ECE 304 Spring '05 First Exam Solutions

Problem 1: Current mirror

Follow the outline procedure at the top of the exam with headings for each major step in the solution. **No points for answer without an outline of the solution. A mish-mash of calculations is not an acceptable outline.**

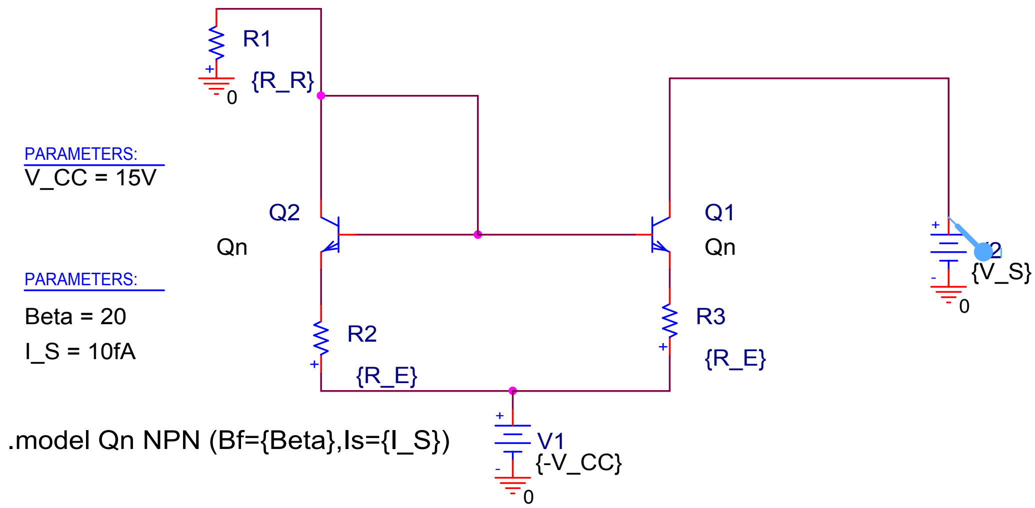


FIGURE 1

Current mirror for Problem 1

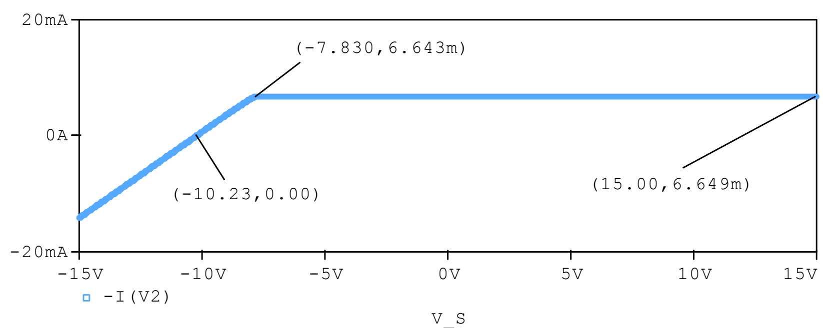


FIGURE 2

Current vs. DC voltage for current mirror of Figure 1

Select values for R_R and R_E so the mirror will have the *I-V* behavior seen in Figure 2. Assume the maximum forward CB bias in saturation is $V_{CB} = -V_{SAT} = -500$ mV. Note that all transistors have infinite Early voltages.

ANSWER: $R_R = 1\ k\Omega$ and $R_E = 998\ \Omega$

OUTLINE:

The break in slope of the mirror occurs when transistor Q1 first saturates, which means when V_{CB} (Q1) = –500 mV. From Figure 2, this occurs when $V_S = -7.83V$. At this value of $V_S = V_C(Q1)$, the base voltage of Q1 is at $V_B = V_C - V_{CB} = -7.83 - (-.5) = -7.33$ V. The current from Figure 2 is 6.649 mA (the transistor will become fully active 500mV above the corner, increasing the current from 6.643 mA at the corner to the fully active value 6.649 mA above the corner). By Ohm's law applied to R_R,

EQ. 1

$$R_R = \frac{7.33}{6.649mA\ (1+2/20)} = 1.002\ k\Omega.$$

Using the given current we find V_{BE} as $V_{BE} = V_{TH}\ \ell n(6.649mA/10fA) = 704.1$ mV. From Ohm's law with R_E we find EQ. 2 below.

