Calculators required, closed book; Friday, February 11, 8 AM – 10 AM

ECE 304: Exam 2 Spring '05 Solutions

NOTE: IN ALL CASES

1. Solve the problem on scratch paper
2. Once you understand your solution, put your answer on the answer sheet
3. Follow your answer with an outline of your solution. No points for answer without an outline of the solution. A mish-mash of computation is not an acceptable outline.

PRINT your name at the top of each answer sheet

Assume V_{TH} = 25.864 mV in all problems
`

Problem 1: Two port

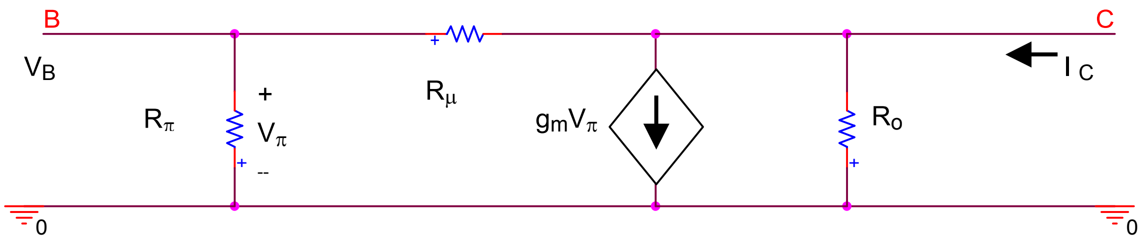

FIGURE 1
Circuit to be analyzed as a two port

Choosing as independent variables V_B and I_C, find the two-port network equivalent to Figure 1.

ANSWER

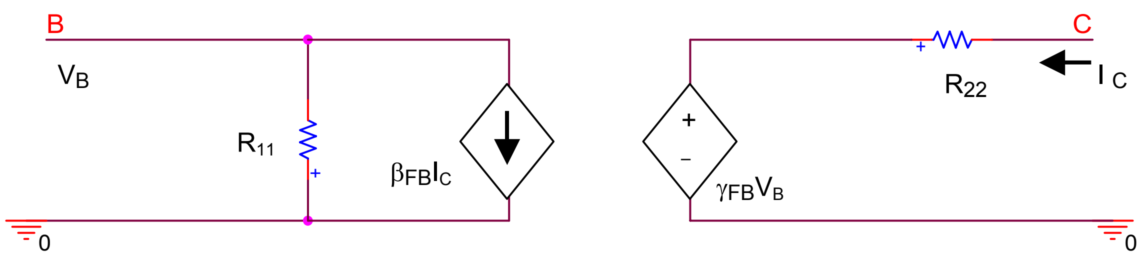

FIGURE 2
Two port equivalent to Figure 1

Components in Figure 2 are related to those in Figure 1 by

EQ. 1

$$R_{11} = R_\pi // \left(\frac{R_\mu + R_O}{1 + g_m R_O} \right), \qquad R_{22} = R_\mu // R_O$$

$$\beta_{FB} = -\frac{R_O}{R_\mu + R_O} \qquad \gamma_{FB} = \frac{R_O}{R_\mu + R_O}\left(1 - g_m R_\mu\right)$$

NOTE:

Some students wanted to interpret this circuit as the small-signal equivalent circuit of a bipolar, and so determined that $g_m = \beta/r_\pi$. I have no problem with that. However, as Figure 2 is a small-signal AC circuit, I_C in Figure 2 is a small-signal AC current. Therefore, I_C in Figure 2 *cannot* be interpreted as the DC collector current I_C that enters the definition $g_m = I_C/V_{TH}$.

OUTLINE

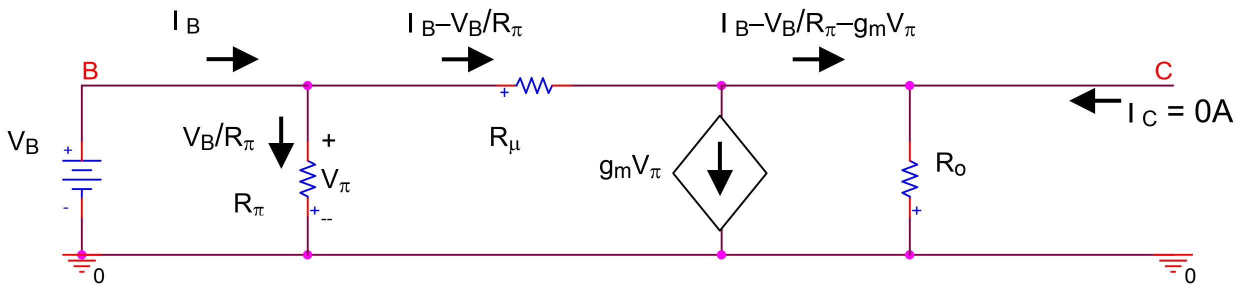

FIGURE 3

Open circuit right side to eliminate dependent current source in two port

Comparing Figure 3 with the two port with open-circuit on right and voltage source on the left we find from KVL

EQ. 2

$$V_B = (I_B – V_B/R_\pi)\,R_\mu + (I_B – V_B/R_\pi – g_m V_B)\,R_O;$$

Combing terms and using $R_{11} = V_B/I_B$, we find the given value for R_{11}.

Comparing the voltage at the right of Figure 3 with the two port we find from Ohm's law

EQ. 3

$$\gamma_{FB} V_B = (I_B – V_B/R_\pi – g_m V_B)\,R_O;$$

Collecting terms and using I_B as found in EQ. 2, we determine γ_{FB} as given.

