Gain and Phase Margin Problem

Problem

Select the frequency f_1 in the gain expression of EQ. 1 below to obtain a two-pole Butterworth step response for a voltage feedback amplifier with β_{FB} = 10 mV/V.

EQ. 1

$$A_{\upsilon}(f) = \frac{A_{\upsilon 0}}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)\left(1 + j\frac{f}{f_3}\right)} .$$

The frequencies $f_2 = 10^6$Hz and $f_3 = 10^7$ Hz, and the low-frequency gain is $A_{\upsilon 0} = 10^5$ V/V. Also, determine the gain and phase margins of this amplifier.

Schematic

The PSPICE circuit for this amplifier is shown in Figure 1.

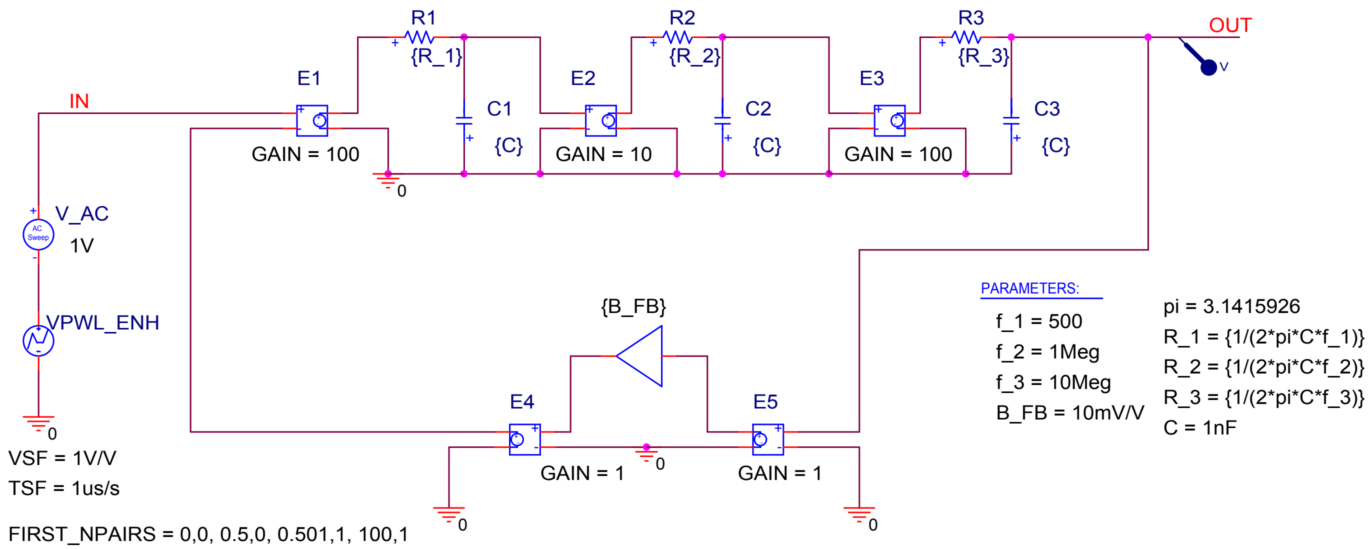

FIGURE 1

PSPICE representation of three-pole amplifier

Using the circuit of Figure 1 we can find the gain and phase plots, as shown in Figure 2 and Figure 3 below.