EQ. 2

$$R_E = \frac{-7.33 + 15 - 0.704}{6.649mA(1 + 1/20)} = 998\ \Omega .$$

Problem 2: CE amp with current mirror bias

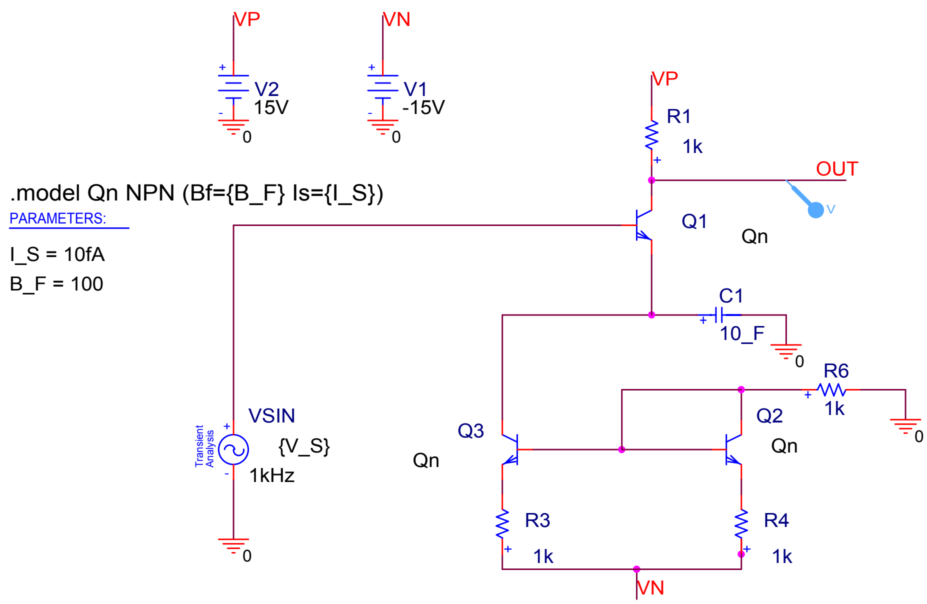


FIGURE 3

CE amplifier with current mirror bias; C_C is a bypass capacitor

All resistors have a value of 1 kΩ. Transistors are ideal (see dot-model statement), with infinite Early voltage. Determine the following:

1. The DC output voltage V_O. Find V_{BE} in the mirror by iteration.

ANSWER: $V_{OUT} = 8.028$ V, and $V_{BE} = 705.6$ mV.

OUTLINE:

First, we find the mirror current $I_C\,(Q3)$; then the collector current of Q1, $I_C\,(Q1)$. Then $V_{OUT} = V_{CC} - I_C R_C$.

- *Mirror current.* By Ohm's law applied to R_R, labeling $I_C(Q3) = I_C$,

EQ. 3

$$V_B = -I_C(1+2/\beta)\ R_R.$$

By Kirchhoff's voltage law along the right side of the mirror

EQ. 4

$$V_B = I_C(1 + 1/\beta)\ R_E - V_{CC} + V_{BE}.$$

Combining EQ. 3 and EQ. 4, we find I_C is given by EQ. 5 below.

EQ. 5

$$I_C = \frac{V_{CC} - V_{BE}}{(1 + 2/\beta)R_R + (1 + 1/\beta)R_E} .$$

However, V_{BE} also is determined from I_C as EQ. 6 below.

EQ. 6

$$V_{BE} = V_{TH}\ \ell n(I_C/I_S).$$

Therefore, we guess V_{BE} = 700mV and find I_C from EQ. 5. Then we use this value of I_C in EQ. 6 to find a new value for V_{BE}. We place this new V_{BE}-value in EQ. 5 to find a new value for I_C, and so on. After a few iterations, the result is V_{BE} = 705.6 mV and $I_C(Q3)$ = 7.042 mA.