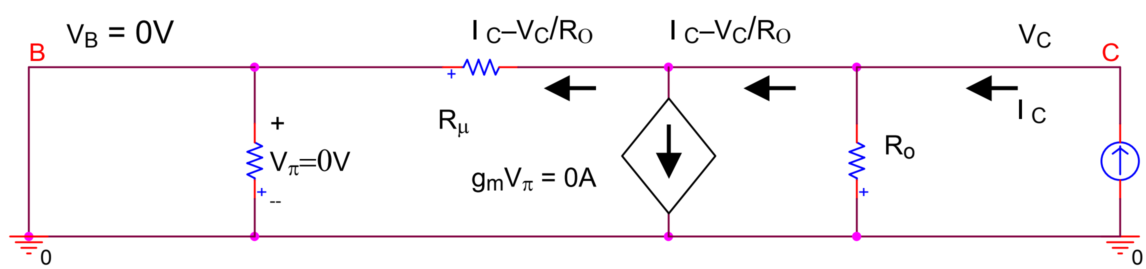

FIGURE 4

Short circuit left side to eliminate dependent voltage source in two port

With the left side shorted, V_π = 0V and the dependent current source in Figure 4 is an open circuit. Evidently R_μ and R_O are in parallel, and $R_{22} = V_C/I_C$ is as given. The short-circuit current from node B to ground is given by the current divider as $I_C\,(R_O/(R_O+R_\mu))$, and flows in the opposite direction to $\beta_{FB}\,I_C$ in the two port. Therefore, β_{FB} is the negative of this divider ratio, as given.

PROBLEM 2: DIFFERENTIAL AMPLIFIER

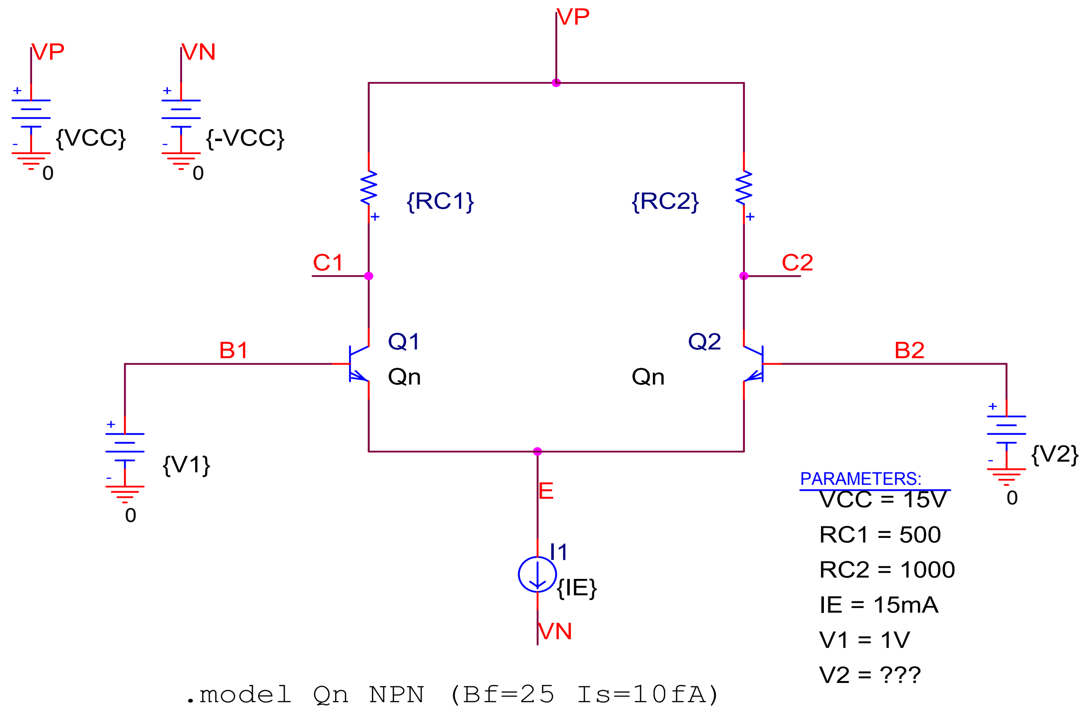

FIGURE 5
Differential amplifier; note that R_{C1} is not the same as R_{C2}, and transistors are ideal with infinite Early voltages

1. Make a table on your answer sheet like the one below, and fill it in. Assume that in the active mode V_{BE} = 650 mV and in saturation V_{CB} = –650 mV.

V2	Mode Q1	Mode Q2	VBE1	VBE2	VE	VC1	VC2	IC1	IC2	IB1	IB2
-4	A	CO	650mV	-4.35V	350 mV	7.79V	15V	14.4mA	0	577μA	0
1	A	A	650mV	650mV	350mV	11.39V	7.79V	7.21 mA	7.21 mA	288 μA	288 μA
2	CO	S	-350mV	650 mV	1.35V	15V	1.35V	0	13.65 mA	0	1.35 mA
4	CO	S	-2.35 V	650 mV	3.35 V	15V	3.35 V	0	11.65 mA	0	3.35 mA

Note

Outline your solution only for V_2 = -4 V and for V_2 = 4 V.

