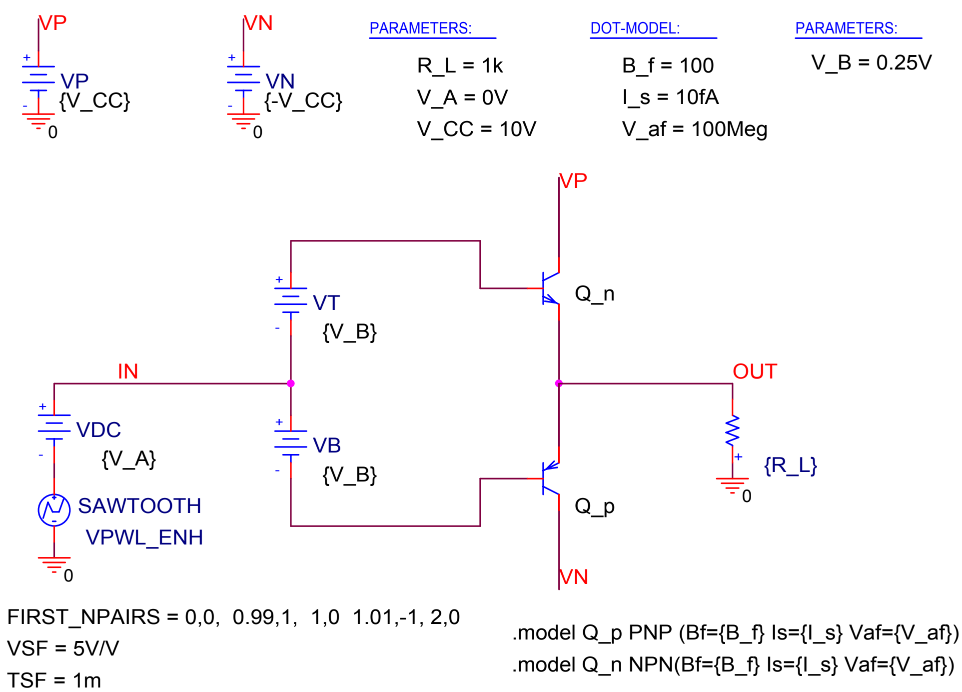

FIGURE 3

Idealized class AB amplifier with battery control of efficiency through V_B

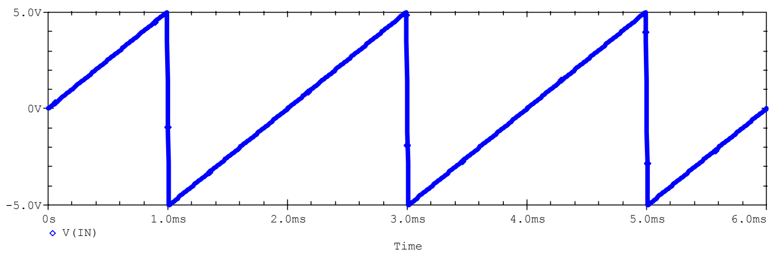

FIGURE 4

Saw tooth input voltage to class AB amplifier; saw tooth runs from – 5V to +5 V in a period of T= 2 ms

For the amplifier of Figure 3, do the following:

1. When V_A = 5V, find V_{OUT} and V_{BE} (NPN)

Answer: V_O = 4.556 V and V_{BE}(NPN) = 0.694 V.

Outline: The output voltage is $V_O = V_A+V_B-V_{BEN}$. However, we do not know V_{BEN} exactly, because the current in the transistor depends on V_O/R_L. Therefore, we do an iterative solution. One way to iterate is to assume V_{BE} = 700 mV, find V_O, determine the current, and get a new estimate of V_{BE} from

$$V_{BE} = V_{TH}\ell n\left(\frac{V_A + V_B - V_{BEN}}{(1+1/\beta_N)R_L I_{SN}}\right),$$

where V_{BEN} inside the logarithm is the guessed value and V_{BE} on the left of the equation is the updated next guess. Once we have a solution for V_{BE}, we also have V_O from the above equation, namely, $V_O = V_A+V_B-V_{BEN}$

2. Sketch the DC voltage transfer curve V_{OUT} vs. V_{IN} and provide numerical coordinates for points of interest.

Answer: See below.