- *Collector current of Q1:* $I_C(Q1) = I_C(Q3)/(1+1/\beta)$ = 6.972 mA.
- *Value of V_{OUT}:* Now $V_{OUT} = V_{CC} - I_C(Q1)R_C$ = 15 – 6.972 mA × 1 kΩ = 8.028 V

2. The small signal voltage gain (formula and numerical value).

ANSWER: The small-signal voltage gain is $A_\upsilon = -I_C R_C/V_{TH}$ = –269.6 V/V.

OUTLINE: The signal voltage V_S is applied directly across r_π so the small-signal collector current is $g_m V_S$. See Figure 4.

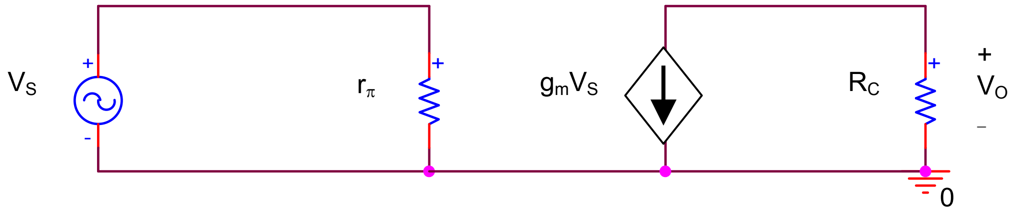


FIGURE 4

Small-signal equivalent circuit for amplifier of Figure 3 with infinite V_{AF} (*i.e.*, $r_O = \infty$)

This current is directed upward through R_C, so the output voltage is $V_O = -g_m R_C V_S$. Thus the voltage gain is ($g_m \approx I_C/V_{TH}$)

EQ. 7

$$\frac{V_O}{V_S} = -g_m R_C \approx -\frac{I_C R_C}{V_{TH}} = -\frac{6.972}{25.864m} = -269.6\ V/V .$$

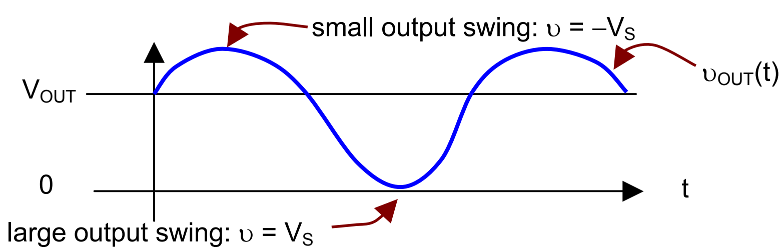
The same result can be found using a CCCS and the relation $r_\pi \approx \beta_{AC} V_{TH}/I_C$.

3. Sketch the output voltage vs. time for the case of a sinusoidal input voltage with the maximum amplitude V_S that leaves the transistor Q1 in active mode throughout the cycle. That is, the input voltage is

$$\upsilon(t) = V\_S \sin(2\pi t/T)$$

Discuss reasons to expect distortion, even if the transistor stays active.

ANSWER: Distortion is expected because the transistor *I-V* relation is the diode law, that is, the current is proportional to $\exp(V_S/V_{TH})$ when $\upsilon(t) = V_S$, and the current is proportional to $\exp(-V_S/V_{TH})$ when $\upsilon(t) = -V_S$ (V_S = amplitude of voltage). Unless $V_S << V_{TH}$, the current swing for $\upsilon(t) = V_S$ is much larger than for $\upsilon(t) = -V_S$, and therefore the AC output downswing is much greater than the AC output upswing. A rough sketch of the output is then shown below.



The device remains active so long as $0 \le \upsilon_{OUT}(t) \le V_{CC}$, the upper limit set by cut off and the lower limit by saturation.

4. Determine the numerical value for the maximum input amplitude V_S that avoids clipping, assuming the above input signal, and the corresponding maximum output upswing and downswing voltage.

