Closed book, calculators required, Monday, March 28: 8AM-10AM

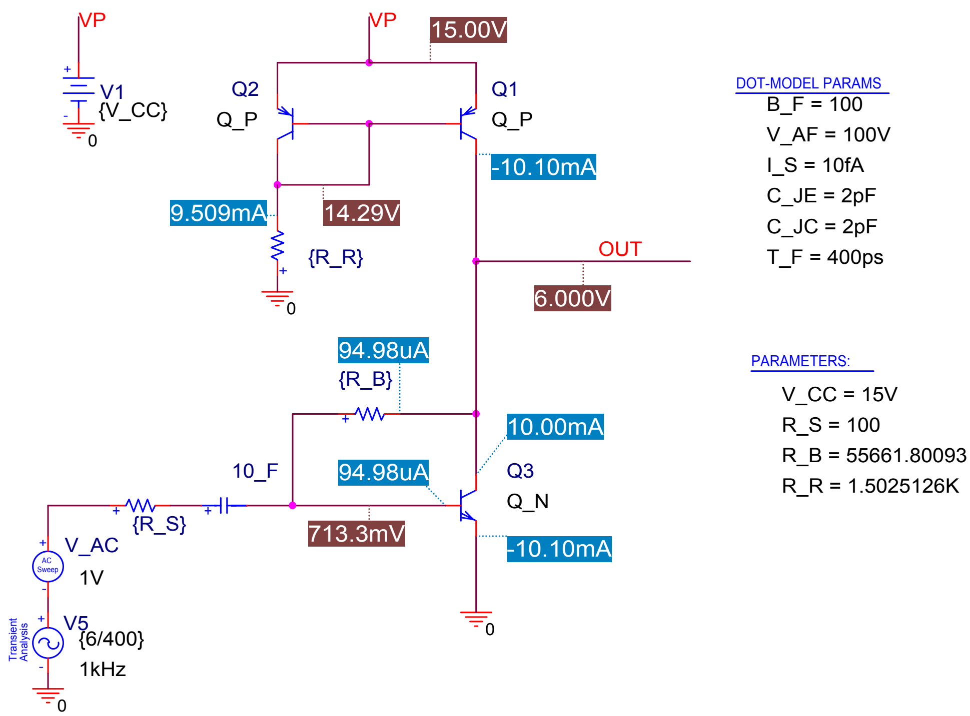
ECE 304 Spring '05 Exam 4 Solutions

NOTE: IN ALL CASES
1. Solve the problem on scratch paper
2. Once you understand your solution, put your answer on the answer sheet
3. Follow your answer with an outline of your solution. No points for answer without an outline of the solution. A mish-mash of computation is not an acceptable outline.

PRINT your name at the top of each answer sheet

Assume V_{TH} = 25.864 mV in all problems and maximum collector-base forward bias in saturation is V_{CB} = 0 V. Where V_{BE} is needed, evaluate it using $V_{BE} = V_{TH}\,\ell n(I_C/I_S)$.

Problem 1: Miller capacitance and Miller approximation



.model Q_P PNP (Bf={B_F} Is={I_S} Vaf={V_AF})

.model Q_N NPN (Bf={B_F} Is={I_S} Vaf={V_AF} Cjc={C_JC} Cje={C_JE} Tf={T_F})

FIGURE 1
Circuit for Problem 1; Early voltages are specified; only Q_N has capacitances

For the circuit in Figure 1, assume $C_\pi = C_{JE} + I_C\tau_F/V_{TH}$ and $C_\mu = C_{JC}$. Find the formulas for the following, and then evaluate them:

1. Determine the amplitudes of the maximum output signal upswing and downswing, and what mode changes limit them.

Answer: The upswing is limited by saturation of Q1 when V_O = 14.29V, an upswing of 8.29V. The downswing is limited by saturation of Q3 when V_O = 713 mV, a downswing of 5.28V.

2. Determine the Miller capacitance using the Miller approximation.

Answer: $C_M = C_\mu \frac{R_B}{R_O + R_B}(1 + g_m R_O)$. C_M = 3.75 nF.

Outline: The Miller capacitance is $C_M = C_\mu(1 - V_O/V_\pi)$ and in the Miller approximation is evaluated using the midband value of V_O/V_π. To find V_O/V_π we use the output side of the midband circuit, as shown in Figure 2 below.

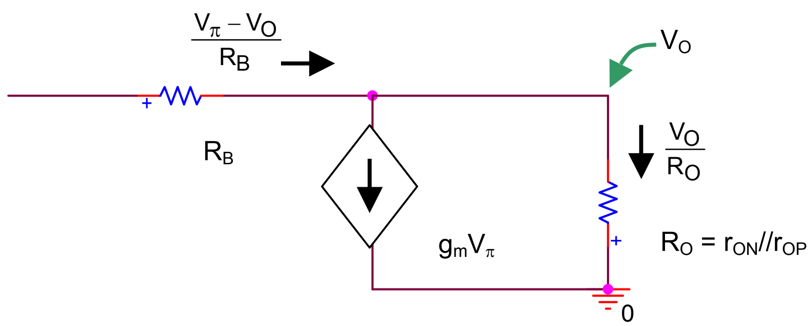


FIGURE 2
Midband output side of circuit

Using KCL at the output node in Figure 2 we find

EQ. 1

$$\frac{V_\pi - V_O}{R_B} = g_m V_\pi + \frac{V_O}{R_O}.$$

Collecting terms,

EQ. 2

$$\frac{V_O}{V_\pi} = -(g_m R_B - 1)\frac{R_O}{R_O + R_B};\ 1 - \frac{V_O}{V_\pi} = \frac{R_B}{R_O + R_B}(1 + g_m R_O);\ C_M = C_\mu \frac{R_B}{R_O + R_B}(1 + g_m R_O).$$

