Closed book, calculators required; Friday, April 14; 8AM-9:50 AM

ECE 304: Exam 5 Solutions Spring '06

NOTE: IN ALL CASES

1. Solve the problem on scratch paper. Then, once you understand your answer, compose your answer sheet as follows:
2. Put your answer first, and
3. Follow your answer with an outline of your solution. Each major step in the outline should
   - 3.1. Begin with a heading that describes the objective of that step, and should
   - 3.2. Have a body where actual work is done, not just hand waving, and should
   - 3.3. Conclude with a quantitative statement of the major result for that step (a number or formula or both).

For all problems take the thermal voltage as V_{TH}= 25.864 mV.

Problem 1: Feedback network design

The open-loop amplifier below is to be hooked up as a closed-loop feedback amplifier satisfying these impedance specifications on the closed-loop input resistance R_{IF} and output resistance R_{OF}.

EQ. 1

$$R_{IF} \geq R_{IS} = 10\ k\Omega \qquad R_{OF} \leq R_{OS} = 1\ \Omega$$

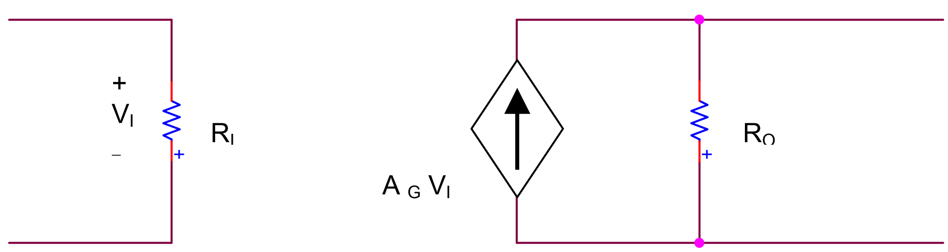

FIGURE 1

Open-loop amplifier; R_I = 1 kΩ; R_O = 10 Ω; gain is A_G = 1 kA/V

1. Sketch the ideal feedback amplifier

Answer:

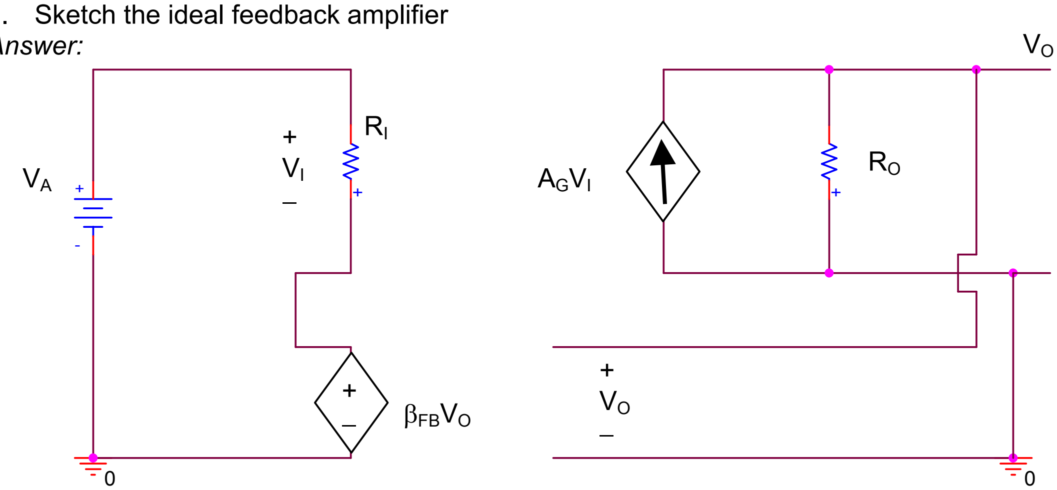

FIGURE 2

Ideal feedback circuit

Outline: Input resistance increases → series connection → V input

Output resistance decreases → shunt connection → V output

Amplifier gain is V/V → dimensions of β_{FB} = V/V → feedback is provided by VCVS

2. What is the β_{FB} (with units) for the ideal feedback network (no feedback resistors)? Explain how you find it in your outline.

Answer: $\beta_{FB} = 9 \times 10^{-4}$ V/V

Outline: Turning off the feedback in Figure 2, we find the loaded voltage gain is

EQ. 2

$$\frac{V_O}{V_A} = A_G R_O$$

Therefore, the performance factor is

EQ. 3

$$PF = 1 + \beta_{FB} A_G R_O \,.$$

The input resistance of the feedback amplifier for a series input is then

EQ. 4

$$R_{IS} = PF \circ R_I = (1 + \beta_{FB} A_G R_O) R_I \,,$$

while the output resistance for shunt connection is

EQ. 5

$$R_{OS} = \frac{R_O}{PF} = \frac{R_O}{(1 + \beta_{FB} A_G R_O)} \,.$$

Solving EQ. 4 and EQ. 5 for the feedback factor β_{FB} we find:

EQ. 6

$$\beta_{FB} \geq \left(\frac{R_{IS}}{R_I} - 1\right)\frac{1}{A_G R_O} \text{ and } \beta_{FB} \geq \left(\frac{R_O}{R_{OS}} - 1\right)\frac{1}{A_G R_O} \,,$$

both of which provide the requirement $\beta_{FB} \geq 9 \times 10^{-4}$ V/V.