ANSWER: Maximum input amplitude is $V_S = 0.8165\ V_{TH}$ = 21.118 mV. Maximum output upswing is 3.891 V and maximum output downswing is 8.028V.

Calculators required, closed book; Friday, January 28, 8 AM – 10 AM

OUTLINE:
When the signal swing is V_S, the collector current increases by $\Delta\iota$ as given by EQ. 8 below.
EQ. 8

$$\Delta\iota = I_S e^{\left(\frac{V_{BE}+V_S}{V_{TH}}\right)} - I_S e^{\left(\frac{V_{BE}}{V_{TH}}\right)} = I_{CQ}\left(e^{\left(\frac{V_S}{V_{TH}}\right)} - 1\right),$$

where I_{CQ} = Q-point collector current.
The increase in current in turn causes an increased voltage drop across R_C of $\Delta\iota\, R_C$, which will cause Q1 to saturate if V_{CB} = 0 V; that is, saturation occurs if
EQ. 9

$$\Delta\iota R_C = I_{CQ}\left(e^{\left(\frac{V_S}{V_{TH}}\right)} - 1\right) R_C = 8.8028\ \text{V}.$$

Solving for V_S we find $V_S = V_{TH}\ell n(8.8028/6.972+1) = 21.118$ mV.
The same value of V_S applies on the upswing, as the amplitude V_S is fixed. Therefore, on the upswing the change in current is
EQ. 10

$$\Delta\iota = I_S e^{\left(\frac{V_{BE}-V_S}{V_{TH}}\right)} - I_S e^{\left(\frac{V_{BE}}{V_{TH}}\right)} = I_{CQ}\left(e^{-\left(\frac{V_S}{V_{TH}}\right)} - 1\right).$$

The exponential is less than 1, so the change in current is negative, and the total current $\iota_C(t) = I_{CQ} + \Delta\iota$ goes down, reducing the voltage drop across R_C and causing the output voltage to swing up by an amount $-\Delta\iota\, R_C$. Substituting the established value of V_S = 21.118 mV, we find the upswing is
EQ. 11

$$-\Delta\iota R_C = I_{CQ}\left(1 - e^{-\left(\frac{V_S}{V_{TH}}\right)}\right) R_C = 3.891\ \text{V}.$$

This swing takes υ_{OUT} to 12.7V, too low to cause cutoff. Therefore, saturation limits V_S.

Problem 3: Active load

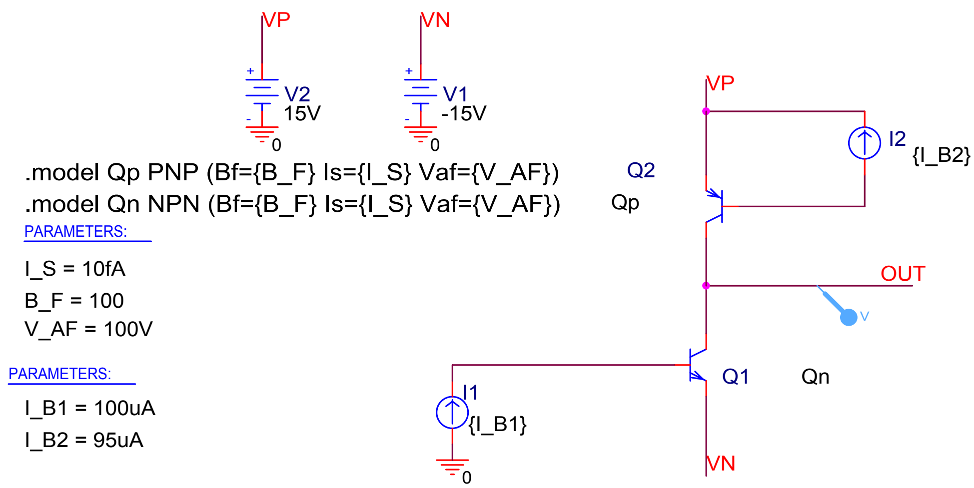


FIGURE 5
CE amplifier with active load; the transistors are matched and have Early voltages of V_{AF} = 100V; I_B1 is fixed at 100 μA and I_B2 is varied in this problem