Evaluating, r_{OP} = 10.72 kΩ, r_{ON} = 10.53 kΩ; $1+g_mR_O$ = 2054, $R_B/(R_O+R_B)$ = 0.913, C_M = 3.75 nF.

3. Determine the upper 3dB corner frequency. *Hint:* find a "Miller resistance" R_M in the input side of the circuit for R_B as well as a Miller capacitance for C_μ.

Answer: $f_C = \frac{1}{2\pi(C_\pi + C_M)(R_S // r_\pi // R_M)}$ = 1.9 MHz

with $C_M = C_\mu \frac{R_B}{R_O + R_B}(1 + g_m R_O)$ and $R_M = \frac{R_O + R_B}{1 + g_m R_O}$.

Outline:

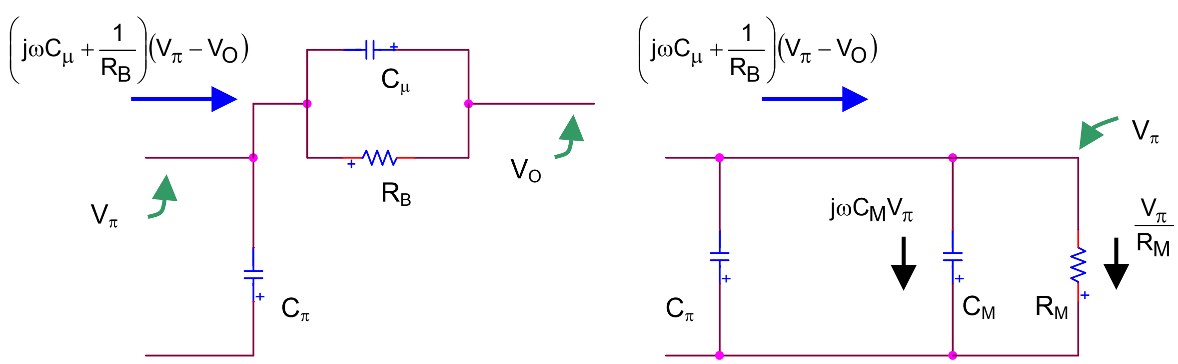


FIGURE 3
Finding the Miller equivalents

The current flowing to the right at the top of the left-hand circuit in Figure 3 is made to agree with that at the top of the right-hand circuit by making C_M and R_M satisfy EQ. 3 below.

EQ. 3

$$C_M = C_\mu\left(1 - \frac{V_O}{V_\pi}\right);\ \frac{1}{R_M} = \frac{\left(1 - \frac{V_O}{V_\pi}\right)}{R_B}.$$

Evaluating R_M using $1-V_O/V_\pi$, we find EQ. 4 below.

EQ. 4

$$R_M = \left(\frac{R_O + R_B}{1 + g_m R_O}\right).$$

Using the circuit on the right side of Figure 3, analysis of the frequency dependence is made as shown in Figure 4 below.

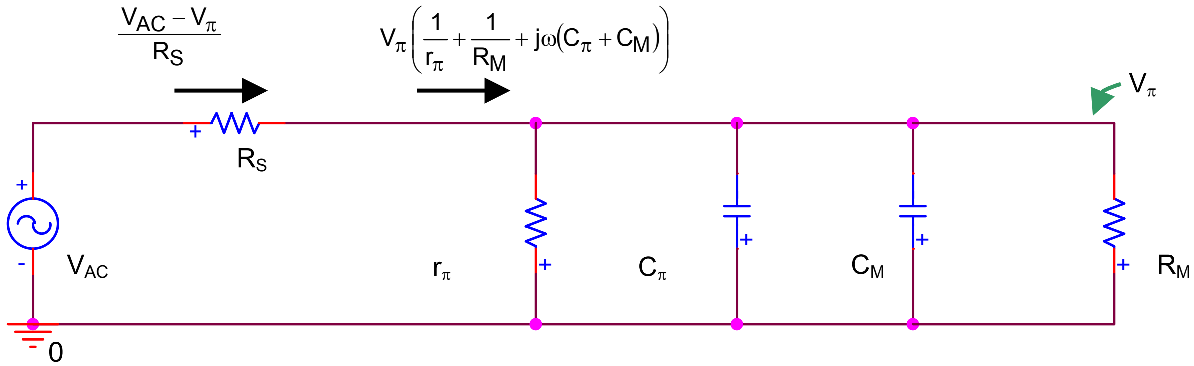


FIGURE 4

Finding the frequency dependence of V_π/V_O

Equating the currents at the top node of Figure 4, we find V_π/V_{AC}:

EQ. 5

$$\frac{V_\pi}{V_{AC}} = \left(\frac{R_S \,//\, r_\pi \,//\, R_M}{R_S}\right)\frac{1}{1 + j\omega(C_\pi + C_M)(R_S \,//\, r_\pi \,//\, R_M)}.$$

As we know the input circuit sets the corner frequency, the corner frequency is

EQ. 6

$$f_C = \frac{1}{2\pi(C_\pi + C_M)(R_S \,//\, r_\pi \,//\, R_M)}$$

with C_M and R_M from EQ. 2 and EQ. 4. We find f_C = 1.9 MHz. (PSPICE says 3.7 MHz because it allows C_μ to change with bias to C_μ = 1 pF).