OUTLINE

V_2 = –4V

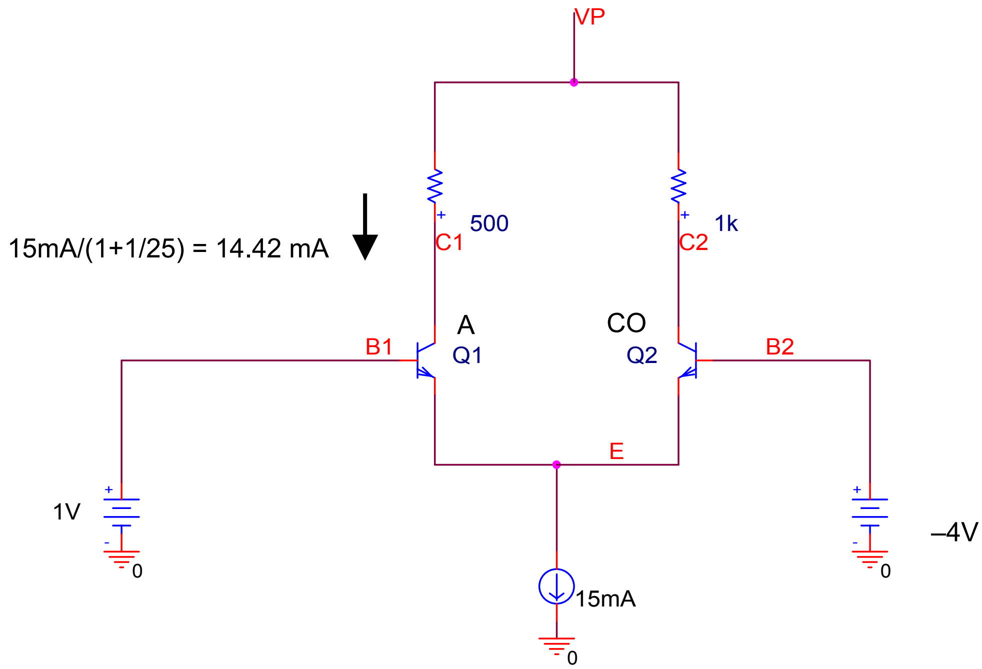

FIGURE 6
DA for V_2 =–4V

Because V_{BE}(Q2) << V_{BE}(Q1), Q2 is cut off. Assuming Q1 is active as a first guess, the collector current of Q1 is 14.42 mA, which implies a voltage drop across 500 Ω of 7.21 V. Hence, V_C(Q1) = 15–7.21 = 7.79V, which means V_{CB}(Q1) > 0, and Q1 is active as assumed, not saturated. Using the given V_{BE} = 650 mV, $V_E = V_1$ –0.65 = 350 mV. V_{BE}(Q2) = –4 – V_E = –4.35 V. As Q2 is cut off, there is no current through its collector resistor, and $V_{C2} = V_{CC}$ = 15V.

V_2 = 4V

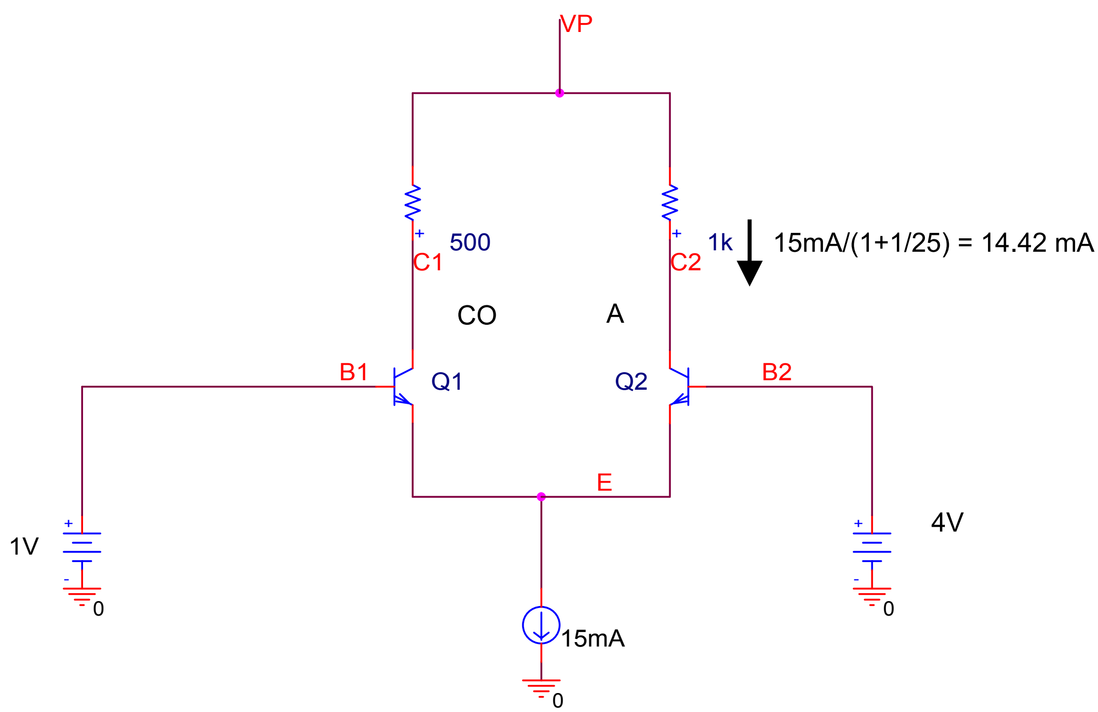

FIGURE 7
DA for V_2 = 4V using initial guess that Q2 is active

Following the same argument as for $V_2 = –4V$, we decide that Q1 is cut off and guess Q2 is active. The voltage V_{C2} is then $V_{C2} = 15 – 14.42mA \times 1\ k\Omega = 0.57V$, making the collector below the base, which is at $V_2 = 4V$. Therefore, Q2 is not active but saturated.