3. Design the T-section of resistors in Figure 3 that provides an amplifier satisfying the specs. Sketch the feedback circuit and derive the equations for R_A and R_B. Choose R_A and R_B for the highest gain of the loaded amplifier consistent with the specifications of EQ. 1.

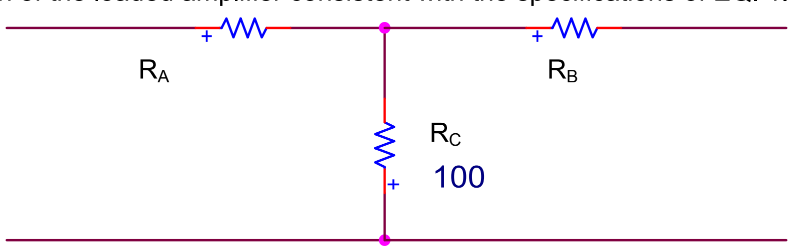

FIGURE 3

T-section of resistors for feedback network; the center resistor has value R_C = 100 Ω

Answer:

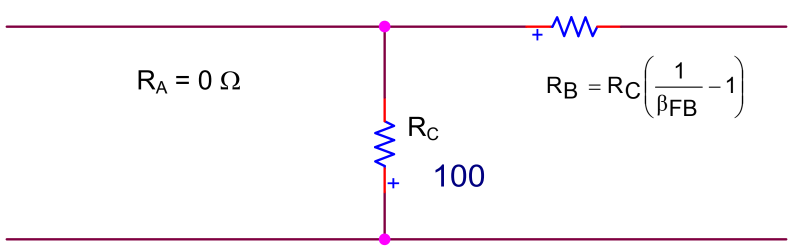

FIGURE 4

T-section for feedback with β_{FB} as feedback factor

Outline: The appropriate two-port has a VCVS on the feedback side. Therefore, the independent variable on the input side is current, and that on the output side is voltage. Therefore, the two port is as shown in Figure 5 below.

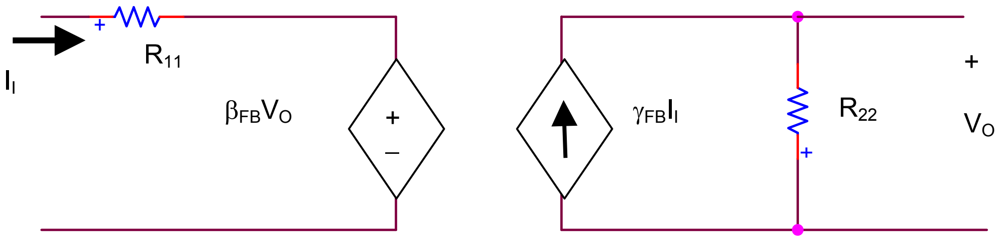

FIGURE 5
Two-port appropriate for feedback using a VCVS

Solving for the two-port parameters we find

EQ. 7

$$\beta_{FB} = \gamma_{FB} = \frac{R_C}{R_B + R_C} = \frac{1}{1 + \frac{R_B}{R_C}},$$

which depends only on the ratio R_B/R_C and is independent of R_A. Consequently we can choose $R_A = 0\ \Omega$, which reduces the loading of the loaded gain due to the feedback network.
As R_C is given, for a selected value of β_{FB} EQ. 7 determines R_B as

EQ. 8

$$R_B = R_C\left(\frac{1}{\beta_{FB}} - 1\right)$$

4. Beginning with the value of β_{FB} from Part 2, describe how you will iterate numerically and do at least one iteration in detail to show how it goes.

Answer: We set up an iteration procedure as follows:

(i) Guess a value for $\beta_{FB.}$ (Start with β_{FB} from Part 2)
(ii) Find R_B using EQ. 8
(iii) Find next value for β_{FB} using the performance factor PF, that is,
(iv) $$\beta_{FB} = \frac{PF - 1}{A_G(R_O // R_{22})\frac{R_I}{R_I + R_{11}}} = \frac{9}{A_G(R_O // (R_B + R_C))\frac{R_I}{R_I + (R_B // R_C)}}$$
(v) Go back to step (i)

Outline:
To find the value of R_B, we find the loaded gain and the performance factor as a function of R_B. Then we implement the required input or output resistance using this performance factor and select R_B to meet the specs. First we find the loaded gain.
Using the two-port we find the resistances to be

EQ. 9

$$R_{11} = R_A + R_B // R_C = R_B // R_C \text{ (recall } R_A = 0\ \Omega\text{) and } R_{22} = R_B + R_C.$$

The loaded gain is found using Figure 6 below.