4. Determine the frequency at which the Miller approximation breaks down

Answer: The frequency where $1 - V_O/V_\pi$ drops by 3dB is $f_M = \frac{1}{2\pi C_\mu (R_O \,//\, R_B)}$ =16.4 MHz.

Outline: We look at the output side of the circuit in Figure 5 and use KCL at the collector to find EQ. 7 below.

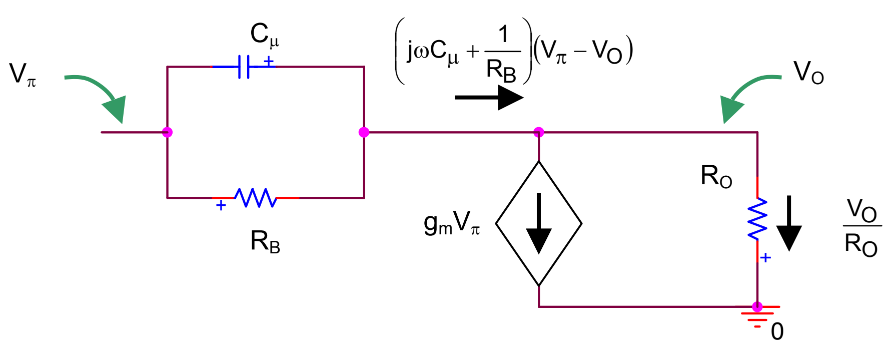


FIGURE 5

Finding the frequency dependence of V_O/V_π

EQ. 7

$$\left(j\omega C_\mu + \frac{1}{R_B}\right)(V_\pi - V_O) = g_m V_\pi + \frac{V_O}{R_O}.$$

Collecting terms we find $1 - V_O/V_\pi$ as given in EQ. 8 below.

EQ. 8

$$1 - \frac{V_O}{V_\pi} = \frac{R_O /\!/ R_B}{R_B}\left(\frac{1 - g_m R_B + j\omega C_\mu R_B}{1 + j\omega C_\mu (R_O /\!/ R_B)}\right),$$

suggesting the Miller capacitance will show frequency dependence for frequencies at or above the frequency f_M given in EQ. 9 below.

EQ. 9

$$f_M = \frac{1}{2\pi C_\mu (R_O /\!/ R_B))} = 16.4 \text{ MHz}$$

5. If R_R were reduced to 1 kΩ, discuss qualitatively what would happen to the 3dB corner frequency?

Answer: If R_R is reduced the mirror current increases, and the current in Q3 also must increase. A larger current lowers R_O (because r_O varies inversely as the current), but the gain $g_m r_O$ does not change much because g_m increases linearly with current. That is,

EQ. 10

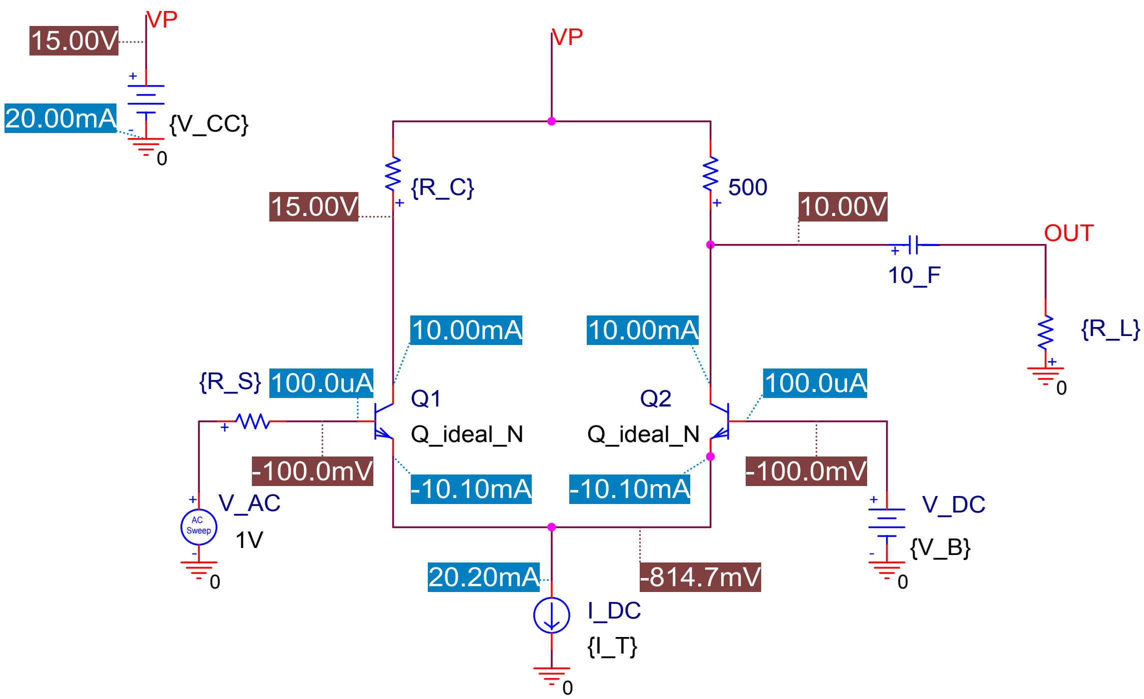
$$g_m r_O = \frac{I_C}{V_{TH}} \frac{(V_{CB} + V_{AF})}{I_C} = \frac{V_{CB} + V_{AF}}{V_{TH}},$$