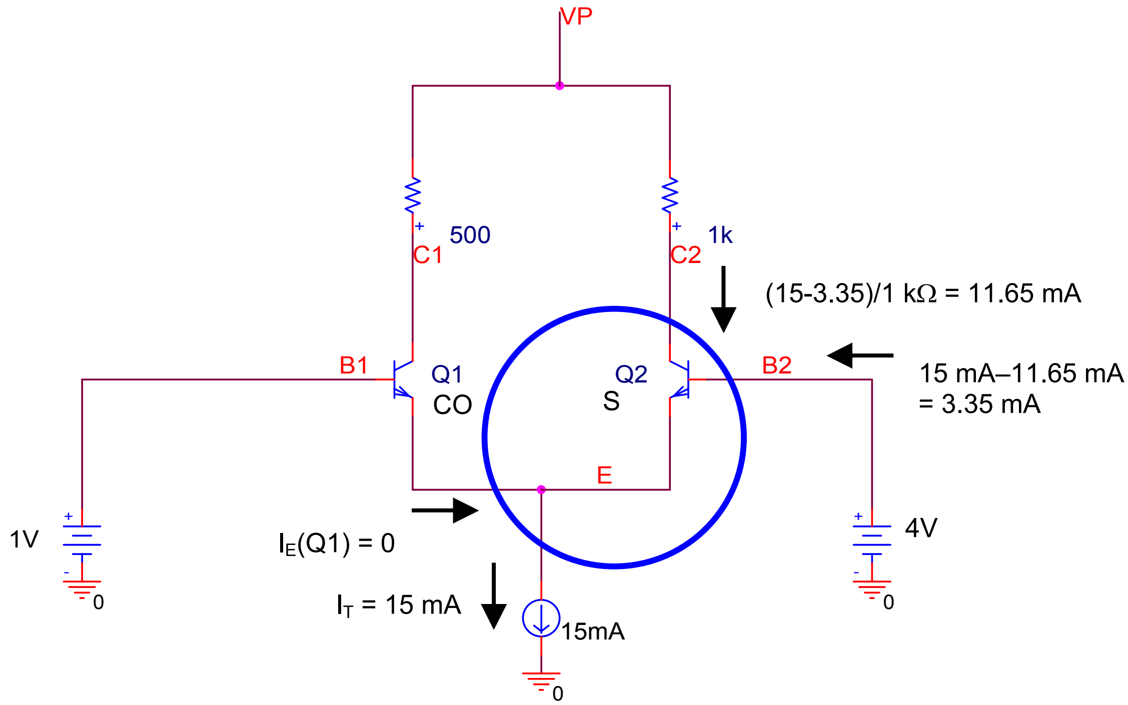

FIGURE 8
Revised version of DA for $V_2 = 4V$ assuming Q2 is saturated

If Q2 is saturated, $V_{CB} = –650mV$, so its collector voltage is $4V –650\ mV = 3.35V$. Therefore, its collector current is $I_{C2} = (15–3.35)/\ 1\ k\Omega = 11.65\ mA$. Using KCL for the surface shown as a circle in Figure 8, $I_B(Q2) + I_C(Q2) = I_T \rightarrow I_B(Q2) = 15\ mA – 11.65\ mA = 3.35\ mA$. No current contribution to I_T comes from Q1 because it is cut off. Also, $V_C(Q1) = V_{CC} = 15V$. The emitter voltage is controlled by Q2, as it carries the current, so $V_E = V_2 – V_{BE2} = 4 – 0.65 = 3.35\ V$. Consequently $V_{BE1} = 1V – 3.35V = –2.35V$.

2. For $V_1 = 1\ V$, voltage V_2 is adjusted to make V_{C1} and V_{C2} the same. Determine the value of V_2 for which $V_{C1} = V_{C2}$, and the corresponding value of $V_C = V_{C1} = V_{C2}$. Assume $V_{TH} = 25.864\ mV$ and use $V_{BE} = V_{TH}\ \ell n(I_C/I_S)$ to find V_{BE}.

ANSWER

$V_2 = 981.9786\ mV$; $V_C = 10.1923\ V$

OUTLINE

We find the collector currents using Ohm's law, and use these currents to find V_{BE} for Q1 and Q2. Then these V_{BE} values are used to find two estimates for the emitter voltage, V_E, determining V_2. The collector currents are

EQ. 4

$$I_{C1} = (15-V_C)/500\ \Omega; \quad I_{C2} = (15-V_C)/1\ k\Omega$$

The V_{BE} values are

EQ. 5

$$V_{BE1} = V_{TH}\ \ell n(I_{C1}/I_S); \quad V_{BE2} = V_{TH}\ell n(I_{C2}/I_S)$$

The emitter voltage is then

EQ. 6

$$V_E = V_1 - V_{BE1} = 1 - V_{TH}\ell n(I_{C1}/I_S) = V_2 - V_{BE2} = V_2 - V_{TH}\ell n(I_{C2}/I_S)$$

Using $\ell n(x) - \ell n(y) = \ell n(x/y)$ we find

EQ. 7

$$1 - V_2 = V_{TH}\ \ell n(I_{C1}/I_{C2}) = V_{TH}\ \ell n\left(\frac{15-VC}{500\ \Omega}\bullet\frac{1\,k\Omega}{15-VC}\right) = V_{TH}\ \ell n2.$$

Hence,

EQ. 8

$$V_2 = 1 - V_{TH}\ \ell n2 = 982.07\ mV.$$

To find V_C we use KCL at the emitter node to find

EQ. 9

$$\left(\frac{15-V_C}{500}+\frac{15-V_C}{1k}\right)\left(1+\frac{1}{\beta}\right) = 15\ mA \ \rightarrow V_C = 10.192V.$$