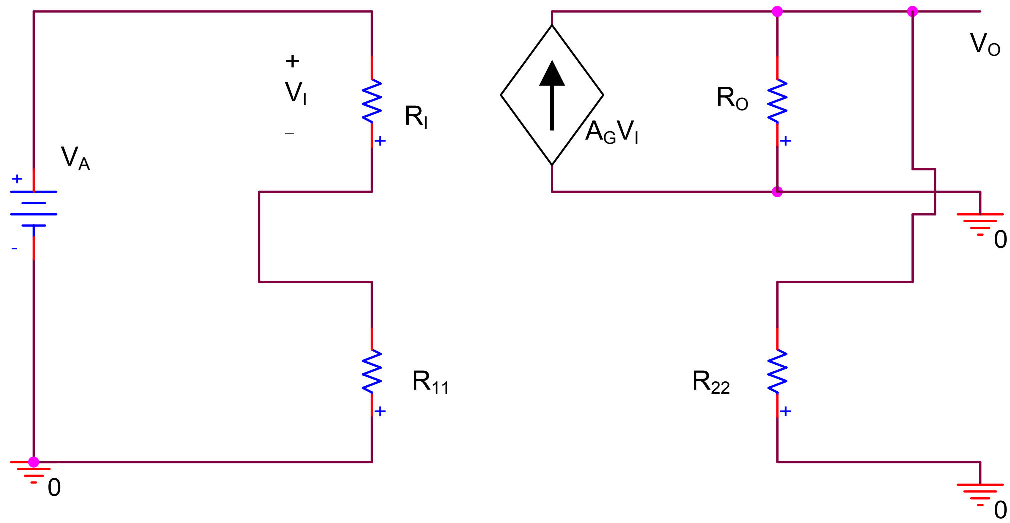

FIGURE 6

Circuit for finding loaded gain with dependent sources turned off

The loaded gain is

EQ. 10

$$\frac{V_O}{V_A} = A_G (R_O // R_{22}) \frac{R_I}{R_I + R_{11}},$$

and the performance factor is

EQ. 11

$$PF = 1 + \beta_{FB} A_G (R_O // R_{22}) \frac{R_I}{R_I + R_{11}}.$$

From EQ. 4 and EQ. 5 we know PF = 10 to satisfy the impedance specs. Therefore, we set up an iteration procedure as follows:

1. Guess a value for $\beta_{FB.}$ (Start with β_{FB} from Part 1)
2. Find R_B using EQ. 8
3. Find next value for β_{FB} using EQ. 11, that is,

$$\beta_{FB} = \frac{PF - 1}{A_G (R_O // R_{22}) \frac{R_I}{R_I + R_{11}}} = \frac{9}{A_G (R_O // (R_B + R_C)) \frac{R_I}{R_I + (R_B // R_C)}}$$

4. Go back to step 1

NUMERICAL WORK

$\beta_{FB} = 9 \times 10^{-4}$ V/V

$R_B = 100\ (10^4/9-1) = 111.11$ kΩ

$$\beta_{FB} = \frac{9}{1\,k \times 10 // 1.111 \times 10^5 \times \frac{1\,k}{1\,k + 99.91}} = 9.9 \times 10^{-4} \text{ V/V}$$

$R_B = 100.91$ kΩ

$R_{11} = R_B//R_C = 99.90$ Ω; $R_{22} = R_B+R_C = 101.01$ kΩ

$$\beta_{FB} = \frac{9}{1\,k \times 10 // 1.0101 \times 10^5 \times \frac{1\,k}{1\,k + 99.90}} = 9.9 \times 10^{-4} \text{ V/V}$$

etc.

Problem 2: Feedback design for linearity

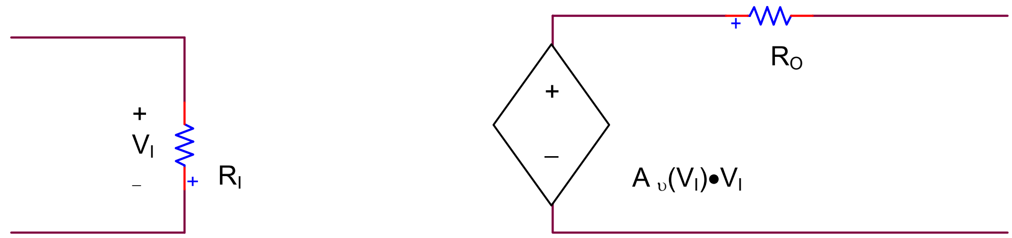

FIGURE 7

Voltage amplifier with nonlinear gain $A_\upsilon(V_I)$

The voltage amplifier of Figure 7 is to be hooked up as a feedback current amplifier that delivers a linear output current I_{OUT} through R_O when driven by a current source I_A at the input. The nonlinear gain of the voltage amplifier is

EQ. 12

$$A_\upsilon(V_I) = A_{\upsilon 0}\frac{1}{1+\dfrac{V_I}{V_A}},$$

with $A_{\upsilon 0}$ = 10 V/V and V_A = 200 mV. The input and output resistances are R_I = 1 kΩ and R_O = 10 Ω.