which is independent of I_C. Therefore, Miller capacitance and Miller R_M won't change much either. So any change in the bandwidth due to changing I_C should not be large. PSPICE suggests about a 10% increase.

6. If R_R were reduced to 1 kΩ, discuss qualitatively what would happen to the amplitudes of the maximum output signal upswing and downswing?

Answer: If R_R is reduced the mirror current increases. To obtain more current from Q3, its V_{CB} must increase, raising the value of V_O. Consequently the maximum output upswing would decrease and the maximum output downswing would increase. Incidentally, the increased reverse bias of the CB junction lowers C_μ of Q3, which in turn reduces the Miller capacitance, and so increases the bandwidth. This is the source of the bandwidth increase in PSPICE mentioned in Part 5 above.

Problem 2: Time-constant method



.model Q_ideal_N NPN (Bf={B_F} Is={I_S} Cjc={C_JC} Cje={C_JE} Tf={T_F})

DOT-MODEL PARAMS	PARAMETERS:
B_F = 100	V_CC = 15V
I_S = 10fA	R_C = 1u
C_JE = 2pF	R_S = 1k
C_JC = 2pF	R_L = 1k
T_F = 400ps	I_T = {20mA*(1+1/B_F)}
	V_B = {-I_T/(2*(B_F+1))*R_S

FIGURE 6

Circuit for problem 2; notice that $R_C = 1\ \mu\Omega$, that is, R_C is negligible, and Early voltages are infinite; DC voltage V_B places Q1 and Q2 at the same Q-point for zero AC signal

The circuit of Figure 6 has an AC signal applied on one side only, at the base of Q1. The base of Q2 is biased with a DC source to make the currents in Q1 and Q2 the same at zero AC input, as shown. For the circuit in Figure 6, assume $C_\pi = C_{JE} + I_C\tau_F/V_{TH}$ and $C_\mu = C_{JC}$. Find formulas for the following, and then evaluate them numerically:

1. Using the method of open- or short-circuit time constants (whichever is appropriate), determine the upper 3dB corner frequency of the amplifier in Figure 6 for the case $R_C = 0\ \Omega$ (as shown in Figure 6).

Answer: f_C = 5.71 MHz; formula is found below.
Outline: See Part 2

2. Using the method of open- or short-circuit time constants (whichever is appropriate), determine the upper 3dB corner frequency of the amplifier in Figure 6 for $R_C = 500\ \Omega$ (*not* shown in Figure 6). Because V_B maintains the same Q-point currents shown in Figure 6, the Q-point collector voltage of Q1 changes to match that of Q2.

Answer: f_C = 1.67 MHz; formula derived below.
Outline: Evaluations of the resistances is easily done in PSPICE, as shown in Figure 7 - Figure 11.

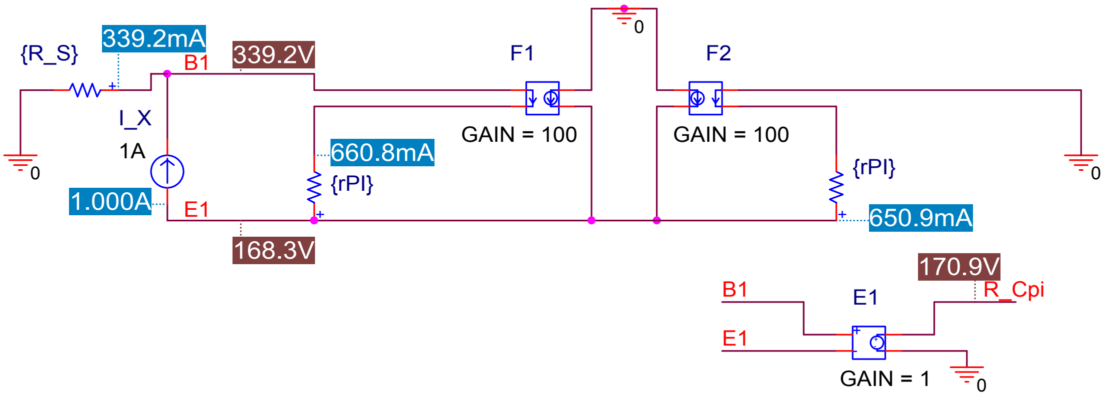


FIGURE 7

Resistance for $C_{\pi 1}$ is $R_{C\pi 1}$ = 170.9 Ω; the collector currents are dictated by the base currents through the CCCS's, so the collector resistors are not involved.