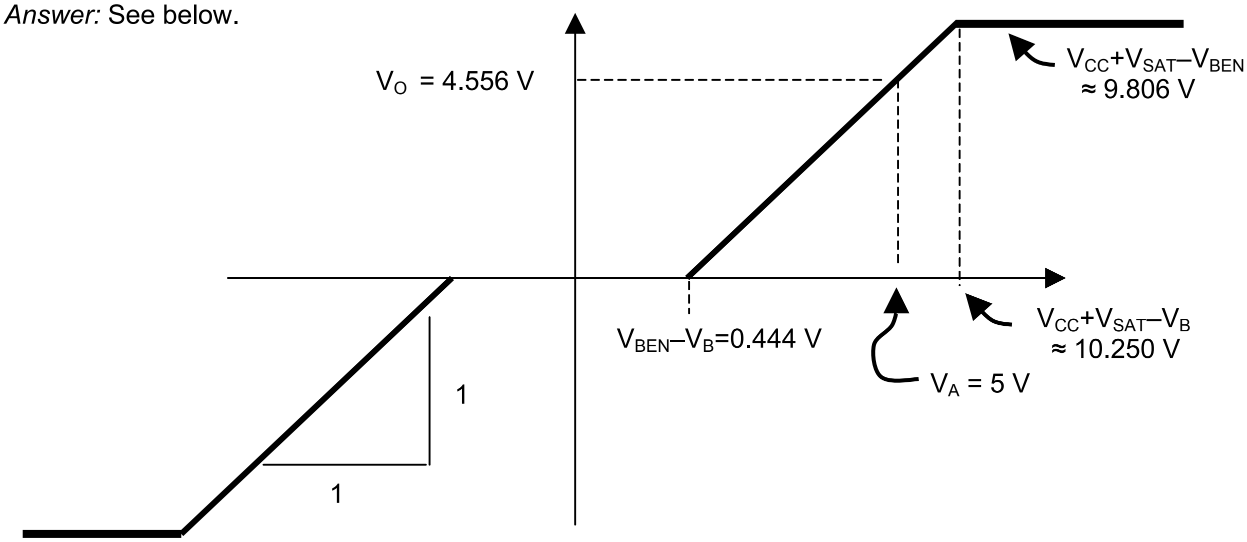

FIGURE 5

Approximate transfer curve V_{OUT} vs. V_{IN} assuming an approximate gain of 1 V/V; saturation voltage V_{SAT} is not known numerically, but probably is about 500 mV

3. What fraction of the period T is Q1 (or Q2) on? (A <u>fraction</u> is requested, not a time).

Answer: The transistors are on for a time, say τ, given as a fraction of a cycle by

$$\frac{\tau}{T} = \left(\frac{V_A + V_B - V_{BEN}}{2V_A}\right), \text{ or } 45.56\% \text{ of a cycle.}$$

Outline:
Assuming the gain is approximately one, we use similar triangles. The input saw tooth and the output saw tooth shapes are the same triangles except for altitude, so equating the tangents of the angle at the base we find

$$\frac{V_A + V_B - V_{BEN}}{\frac{T}{2} - t_0} = \frac{V_A}{\frac{T}{2}},$$

where t_0 is the time at which the output begins to increase from zero volts. We rearrange to solve for the ratio $(T/2-t_0)/T = \tau/T$, obtaining the formula in the answer.

4. Sketch the emitter currents in Q1 and Q2 vs. time on a combined plot using as time variable the ratio (time/T), where T=period, and provide numerical coordinates for key points. Indicate on these plots the currents at $V_O = 0$ V (current I_Q).