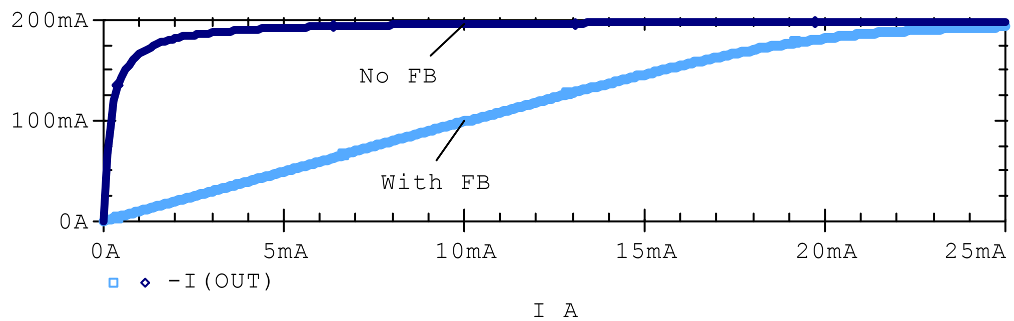

FIGURE 8

Comparison of output current vs. input current for circuit with zero feedback (top) and with feedback (bottom); the transfer characteristic is made more linear by the feedback

Design the feedback amplifier so the current I_{OUT} through R_O vs. input drive current I_A resembles Figure 8 for small loads across the amplifier (for example, a short circuit). For design purposes, the feedback amplifier follows the relation

EQ. 13

$$I_{OUT} = 10 \bullet I_A .$$

1. Sketch your feedback amplifier topology with current driver attached. Use ideal feedback (not using resistors). Show where a load resistor would be attached.

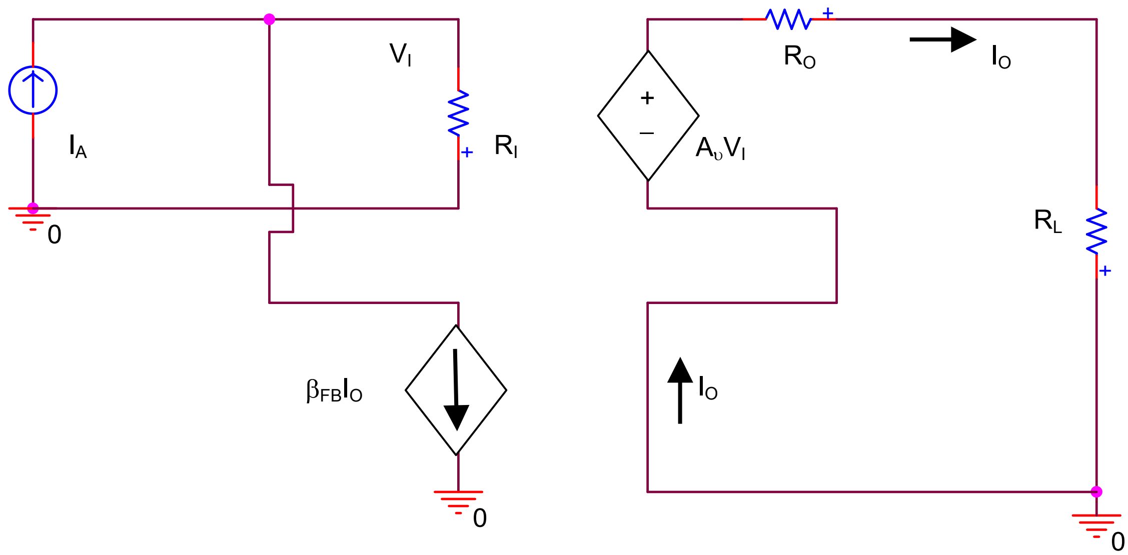

FIGURE 9

Feedback current amplifier made from a voltage amplifier; potential load resistor is R_L

Outline: We want a current amplifier. This amplifier type implies high output resistance (series connection) and low input resistances (shunt connection). The gain of the current amp has dimensions of A/A, so the feedback factor has dimensions of A/A → CCCS.

2. Find the necessary feedback β_{FB}. State value *and* units. Explain your procedure in your outline.

Answer: β_{FB} = 0.099 A/A.