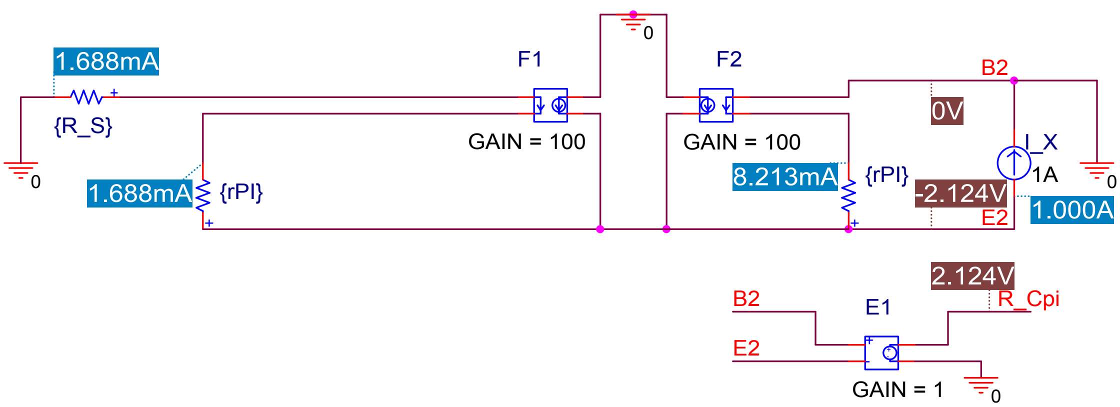


FIGURE 8

Resistance for $C_{\pi 2}$ is $R_{C\pi 2}$ = 2.124 Ω

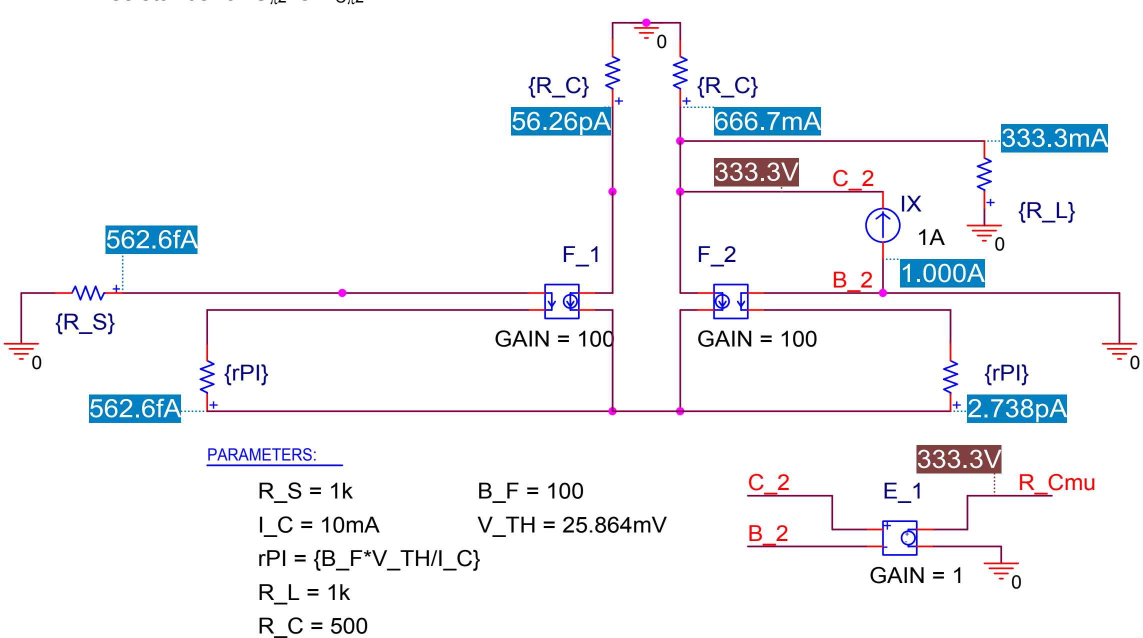


FIGURE 9

Resistance for $C_{\mu 2}$ is $R_{C\mu 2}$ = 333.3 Ω

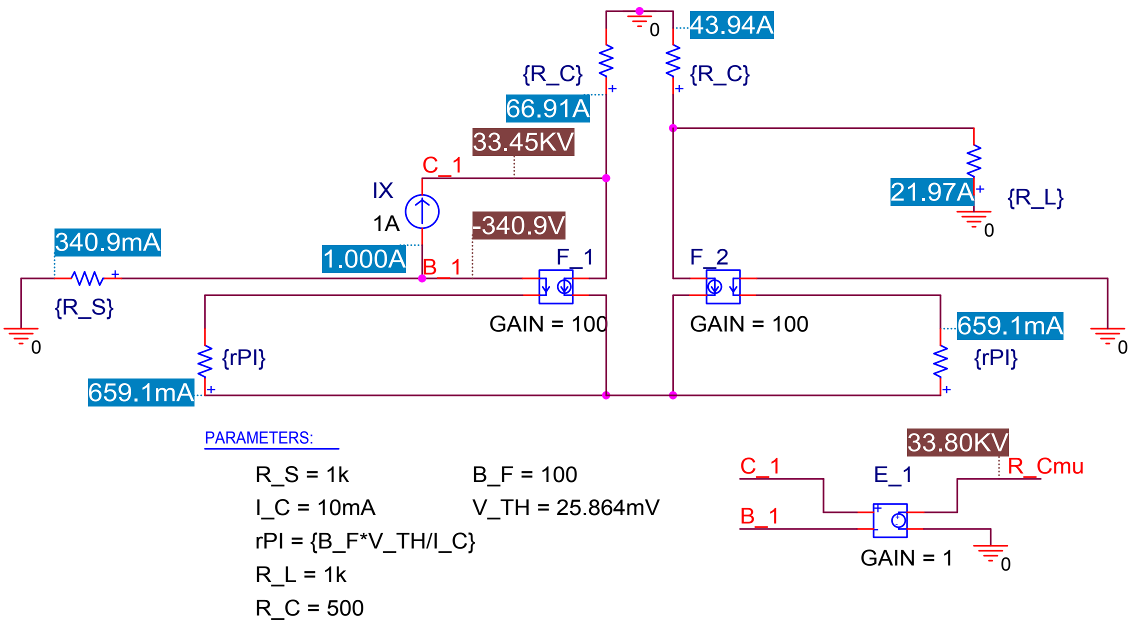


FIGURE 10

Resistance for $C_{\mu 1}$ is $R_{C\mu 1}$ = 33.8 kΩ

Time constant is $\tau = C_\pi(170.9\ \Omega + 2.124\ \Omega) + C_\mu(333.3\ \Omega + 33.8\ k\Omega)$. $C_\pi = C_{JE} + I_C\tau_F/V_{TH}$ = 156.7 pF, $C_\mu = C_{JC}$ = 2 pF → τ = 95.36 ns → f_C = 1.67 MHz