Answer:
First, we find the current at $V_O = 0$ V from the diode law as

$$I_Q = I_S\left(e^{V_B/V_{TH}} - 1\right)\left(1 + 1/\beta_{fN}\right) = 159.3 \text{ pA}$$

Then we find the maximum emitter current as

$$I_{max} = \frac{V_A + V_B - V_{BEN}}{R_L} = \frac{4.556}{1000} = 4.556 \text{ mA}$$

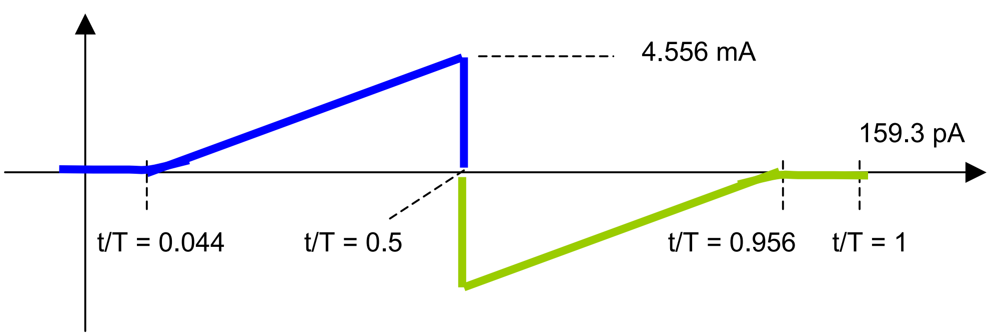

FIGURE 6
Emitter current vs. time in fractions of a period T

5. Find the efficiency η (useful power/input power) as a percentage.

Answer: The efficiency is

$$\eta = \frac{2}{3}\left(1 + \frac{1}{\beta}\right)\left(\frac{V_A + V_B - V_{BEN}}{V_{CC}}\right) = 30.7 \text{ \%}$$

Outline:
The instantaneous useful power $p_{out}(t)$ is

$$p_{out}(t) = \frac{\upsilon_{OUT}^2(t)}{R_L}.$$

The output voltage as a function of time is given by

$$\upsilon_{OUT}(t) = V_O\left(\frac{t - t_0}{\frac{T}{2} - t_0}\right),$$

as may be checked by setting $t=t_0$, where $\upsilon_{OUT} = 0$, and $t=T/2$, where $\upsilon_{OUT} = V_O$, the maximum output voltage. This relation applies only in the time interval $t_0 \le t \le T/2$. In the interval $T/2 \le t \le T-t_0$, the output voltage $\upsilon_{OUT}(t)$ flips sign and it is zero in the rest of the cycle. The average output power is then

$$P_{OUT} = \frac{2}{T}\int_{t_0}^{T/2} dt \left(\frac{V_O^2}{R_L}\right)\left(\frac{t-t_0}{\frac{T}{2}-t_0}\right)^2 .$$

We can simplify this integral using the substitutions

$$x = \left(\frac{t-t_0}{\frac{T}{2}-t_0}\right) \text{ and } dx = \left(\frac{dt}{\frac{T}{2}-t_0}\right).$$

The variable x ranges from 0 to 1 as t varies from t_0 to T/2. Therefore, the integral becomes

$$P_{OUT} = \frac{2\left(\frac{T}{2}-t_0\right)}{T}\int_0^1 dx \left(\frac{V_O^2}{R_L}\right)(x)^2 = \frac{2}{3}\frac{V_O^2}{R_L}\left(\frac{1}{2}-\frac{t_0}{T}\right).$$