Outline: For a feedback current amplifier the gain is given by

EQ. 14

$$\frac{I_O}{I_A} = \frac{A_{LOADED}}{1+\beta_{FB}A_{LOADED}} .$$

Therefore, the approach is to find A_{LOADED} and solve for β_{FB} using the given result I_O/I_A = 10 A/A. Setting the dependent source to zero, we find $A_{LOADED} = A_\upsilon R_I/R_O$. Solving EQ. 14 for β_{FB} we find for small voltages V_I:

EQ. 15

$$\beta_{FB} = \frac{I_A}{I_O} - \frac{1}{A_{\upsilon 0}R_I/R_O} = \frac{1}{10} - \frac{1}{10\,\frac{1\,k}{10}} = 0.099 \quad A/A$$

3. Find the deviation of your feedback amplifier from EQ. 13 at I_A = 20 mA with short-circuit load

Answer: The current from the feedback amplifier falls 18.2 mA below the ideal 200 mA from EQ. 13, an error of 9.1%

Outline:

According to EQ. 13, the ideal output at I_A = 20 mA is I_O = 200 mA. According to EQ. 14,

EQ. 16

$$\frac{I_O}{I_A} = \frac{A_{LOADED}}{1+\beta_{FB}A_{LOADED}} = \frac{\frac{A_{\upsilon 0}}{1+V_I/V_A}\frac{R_I}{R_O}}{1+\beta_{FB}\frac{A_{\upsilon 0}}{1+V_I/V_A}\frac{R_I}{R_O}} = \left(\frac{A_{\upsilon 0}\frac{R_I}{R_O}}{1+\beta_{FB}A_{\upsilon 0}\frac{R_I}{R_O}}\right)\frac{1}{1+\frac{V_I/V_A}{1+\beta_{FB}A_{\upsilon 0}\frac{R_I}{R_O}}}$$

or, because the leading factor is 10 A/A,

EQ. 17

$$\frac{I_O}{I_A} = \frac{10}{1 + \dfrac{V_I / V_A}{1 + \beta_{FB} A_{\upsilon 0} \dfrac{R_I}{R_O}}} = \frac{10}{1 + \dfrac{V_I / V_A}{PF}} .$$

Voltage V_I is given as

EQ. 18

$$V_I = \left(I_A - \beta_{FB} I_O\right) R_I = I_A R_I \left(1 - \beta_{FB} \frac{10}{1 + \dfrac{V_I / V_A}{PF}} \right) .$$

This equation can be rearranged as a quadratic for V_I, as shown in EQ. 19 below.

EQ. 19

$$V_I^2 + V_I \left(V_A PF - I_A R_I\right) - I_A R_I V_A PF \left(1 - 10\beta_{FB}\right) = 0 .$$

Substituting values, the linear term is zero, leaving

EQ. 20

$$V_I = \sqrt{I_A R_I V_A PF \left(1 - 10\beta_{FB}\right)} = 2 \text{ V} .$$

Then EQ. 17 provides I_O = 181.82 mA, or ΔI_O = 200 mA – 181.82 mA = 18.2 mA, an error of 9.1%.

4. Find the performance factor of your amplifier

Answer: PF = 1 + $\beta_{FB} A_{LOADED}$ = 100

5. Find the input and output resistances of the feedback amplifier

Answer: $R_I(FB) = R_I/PF = 10\ \Omega$; $R_O(FB) = R_O \times PF = 1\ k\Omega$.