If the left-hand collector resistor is removed, Figure 10 is replaced by Figure 11 below.

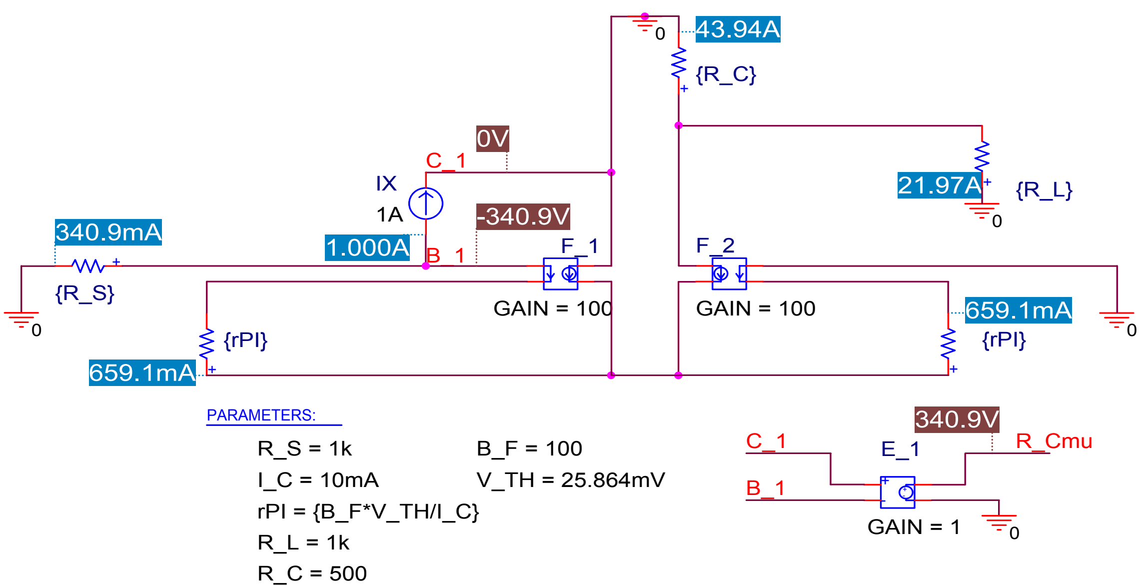


FIGURE 11

Resistance for $C_{\mu 1}$ with R_C for Q1 removed is $R_{C\mu 1}$ = 340.9 Ω

Using Figure 11, $\tau = C_\pi(170.9\ \Omega + 2.124\ \Omega) + C_\mu(333.3\ \Omega + 340.9\ \Omega)$ = 1.35 ns → f_C = 5.59 MHz.

FORMULAS

The above figures show the basic circuits that must be solved and provide the numerical results. Below, the formulas for the time constants obtained by solving these circuits are derived.