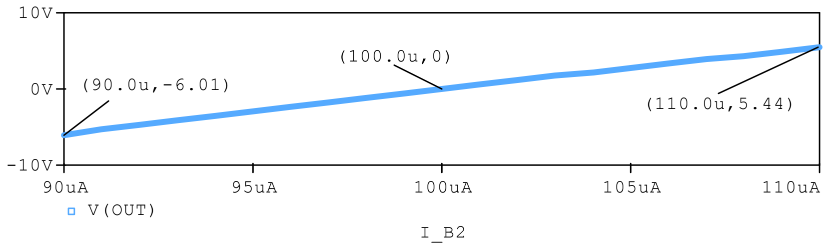


FIGURE 6

Provided plot of output voltage as a function of I_B2 when I_B1 = 100 μA

1. Explain using formulas why the output voltage of the amplifier increases approximately linearly with the base current I_{B2} biasing the load, as shown in Figure 6.

ANSWER:

EQ. 12

$$V_{OUT} = \frac{(I_{B2}/I_{B1})(V_{AF} + V_{CC} - V_{EB}) - V_{AF} - V_{CC} + V_{BE}}{1 + (I_{B2}/I_{B1})}.$$

Over the range of I_{B2} shown in Figure 6, the ratio I_{B2}/I_{B1} in this equation changes only slightly from unity. In the numerator this change is important because the negative terms very nearly cancel the positive terms (they cancel exactly when $I_{B2}/I_{B1} = 1$ according to Figure 6), so the linear I_{B2}/I_{B1} dependence of the numerator is significant. However in the denominator the sum is nearly 2 over the range of the plot, so the I_{B2}/I_{B1} dependence in the denominator is almost negligible. With $V_{BE} \approx V_{EB} \approx 700$ mV, EQ. 12 agrees with the labeled points in the provided Figure 6.

OUTLINE

The value of V_{OUT} is set by the condition that the collector current from Q2 be equal to the collector current going to Q1. This condition is

EQ. 13

$$I_{C1} = \beta_0 (1 + \frac{V_O + V_{CC} - V_{BE}}{V_{AF}}) I_{B1} = I_{C2} = \beta_0 (1 + \frac{V_{CC} - V_{EB} - V_O}{V_{AF}}) I_{B2},$$

where we have used the V_{CB} dependence of $\beta=\beta_0(1+V_{CB}/V_{AF})$ and the relation $I_C = \beta I_B$. Solving for V_O (representing the output voltage), we obtain the stated result.

2. Determine a formula for the slope $\partial V_{OUT}/\partial I_{B2}$ when I_{B1} is held fixed.

ANSWER: By differentiation, the slope is given by

$$\frac{\partial V_O}{\partial I_{B2}} = -\frac{V_O - (V_{AF} + V_{CC} - V_{EB})}{I_{B1} + I_{B2}};$$

3. Evaluate your slope formula and compare it with Figure 7; the slope does decrease somewhat with increasing I_{B2}.

ANSWER: Using $V_{BE} \approx V_{EB} \approx 700$mV, the formula agrees almost exactly with the labeled points in the provided Figure 7.

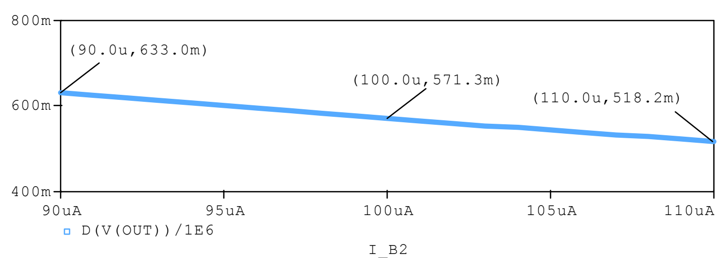


FIGURE 7

Provided plot of slope of Figure 6 in V/μA; I_{B1} is fixed at I_{B1} = 100 μA