Problem 3: High corner-frequency design

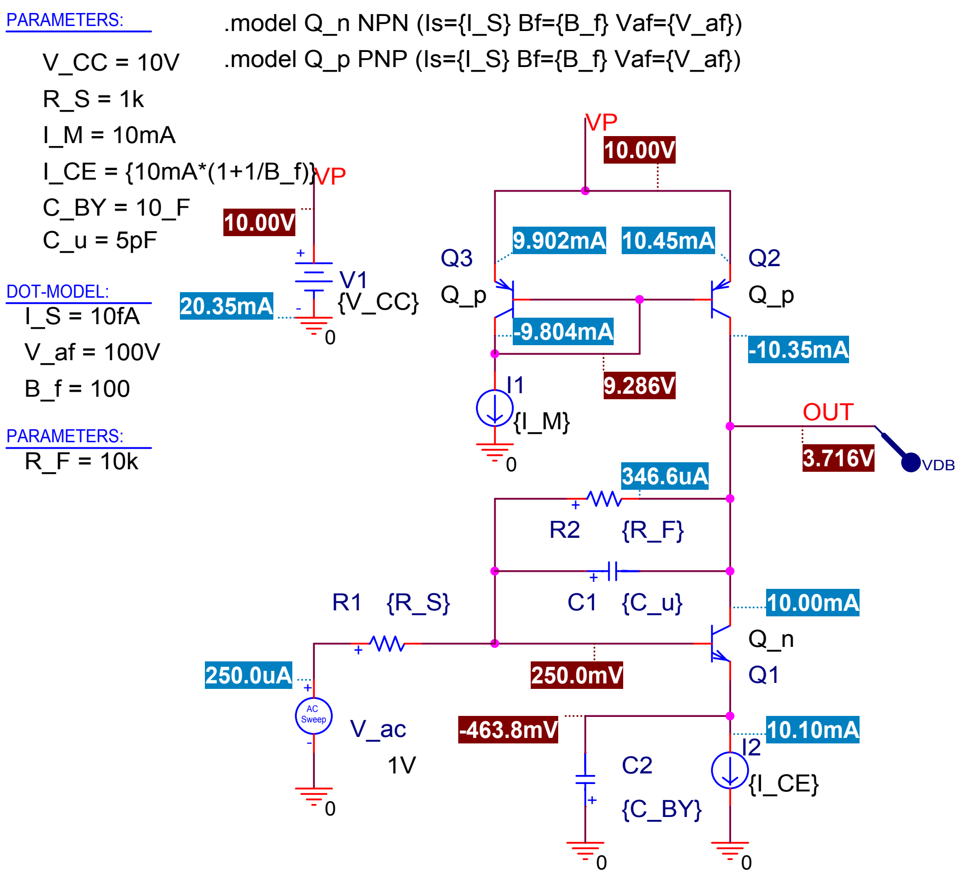

FIGURE 10
Amplifier for Problem 3; notice that the dot-model statements contain no capacitances

In Figure 10 all transistors exhibit Early effect ($r_O \neq \infty$, and β is a function of V_{CB}). None have internal capacitances. Feedback resistor R_F has value R_F = 10 kΩ.

For frequencies at midband and above, sketch the Bode magnitude plot of gain (in dB) vs. frequency for the amplifier with the dependent sources of the feedback *off* and with them *on*, using the same axes. Label the 3dB frequencies and midband gains.
Answer:
The Bode magnitude plot for voltage gain is shown in Figure 11 below.

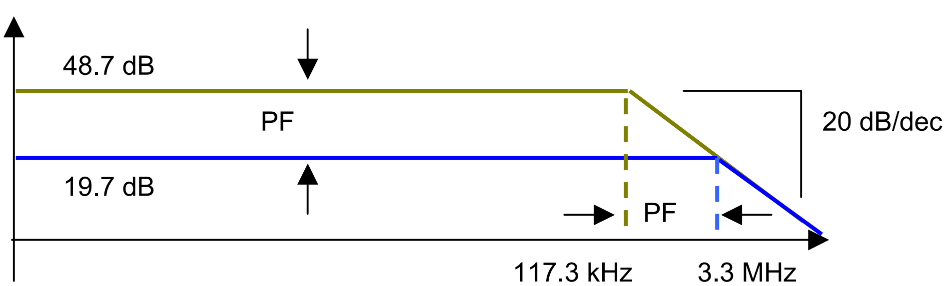

FIGURE 11
Bode magnitude plot for voltage gain

Follow these steps:

1. Make a small-signal *midband* circuit for the amplifier

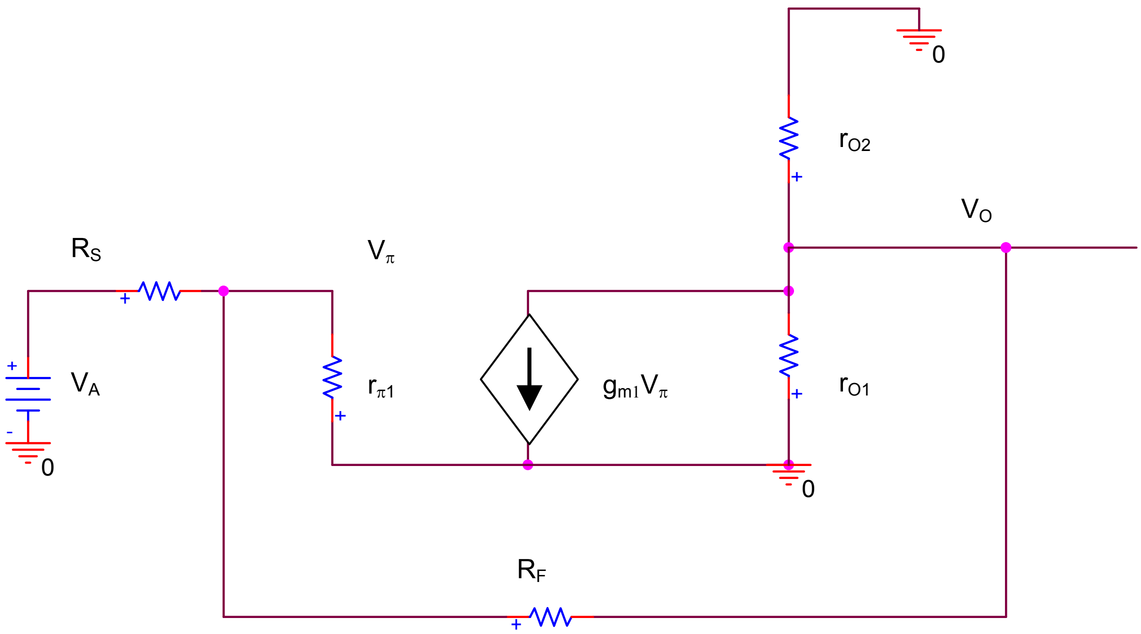

FIGURE 12
Small-signal midband circuit

2. Make a two-port for the feedback due to R_F

Answer:

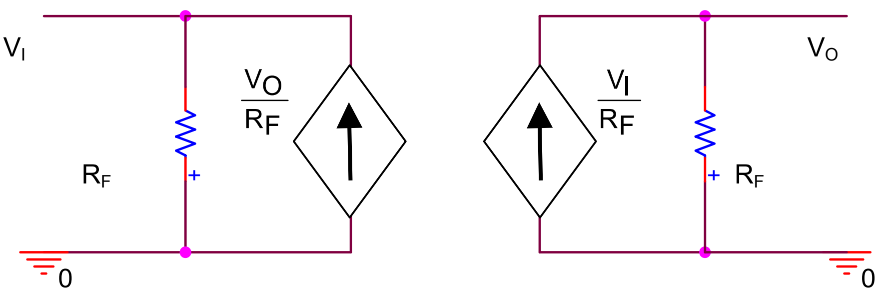

FIGURE 13
Two-port for feedback circuit R_F; $\beta_{FB} = -1/R_F$

Outline: The output connection is shunt → voltage output
Input connection is shunt → current input
Therefore, feedback amplifier is V_{OUT}/I_{IN} = transR; gain has dimensions of Ohms.
Therefore, the feedback factor has dimensions of $1/\Omega$ = I/V → VCCS. Therefore the feedback two-port has the form shown in Figure 14 below.

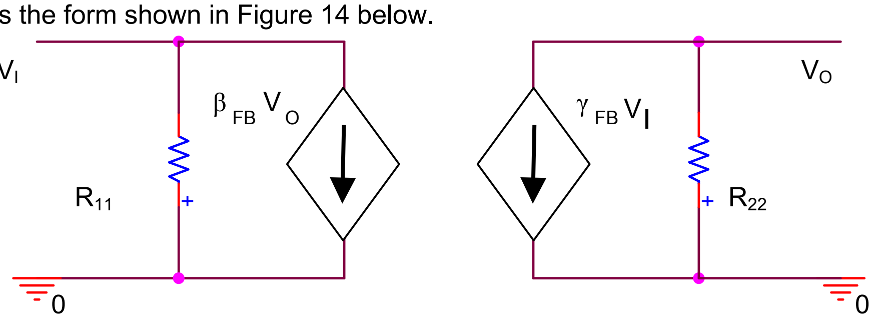

FIGURE 14
Two-port for trans R feedback amplifier

Following the usual procedure with terminations that eliminate one dependent source at a time, we arrive at Figure 13.

3. Turn *off* the dependent sources
4. Find the Miller capacitance corresponding to C_μ for the loaded amplifier using the Miller approximation

Answer: $C_M = C_\mu\left(1 - \frac{V_O}{V_\pi}\right) = C_\mu\left(1 + g_m\left(r_{O1} // r_{O2} // R_F\right)\right)$.

Outline: Using the midband circuit of Figure 12 and the two-port of Figure 14 with the dependent sources turned off, we arrive at the circuit of Figure 15, where C_μ is inserted in the midband circuit only to show how to calculate the Miller capacitance.