The instantaneous power input from the top supply voltage $p_{in}(t)$ is

$$p_{in}(t) = \iota_C(t)V_{CC} = \frac{\upsilon_{OUT}(t)}{R_L(1+1/\beta)}V_{CC} .$$

leading to an average power input from both top and bottom supply voltages of

$$P_{IN} = \frac{2}{T}\int_{t_0}^{T/2} dt \left(\frac{V_O V_{CC}}{(1+1/\beta)R_L}\right)\left(\frac{t-t_0}{\frac{T}{2}-t_0}\right) = \frac{2\left(\frac{T}{2}-t_0\right)}{T}\int_0^1 dx \left(\frac{V_O V_{CC}}{(1+1/\beta)R_L}\right)(x) = \frac{V_O V_{CC}}{(1+1/\beta)R_L}\left(\frac{1}{2}-\frac{t_0}{T}\right).$$

Taking the ratio of average useful output power to average input power we find the efficiency η to be

$$\eta = \frac{P_{OUT}}{P_{IN}} = \frac{2}{3}\left((1+1/\beta)\frac{V_O}{V_{CC}}\right) = \frac{2}{3}\left((1+1/\beta)\frac{V_A+V_B-V_{BEN}}{V_{CC}}\right) = 30.7\%$$

6. If V_B increases, what happens to the efficiency? Explain.

Answer: From Part 5, the efficiency increases as V_B increases. That result contradicts the idea that the amplifier becomes more and more Class A as V_B increases. The reason for the improvement in efficiency with V_B is that, in this example, the input voltage is maintained at V_A, while the output voltage actually becomes larger as V_B increases. A more fundamental reason is that our analysis is based on Class B operation, with each transistor off more than half the time.

However, if we increase V_B sufficiently, no transistor ever cuts off, and the analysis of Part 5 becomes inaccurate because it assumes each transistor is off for at least half a period. At large V_B, both transistors are on all the time, the current I_Q is substantial, and each exhibits a current waveform that is a complete saw tooth. DC current is drawn. Efficiency is low in this case.

An example is shown in Figure 7. It shows that at large V_B each device is "on" all the time, contributing a DC power loss as is typical of Class A operation. It also shows that the transition from Class B operation (Part 5 analysis) to Class A operation occurs over a very small range of V_B, so intermediate Class AB operation exists only for a small range of V_B.

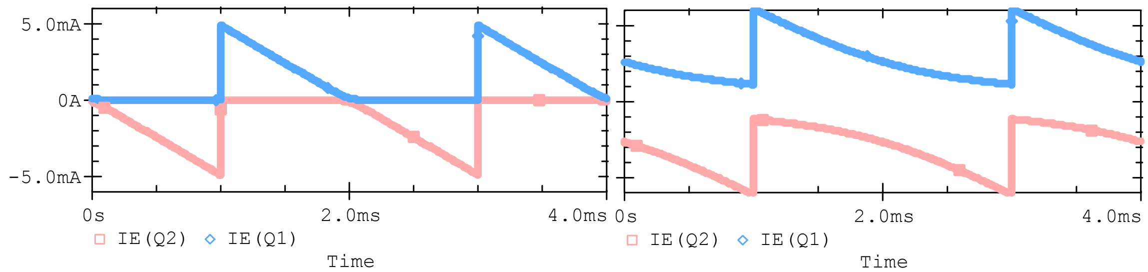

FIGURE 7

Current waveforms for Class B: V_B = 0.6V (left); and Class A: V_B = 0.68 V (right)