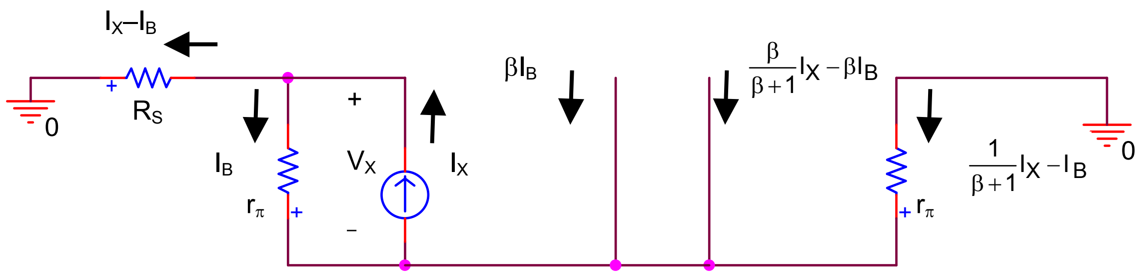


FIGURE 12

Finding $R_{C\pi 1}$

From Figure 12, KVL provides

EQ. 11

$$\left(I_X - I_B\right)R_S - I_B r_\pi + \left(\frac{I_X - (\beta+1)I_B}{\beta+1}\right) r_\pi = 0\ ;\quad I_B = \frac{R_S + r_E}{2r_\pi + R_S} I_X\ .$$

EQ. 12

$$V_X = I_B r_\pi \ \rightarrow\ R_{C\pi 1} = \frac{(R_S + r_E) r_\pi}{2r_\pi + R_S}$$

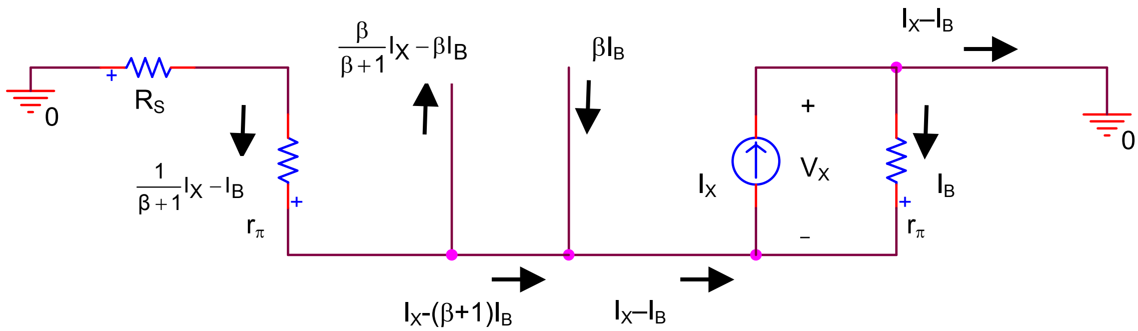


FIGURE 13

Finding $R_{C\pi 2}$

From Figure 13,

EQ. 13

$$\left(\frac{I_X - (\beta+1)I_B}{\beta+1}\right)\left(r_\pi + R_S\right) - I_B r_\pi = 0\ ;\quad I_B = \frac{R_S/(\beta+1) + r_E}{2r_\pi + R_S} I_X\ .$$

EQ. 14

$$V_X = I_B r_\pi \ \rightarrow\ R_{C\pi 2} = \frac{\left(\frac{R_S}{\beta+1} + r_E\right) r_\pi}{2r_\pi + R_S}\ .$$

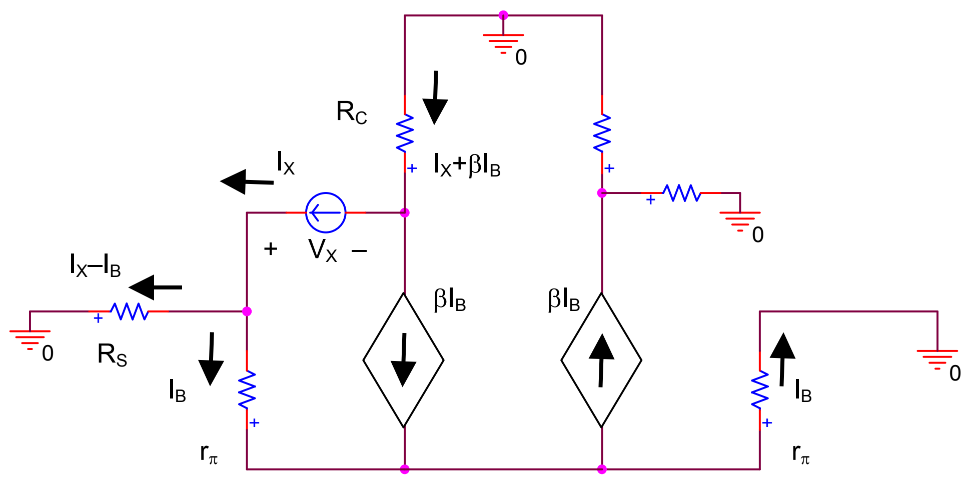


FIGURE 14

Finding $R_{C\mu1}$

KVL along the bottom of Figure 14 provides

EQ. 15

$$I_B = I_X \frac{R_S}{2r_\pi + R_S}\,.$$

The voltage on the left of I_X is $I_X(2r_\pi//R_S)$. The voltage on the right of I_X is $-(I_X+\beta I_B)R_C$. Therefore,