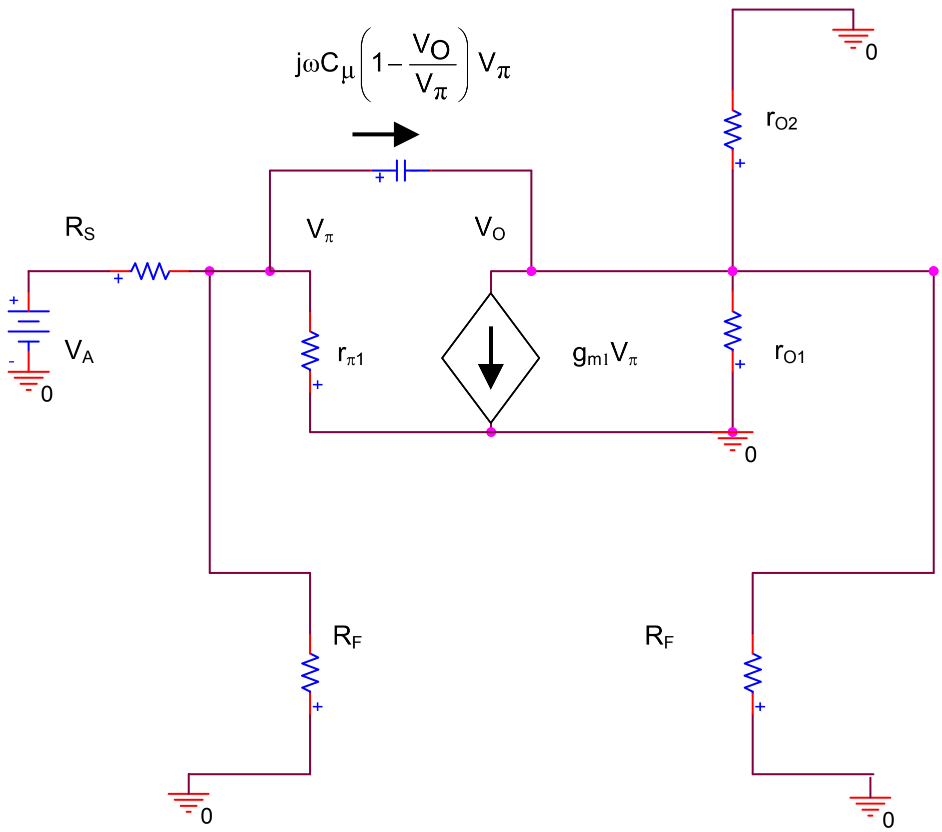

FIGURE 15

Midband circuit with loading; capacitance C_μ is added to show that $C_M = C_\mu(1-V_O/V_\pi)$

Figure 15 shows the voltage across C_μ is $V_\pi – V_O$, so the Miller capacitance is $C_\mu(1-V_O/V_\pi)$. Using Figure 15 with C_μ removed, we find the midband value of V_O/V_π given in EQ. 21.

EQ. 21

$$\frac{V_O}{V_\pi} = -g_{m1}\left(r_{O1} // r_{O2} // R_F\right),$$

with

$$g_{m1} = \frac{I_{C1}}{V_{TH}} = 0.3866 \text{ A/V}$$

$$r_{O1} = \frac{V_{AF} + V_{CB}}{I_{C1}} = 10.347 \text{ k}\Omega$$

$$r_{O2} = \frac{V_{AF} + V_{BC}}{I_{C2}} = 10.2\ k\Omega.$$

These values result in the Miller capacitance given by

$$C_M = C_\mu\left(1 - \frac{V_O}{V_\pi}\right) = 5pF[1 + .3866(10.347k // 10.20k // 10k)] = 6.565\ nF.$$

5. Using the Miller capacitance, find the upper 3dB corner frequency of the loaded amplifier with dependent sources of feedback turned *off*

Answer: f_{3dB} = 117.3 kHz (PSPICE 115.7 kHz, difference due to Miller approximation)

Outline: $\tau = C_M(R_S // r_{\pi 1} // R_F)$ = 1.357 μs, using

EQ. 22

$$r_{\pi 1} = \frac{\beta\left(1 + \frac{V_{CB}}{V_{AF}}\right)V_{TH}}{I_{C1}} = 267.6\ \Omega$$

we find

$$f_{3dB} = \frac{1}{2\pi\tau} = 117.3\ kHz$$

6. Find the upper 3dB corner frequency of the *feedback* amplifier

Answer: f_{3dB} (FB) = $f_{3dB} \times$ PF = 3.30 MHz (PSPICE 3.25 MHz)

Outline: we have to find the performance factor. So we need the loaded midband transR gain, which is found using Figure 15 with C_μ removed. We treat the amplifier as a transR amplifier and find

EQ. 23

$$A_R = -g_{m1}(r_{O1} // r_{O2} // R_F)(R_S // r_{\pi 1} // R_F) = -271.2\ k\Omega$$

Hence the performance factor is

EQ. 24

$$PF = 1 + \left(-\frac{1}{R_F}\right)A_R = 1 + 271.2k/10k = 28.12.$$

7. Find the *midband* gains with the dependent sources turned *off* and turned *on*.

Answer: We use Figure 15 without C_μ. The midband voltage gain of the loaded amplifier with dependent sources set to zero is

EQ. 25

$$\frac{V_O}{V_A} = \frac{V_O}{I_A R_S} = \frac{A_R}{R_S} = 48.7\ dB.$$

The gain with feedback turned on is reduced by the PF, so with feedback,

EQ. 26

$$\frac{V_O}{V_A}(FB) == \frac{A_R}{R_S \times PF} = 19.7\ dB$$

8. Construct the Bode plots

Answer: See Figure 11.