Problem 3: Noninverting amplifier

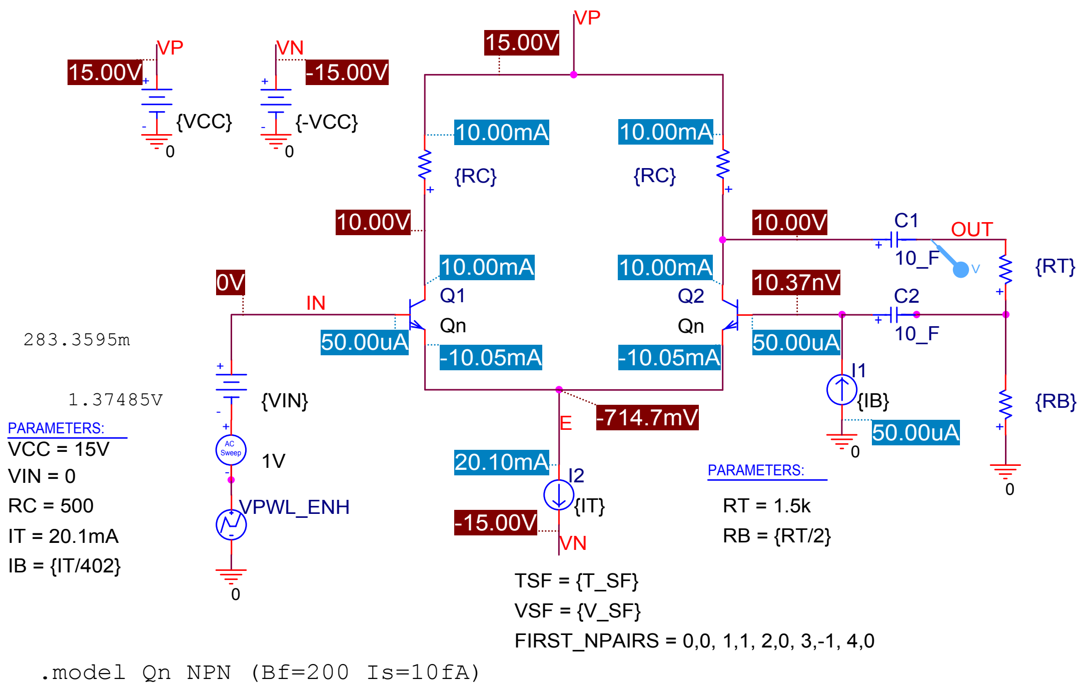

FIGURE 9
Noninverting amplifier

Calculators required, closed book; Friday, February 11, 8 AM – 10 AM

1. Assuming the DA behaves like an ideal operational amplifier, derive a formula that relates a change in V_{IN}, say ΔV_{IN}, and a change in V_{OUT}, say ΔV_{OUT}, as a function of R_T and R_B.

ANSWER

EQ. 10

$$\Delta V_{OUT} = (1 + R_T/R_B)\,\Delta V_{IN}$$

OUTLINE

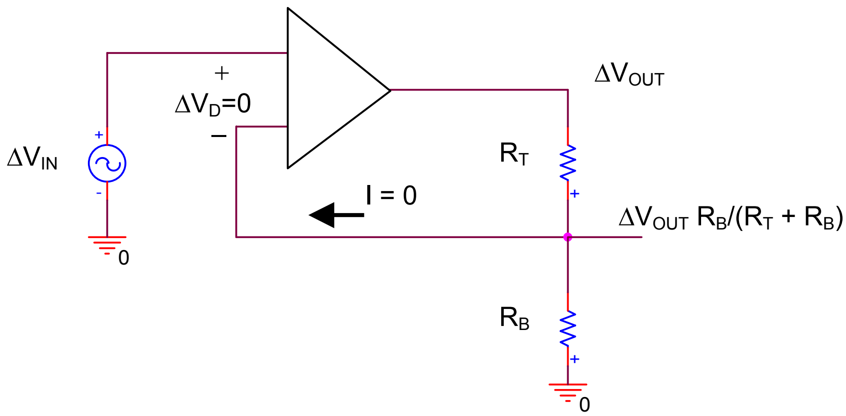

FIGURE 10

Ideal op amp circuit with infinite input R (I=0A) and infinite gain (ΔV_D = 0V)

Because ΔV_D = 0V, $\Delta V_{IN} = \Delta V_{OUT}\, R_B/(R_T + R_B) \rightarrow \Delta V_{OUT} = (1 + R_T/R_B)\,\Delta V_{IN}$

2. Assuming an input transient saw tooth input like Figure 11, and approximating the noninverting amplifier gain as $\Delta V_{OUT} = 3\,\Delta V_{IN}$, what is a formula for the maximum saw tooth amplitude for which the output and input waveforms from the circuit of Figure 9 approximately satisfy this ideal gain relation? What is its numerical value? What is the mechanism that limits this amplitude?

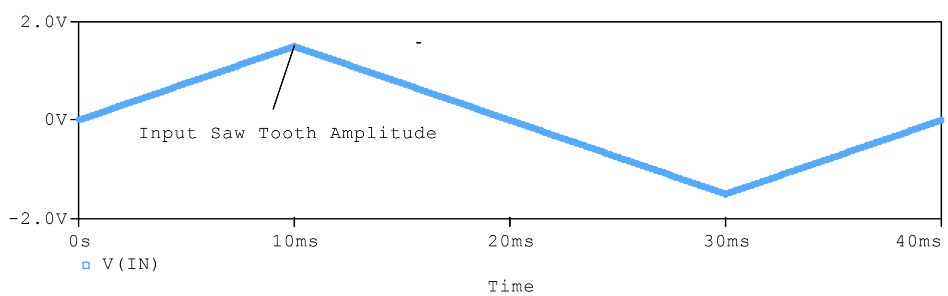

FIGURE 11

Example of input saw tooth, showing definition of amplitude

ANSWER

$$\hat{V} = \frac{I_C}{\frac{2}{R_T} + \frac{3}{R_C}}$$ =1.36 V limited by cut off.

OUTLINE

CUTOFF ANALYSIS

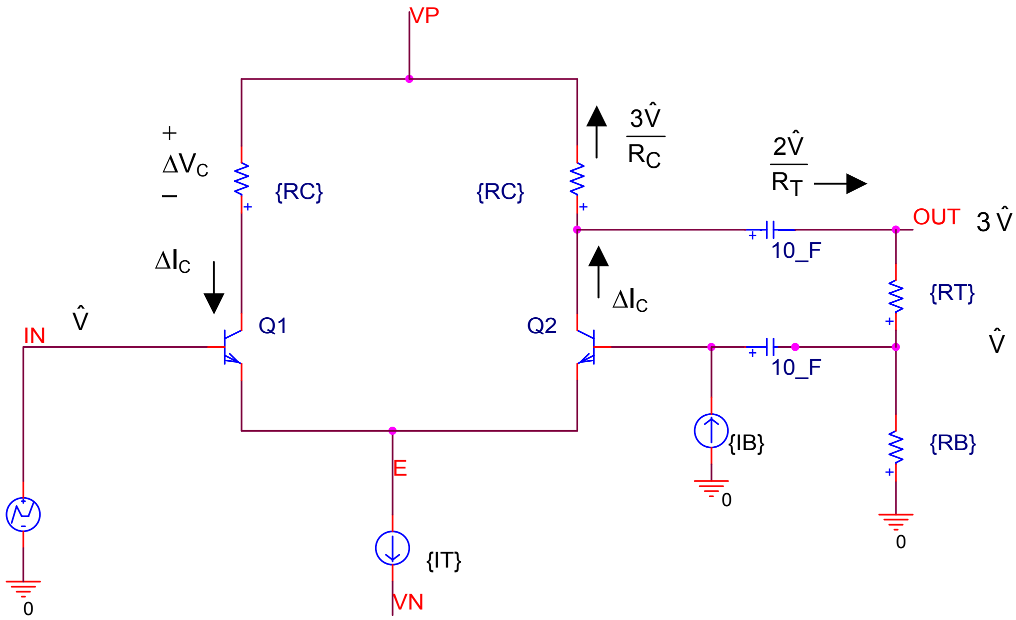

FIGURE 12
Cutoff analysis

The value of ΔI_C is found by analyzing the AC current at the collector of Q2. The output swings up by $3\hat{V}$, and the base of Q2 by $\hat{V}$, placing a drop of $2\hat{V}$ across R_T. Also the voltage across R_C above Q2 drops by $3\hat{V}$. Consequently the change in collector current of Q2 is an upward current