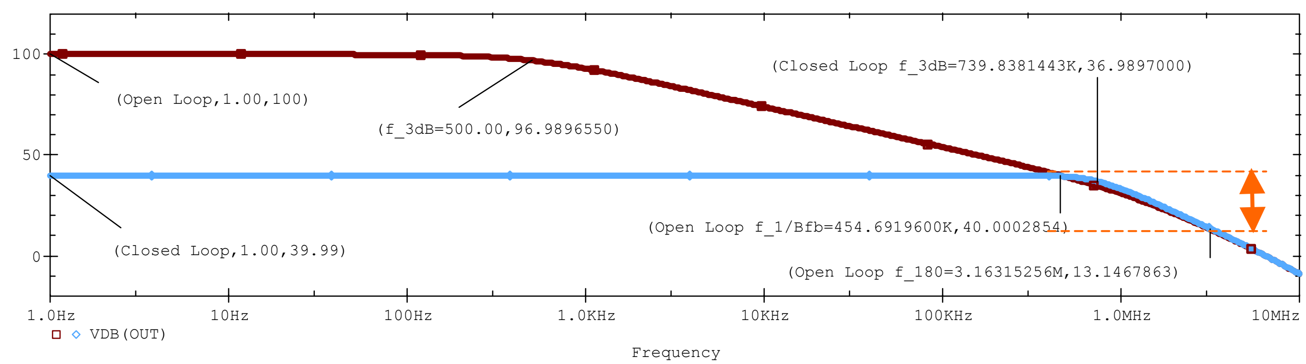

FIGURE 2

PSPICE gain plots for open lop and closed loop amplifier; this plot determines the frequency f_{1/β_FB} where the open-loop gain is $1/\beta_{FB}$; gain margin (26.8dB) is labeled with double arrow

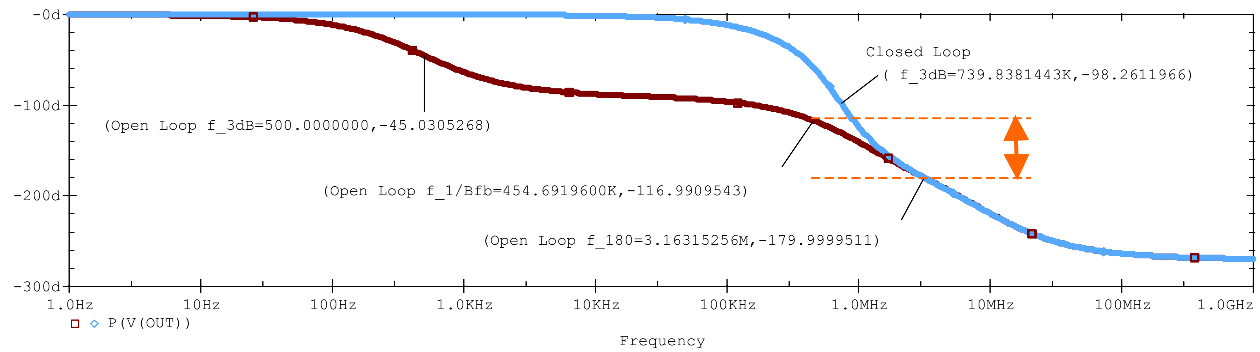

FIGURE 3

PSPICE phase plots for open lop and closed loop amplifier; this plot determines the frequency f_{180} where the phase is –180°; phase margin (63°) is labeled with double arrow

Transient response

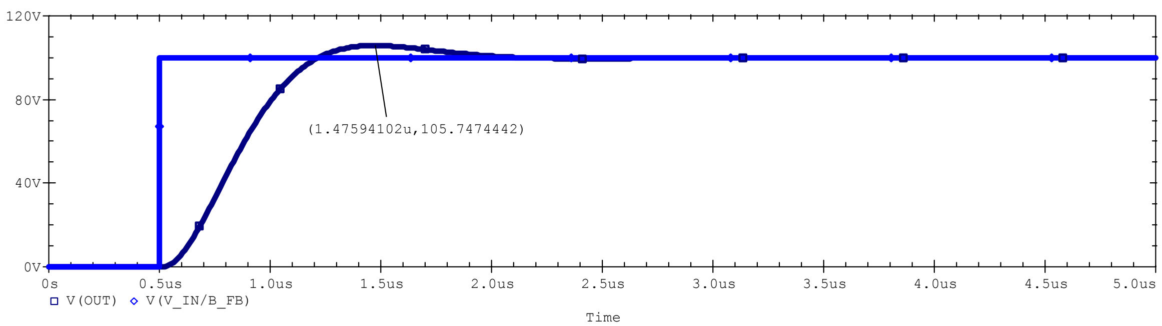

FIGURE 4

Step response of closed loop amplifier; time to first maximum is 0.98 μs and overshoot is 5.7%

The step response in Figure 4 can be compared with the *two*-pole estimates for a Butterworth design of $t_{MAX} = 1/(f_1 + f_2) = 1$ μs and overshoot of 4.3%.