EQ. 16

$$V_X = I_X(2r_\pi//R_S) + (I_X+\beta I_B)R_C = I_X\left(R_S // 2r_\pi\right) + \left(1+\beta\frac{R_S}{2r_\pi + R_S}\right)R_C \rightarrow$$

$$R_{C\mu1} = \left(R_S//2r_\pi\right) + \left(1+\beta\frac{R_S}{2r_\pi + R_S}\right)R_C\,.$$

In the case that R_C is zero (circuit of Figure 6) we have

EQ. 17

$$R_{C\mu1} = \left(R_S//2r_\pi\right).$$

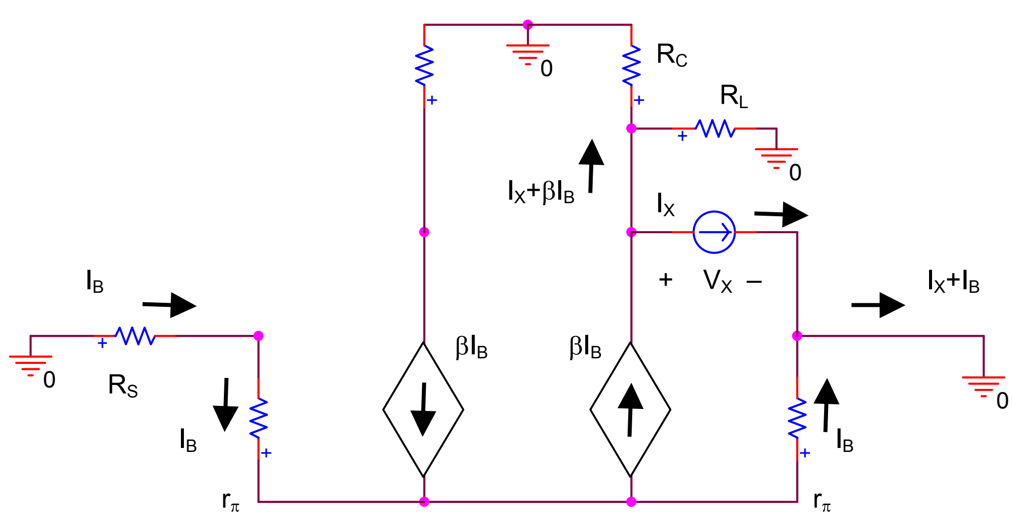


FIGURE 15

Finding $R_{C\mu2}$

KVL along the bottom of Figure 15 shows that I_B = 0 A. Therefore, $R_{C\mu2}$ is given by EQ. 18 below.

EQ. 18

$$R_{C\mu 2} = (R_C//R_L).$$

The total time constant is then
EQ. 19

$$\tau = C_\pi(R_{C\pi 1} + R_{C\pi 2}) + C_\mu(R_{C\mu 1} + R_{C\mu 2})$$

which becomes in the case of Figure 6
EQ. 20

$$\tau = C_\pi\left(\frac{R_S(r_\pi + r_E) + 2r_\pi r_E}{2r_\pi + R_S}\right) + C_\mu\big((R_C // R_L) + (R_S //(2r_\pi)\big),$$

and in the case of the circuit like Figure 6 but with the extra R_C on the input side,
EQ. 21

$$\tau = C_\pi\left(\frac{R_S(r_\pi + r_E) + 2r_\pi r_E}{2r_\pi + R_S}\right) + C_\mu\left((R_C // R_L) + (R_S //(2r_\pi) + \left(1 + \beta\frac{R_S}{R_S + 2r_\pi}\right)R_C\right).$$

The difference in time constants is
EQ. 22

$$\Delta\tau = C_\mu\left(1 + \beta\frac{R_S}{R_S + 2r_\pi}\right)R_C \approx C_\mu\left(\beta\frac{R_C}{2r_\pi}\right)\frac{R_S 2r_\pi}{R_S + 2r_\pi} = C_\mu\left(1 - \frac{V_O}{V_\pi}\right)\frac{R_S 2r_\pi}{R_S + 2r_\pi} = C_M(R_S//(2r_\pi)),$$

which is the delay due to the Miller capacitance C_M, which sees resistance $R_S//(2r_\pi)$.

3. Compare the two bandwidths, and discuss why they are different.

Answer: From the results of Parts 1 and 2 the bandwidth of the diff amp with no collector resistor on the input side is 5.6 MHz compared to 1.7 MHz, the difference being due to the change in time constant of C_μ on the input side. The increase in time constant is due to the Miller effect when R_C is inserted, because the added resistor means a time varying voltage is applied on *both* sides of C_μ, which means there is an AC gain V_O/V_π across this capacitor, magnifying it (the Miller effect: $C_M = C_\mu(1–V_O/V_\pi)$). Without the resistor, one side of the input C_μ is at AC ground, and the time constant of C_μ is greatly reduced.

4. Would your discussion of Part 3 change if τ_F = 400 ns instead of τ_F = 400 ps? Why or why not?

Answer: Yes, it would change because the time constant introduced by C_π would then be so large, due to the increased value of C_π from the $I_C\tau_F/V_{TH}$ term, that the corner frequency is dictated by C_π, not the Miller effect. In this case, both circuits will have the same corner frequency regardless of the presence or the absence of the collector resistor on the input side.