EQ. 11

$$\Delta I_C = \frac{2\hat{V}}{R_T} + \frac{3\hat{V}}{R_C}.$$

Cutoff of Q2 will occur if this AC current cancels the DC collector current of Q2, I_C. Therefore, the condition for cut off of Q2 is

EQ. 12

$$I_C = \Delta I_C = \frac{2\hat{V}}{R_T} + \frac{3\hat{V}}{R_C}.$$

Solving for $\hat{V}$ and using the given value of I_C = 10 mA, the maximum swing that will not cause cut off is

EQ. 13

$$\hat{V} = \frac{I_C}{\frac{2}{R_T} + \frac{3}{R_C}} = \frac{10\text{ mA}}{\frac{2}{1.5\text{ k}\Omega} + \frac{3}{500\ \Omega}} = 1.36\text{ V}.$$

SATURATION ANALYSIS

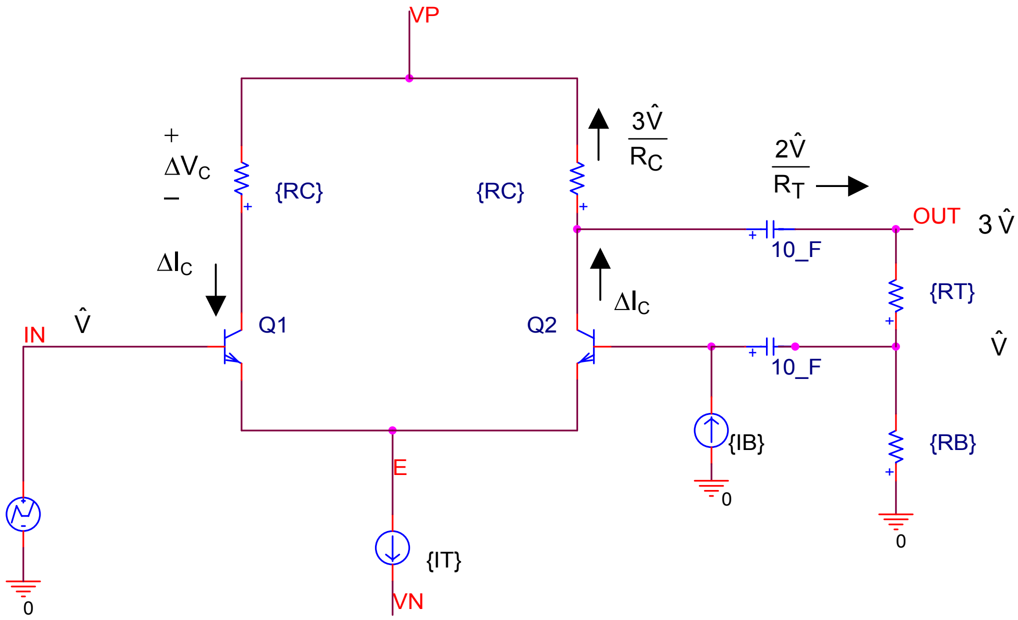

FIGURE 13
Saturation analysis

When the input swings up by $\hat{V}$, the collector current of Q1 increases by ΔI_C. This increase in current causes an increased voltage drop across R_C of $\Delta V_C = \Delta I_C R_C$. If this drop is large enough, Q1 will saturate. The condition for saturation is $V_{CB}(Q1) = 0V$, or

EQ. 14

$$V_C - \Delta V_C = \hat{V} .$$

Here V_C = Q-point value of V_C = 10 V. To find a value for $\Delta V_C = \Delta I_C R_C$ we need a value for ΔI_C. The value of ΔI_C is found by analyzing the AC current at the collector of Q2. The output swings up by $3\hat{V}$, and the base of Q2 by $\hat{V}$, placing a drop of $2\hat{V}$ across R_T. Also the voltage across R_C above Q2 drops by $3\hat{V}$. Consequently the change in collector current of Q2 is

EQ. 15

$$\Delta I_C = \frac{2\hat{V}}{R_T} + \frac{3\hat{V}}{R_C} .$$

Because the sum of the emitter currents of Q1 and Q2 is a constant value I_T, the decrease in collector current of Q2 is the same as the increase in collector current of Q1. Hence, we have determined ΔI_C. Substituting into EQ. 14 we find

EQ. 16

$$V_C - \Delta V_C = V_C - \Delta I_C R_C = V_C - \left(\frac{2\hat{V}}{R_T} + \frac{3\hat{V}}{R_C}\right) R_C = \hat{V} .$$

Collecting terms we find the maximum swing that will not cause saturation of Q1, namely

EQ. 17

$$\hat{V} = \frac{V_C}{4 + 2\frac{R_C}{R_T}} = \frac{10}{4 + 2\frac{500\ \Omega}{1.5\ k\Omega}} = 2.14\ V$$

Comparing EQ. 13 with EQ. 17 we see that it is cutoff that provides the more serious limitation, so the maximum swing will be $\hat{V}$ = 1.36V.

3. What is the formula for the small-signal gain of the amplifier at the bias point shown in Figure 9, and its numerical value?

ANSWER

The gain V_O/V_I = $\dfrac{\beta R_T + (\beta+1)R_B}{2r_\pi\left(1+\dfrac{R_T+R_B}{R_C}\right)+R_B\left(\beta+1+\dfrac{R_T}{R_C}\right)}$ = 2.84 V/V.