Design procedure

For a Butterworth response we require a time constant separation factor of $2\beta_{FB}A_{\upsilon 0}$, so

EQ. 2

$$\frac{\tau_1}{\tau_2} = \frac{f_2}{f_1} \approx 2\beta_{FB}A_{\upsilon 0}\,.$$

That is, $f_1 = f_2/(2 \times 10^{-2} \times 10^5) = 500$ Hz.

We then draw the Bode plots for EQ. 1 with the poles f_1, f_2, f_3 that approximate Figure 2 and Figure 3, and determine the frequencies f_{1/β_FB} and f_{180}. The gain and phase margins are then

EQ. 3

$$\text{phase margin} = \arg\left[A_\upsilon(f_{1/\beta_{FB}})\right] - (-180^\circ) = 63^\circ$$

EQ. 4

$$\text{gain margin} = 20\ell og_{10}\left(\frac{1}{\beta_{FB}}\right) - 20\ell og_{10}\left(\left|A_\upsilon(f_{180})\right|\right) = 26.9 \text{ dB}$$

The numerical work can be done very conveniently using a spreadsheet, as shown in Figure 5 below.

				Open-Loop Amplifier			
Input	pi	3.1415926		Frequency	Phase	Gain	Gain(dB)
	A_v0	1.00E+05	f_1/Bfb	4.55E+05	-116.9913	1.00E+02	39.99993
	f_1	500	f_180	3.16E+06	-180.0001	4.54E+00	13.14674
	f_2	1.00E+06	f_3dB	5.00E+02	-45.03151	7.07E+04	96.9897
	f_3	1.00E+07					
	C_1	1.00E-09					
	B_FB	1.00E-02					
				Feedback Amplifier			
Calculated	R_1	318309.89	Phase margin	63.00875			
	R_2	159.15495	Gain Margin (dB)	26.85318			
	R_3	15.915495					
	A_v0 (dB)	100					
	A_v (f_3dB)	96.9897					
	A_vFB (f_3dB)	36.9897					
	Phase=-(ATAN2(1,Frequency/f_1)+ATAN2(1,Frequency/f_2)+ATAN2(1,Frequency/f_3))*180/pi						
	Gain=A_v0/(SQRT(1+(Frequency/f_1)^2)*SQRT(1+(Frequency/f_2)^2)*SQRT(1+(Frequency/f_3)^2))						

FIGURE 5

Spreadsheet for gain and phase margin calculations

With the spreadsheet of Figure 5 the frequencies f_{1/β_FB} and f_{180} are readily found using GOAL SEEK to set the magnitude to $1/\beta_{FB}$ and the phase to –180° by varying the frequency.

Comment on the two-pole approximation

Figure 4 shows that the two-pole approximation to design for a Butterworth amplifier provides a good approximation for setting the lowest pole at f_1 provided the higher poles are not too close to f_2. In this example the overshoot in step response (Figure 4) is a bit larger than Butterworth because the third pole makes the gain margin a little lower than for the two-pole system comprised of only of poles at f_1 and f_2. The time to maximum overshoot is very nearly as expected.

If we move f_3 to very high frequency, the modified system approaches a two-pole system with phase margin 65.6°, a bit larger than our original system with 63° margin. So a two-pole estimate of phase margin is not a bad approximation, and accounts for the success of the two-pole Butterworth design.

However, the two-pole estimate of gain margin is terrible, as explained next. You may recall that a two-pole system is always stable, with $f_{180} = \infty$. Of course, no real amplifier has $f_{180} = \infty$, so this two-pole estimate of f_{180} is hopelessly inaccurate. Because the gain of any amplifier tends to zero at very high frequencies, the gain $\rightarrow 0$ as $f_{180} \rightarrow \infty$. That is, the poor estimate of f_{180} using a two-pole system makes the two-pole estimate of gain margin hopelessly inaccurate for any real amplifier ($\ell og_{10}(0)$ = – infinity), even if the two-pole system approximates the gain curve quite well over a range of frequencies from low values to somewhere above the second pole.

For these reasons, it is more useful to focus on phase margin as a stability estimate when using a two-pole approximation, not gain margin.