Problem 3: V_{BE} Multiplier

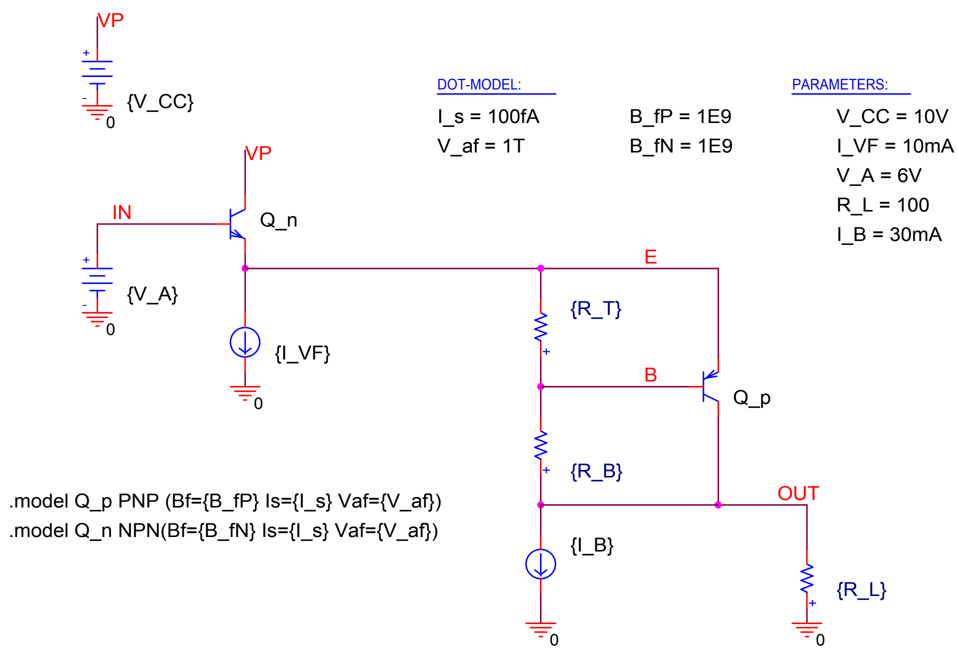

FIGURE 8

Multiplier for Problem 3; DC base currents are negligible

Notice that transistor Early effects and base currents can be neglected.

1. Select values for R_T and R_B so the output voltage is V_{OUT} =0 V when V_A = 2 V. Assume the multiplier current is divided equally between the transistor branch Q_P and the resistor branch made up of R_T and R_B.

Answer: R_T = 44.4 Ω and R_B = 42.9 Ω.

Outline: When V_O = 0, no current flows through the load. Half the current flows through the transistor so $V_{EB} = V_{TH}\ell n[I_B/(2I_s)]$ = 665.5 mV. The current in R_T is also $I_B/2$, so Ohm's law provides $R_T = V_{BE}/(I_B/2)$ = 42.9 Ω.

To find the value of R_B, we need the voltage drop across the multiplier. That drop is the total drop of 2 V, less the drop in the VF. The V_{BE} drop in the VF is $V_{TH}\ell n[(I_B+I_{VF})/I_S]$ = 691.0 mV. Therefore the drop across the multiplier is V_M = 2 – 691mV = 1.309 V and the drop across R_B is 1.309 – V_{EB} = 643.5 mV. Therefore, R_B = 643.5m/(I_B/2) = 42.9 Ω.

2. Find the output voltage when V_A = 6 V. For V_A = 6V, the drop across the multiplier will increase compared to V_A = 2V, affecting the voltage follower.

Answer: V_O = 3.92 V.

Outline: The current in the VF depends on the output voltage because the follower must supply the load current V_O/R_L. But the output voltage is not known. Therefore, an iterative solution is necessary. The iteration algorithm I used is listed below.

We begin by assuming the multiplier drop is still 2V, as in Part 1. So the first guess is V_O = 4 V, leading to a load current of V_O/R_L = 40 mA.

The iteration proceeds as follows:

Guess V_O (begin with V_O = 4 V)

1. Find load current V_O/R_L

Find $V_{BE} = V_{TH}\ell n\{(I_{VF}+I_B+V_O/R_L)/[(1+1/\beta)I_S]\}$ for NPN

Guess V_{EB} for PNP. Start with V_{EB} = 700 mV

Find current in multiplier transistor $I_E = I_B + V_O/R_L - V_{EB}/R_T$

Find new value for $V_{EB} = V_{TH}\ell n\{I_E/[(1+1/\beta)I_S]\}$

Iterate to final V_{EB}.

Find new $V_O = V_A - V_{BE} - V_{EB}(1+R_B/R_T)$

Go back to 1