OUTLINE

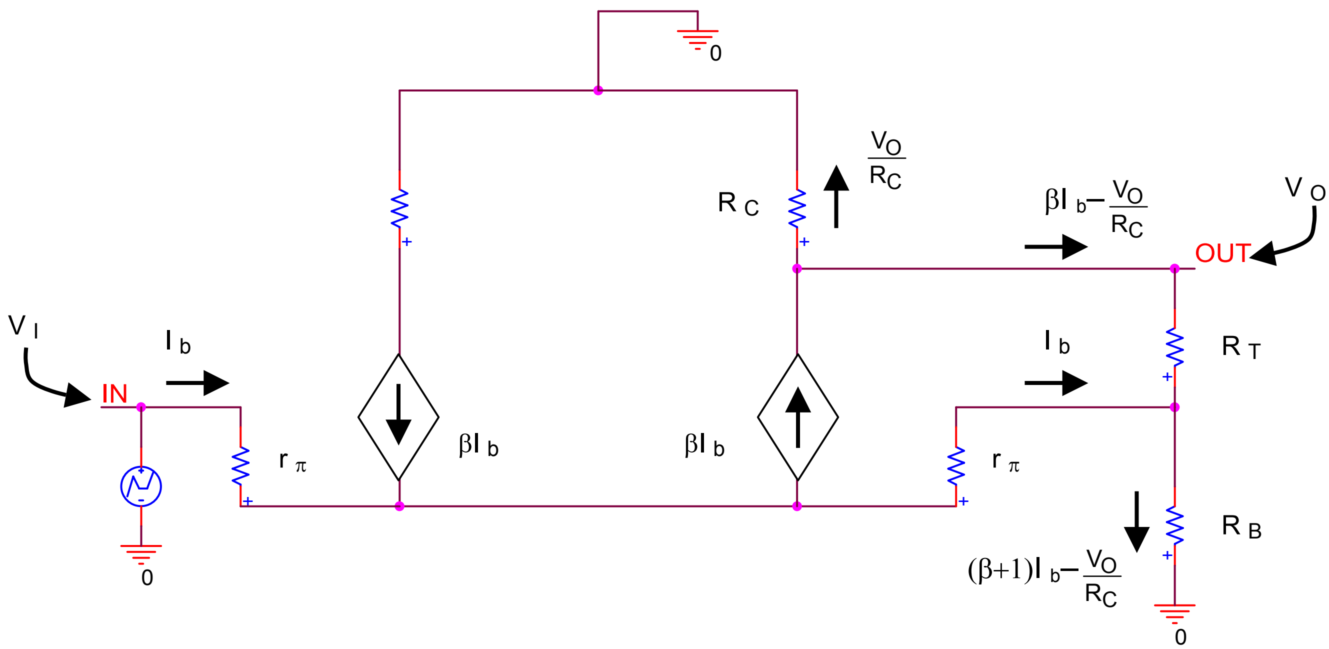

FIGURE 14
Small-signal circuit

KVL down the feedback network provides

EQ. 18

$$V_O = \left(\beta I_b - \frac{V_O}{R_C}\right)\cdot\left(R_T + R_B\right) + I_b R_B\,,$$

which can be solved for I_b.

EQ. 19

$$I_b = V_O\left(\frac{1+\dfrac{R_T+R_B}{R_C}}{\beta R_T + (\beta+1)R_B}\right)$$

KVL from node IN through R_B to ground provides

EQ. 20

$$V_I = I_b(2r_\pi) + \left((\beta+1)I_b - \frac{V_O}{R_C}\right)R_B\,.$$

Substituting for I_b from EQ. 19 and collecting terms we find the reciprocal of the gain is

EQ. 21

$$\frac{V_I}{V_O} = \left(2r_\pi + (\beta+1)R_B\right)\cdot\frac{1+\dfrac{R_T+R_B}{R_C}}{\beta R_T + (\beta+1)R_B} - \frac{R_B}{R_C}\,.$$

Using the given collector current, I_C = 10 mA, we find $r_\pi = \beta V_{TH}/I_C$ = 200 × 25.864 mV/10 mA = 517 Ω. Therefore, the reciprocal gain is

EQ. 22

$$\frac{V_I}{V_O} = (2r_\pi + (\beta+1)R_B)\cdot\frac{1+\frac{R_T+R_B}{R_C}}{\beta R_T+(\beta+1)R_B} - \frac{R_B}{R_C} = \frac{2r_\pi\left(1+\frac{R_T+R_B}{R_C}\right)+R_B\left(\beta+1+\frac{R_T}{R_C}\right)}{\beta R_T+(\beta+1)R_B} = \frac{1}{2.84\,V/V}.$$

TWO-PORT APPROACH

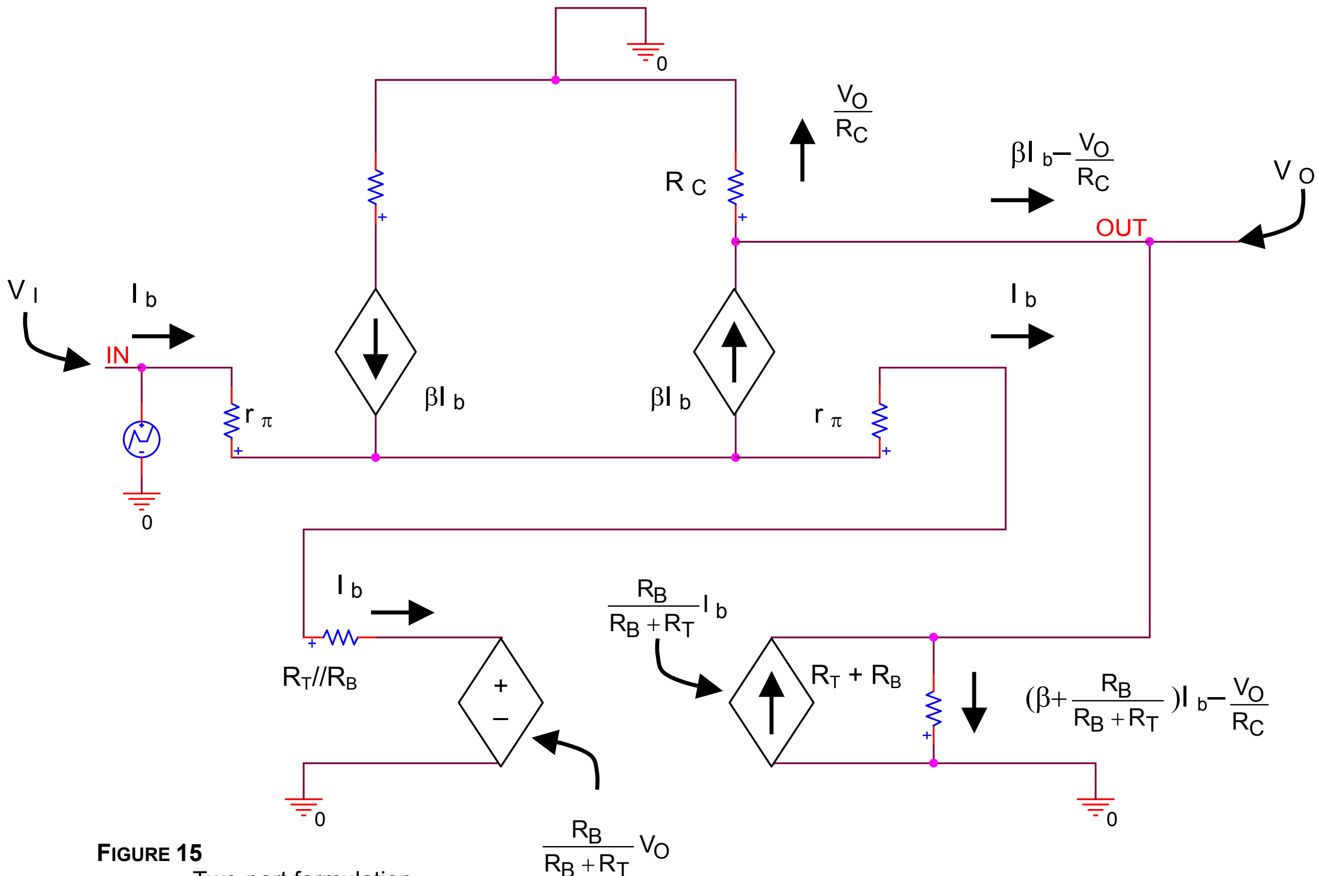

FIGURE 15
Two-port formulation

Figure 15 shows the two-port treatment of the feedback network, another way to find the gain. KVL from the output to ground provides:

EQ. 23

$$V_O = \left[\left(\beta + \frac{R_B}{R_B+R_T}\right) I_b - \frac{V_O}{R_C}\right](R_T + R_B).$$

EQ. 23 results in the same expression for I_b as EQ. 19. KVL from the input to ground provides

EQ. 24

$$V_I = I_b(2r_\pi + R_T // R_B) + \frac{R_B}{R_B+R_T}V_O = \left(\frac{1+\frac{R_T+R_B}{R_C}}{\beta R_T+(\beta+1)R_B}\right)\cdot(2r_\pi + R_T // R_B)V_O + \frac{R_B}{R_B+R_T}V_O,$$

where EQ. 19 was used for I_b. The reciprocal of the gain is then

EQ. 25

$$\frac{V_I}{V_O} = \left(\frac{1 + \dfrac{R_T + R_B}{R_C}}{\beta R_T + (\beta+1) R_B} \right) \cdot \left(2r_\pi + R_T // R_B\right) + \frac{R_B}{R_B + R_T} ,$$

which is equivalent to EQ. 22.

DIGRESSION

That answers the question. Below is some discussion intended to make connection with work in class on the effects of feedback and the effects of loading factors.

Using some algebra, the gain also can be rewritten in terms of a loaded gain and a product of loading factors, as below. The gain with no loading factors is

EQ. 26

$$A_\upsilon = \frac{\left(\beta + \dfrac{R_B}{R_B + R_C}\right) R_C}{2r_\pi} \approx \frac{I_C R_C}{2V_{TH}} \left(1 + \frac{R_B}{\beta\left(R_B + R_C\right)}\right) .$$

This gain is very nearly the same as the diff amp gain without the feedback network. The loaded gain is

EQ. 27

$$A_\upsilon(\text{loaded}) = A_\upsilon \left(\frac{R_C //(R_T + R_B)}{R_C} \right) \cdot \left(\frac{2r_\pi}{2r_\pi + R_T + R_B} \right)$$

The loaded gain shows the importance of the loading factors in producing a maximum in the gain for some choice of R_T. A maximum occurs because the first loading factor increases from zero to one with increasing R_T (remember $R_T \rightarrow 0$ means $R_B \rightarrow 0$ too, because their ratio is fixed), and the second factor decreases from one to zero as R_T increases, as we have seen before in class and in the lab spreadsheet. Finally, the gain with feedback can be written as

EQ. 28

$$\frac{V_O}{V_I} = \frac{A_\upsilon(\text{loaded})}{1 + \dfrac{R_B}{R_T + R_B} A_\upsilon(\text{loaded})} ,$$

where the ratio $R_B/(R_T+R_B)$ is the voltage feedback factor β_{FB}. EQ. 28 shows that, for large loaded gain, the gain with feedback of the noninverting amplifier becomes the ideal value of the circuit with an ideal op amp, that is,

EQ. 29

$$\frac{V_O}{V_I} = \frac{1}{\dfrac{1}{A_\upsilon(\text{loaded})} + \dfrac{R_B}{R_T + R_B}} = \left(1 + \frac{R_T}{R_B}\right) \cdot \left(\frac{1}{1 + \dfrac{1 + \dfrac{R_T}{R_B}}{A_\upsilon(\text{loaded})}} \right) \approx 1 + \frac{R_T}{R_B} ,$$

with the deviation from ideal gain determined by the ratio of the ideal gain to the loaded gain.