ECE 304: Exam 4 Spring '06 Solutions

NOTE: IN ALL CASES

1. Solve the problem on scratch paper. Then, once you understand your answer, compose your answer sheet as follows:
2. Put your answer first, and
3. Follow your answer with an outline of your solution. Each major step in the outline should
3.1. Begin with a heading that describes the objective of that step, and should
3.2. Have a body where actual work is done, not just hand waving, and should
3.3. Conclude with a quantitative statement of the major result for that step (a number or formula or both).

For all problems take the thermal voltage as V_{TH} = 25.864 mV.

Problem 1: Bode plot

1. Make a Bode dB gain plot vs. frequency in Hz (in powers of ten) for the expression below. Be sure to label (frequency, gain) coordinates at each breakpoint and provide the slopes of all segments in dB/decade.

2. Make a Bode phase plot in degrees vs. frequency in Hz (in powers of ten) for the expression below. Be sure to label (frequency, phase) coordinates at each breakpoint and provide the slopes of all segments in degrees/decade.

$$A_{\upsilon}(f) = 1000\frac{200jf(20+jf)(10k+jf)}{(10+jf)(100+jf)(5k+jf)(50k+jf)}$$

where f=frequency in Hz, and k=1000

Answer: The PSPICE gain and phase plots are shown in Figure 2 and Figure 3

Cascade of stages

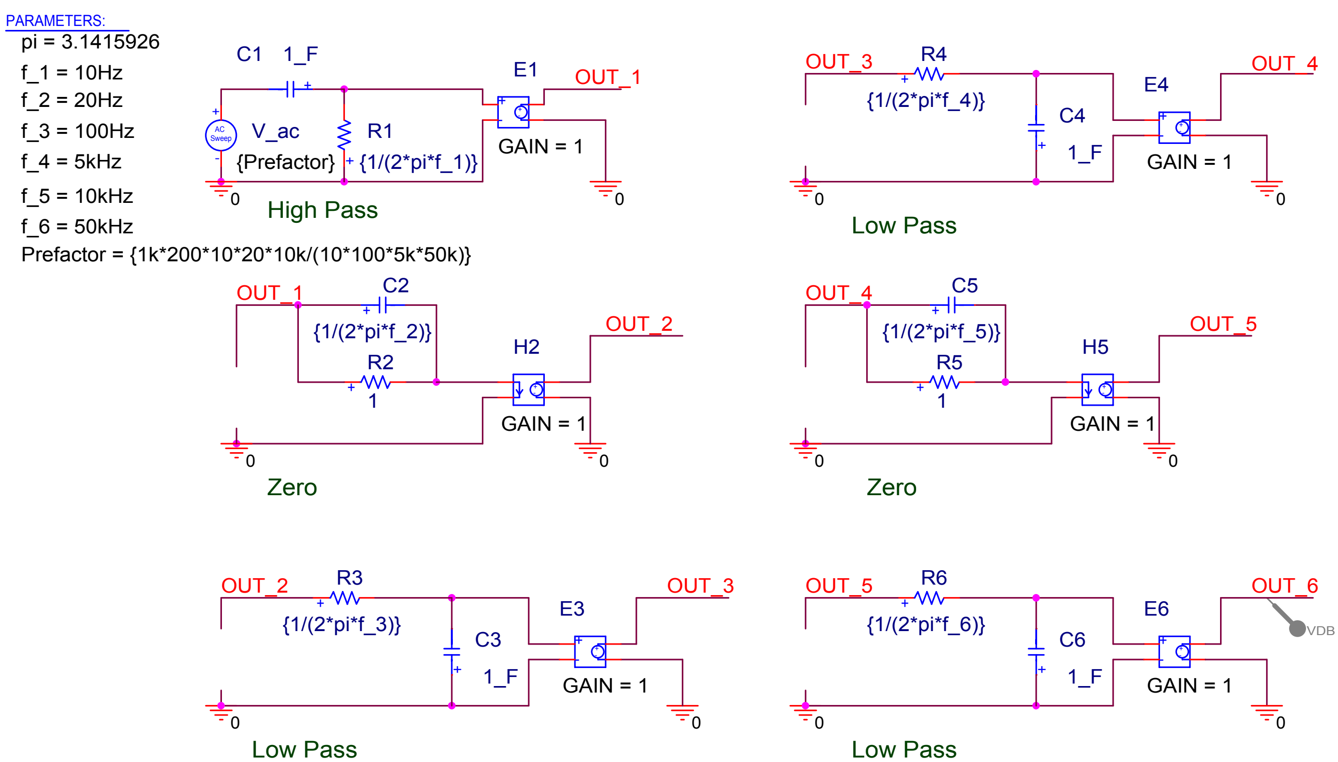


FIGURE 1
PSPICE setup for calculating gain

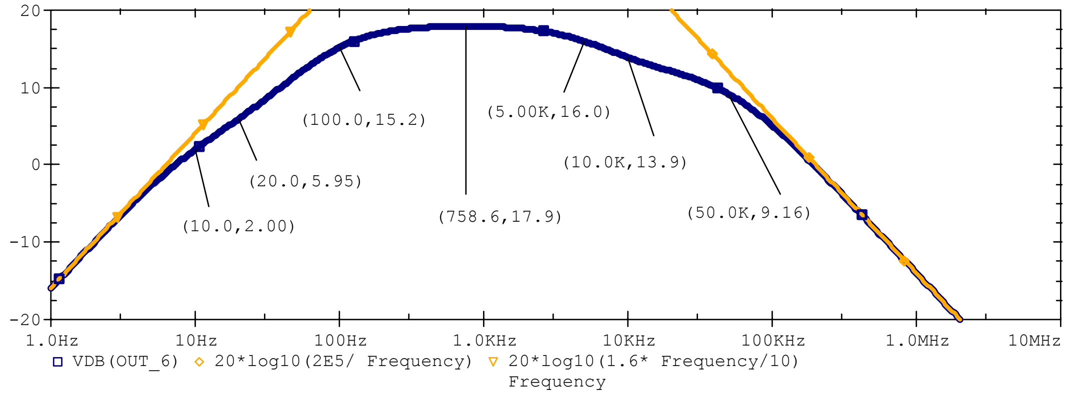


FIGURE 2

Gain magnitude plot for Problem 1; low- and high-frequency asymptotes are shown (EQ. 1 and EQ. 2)

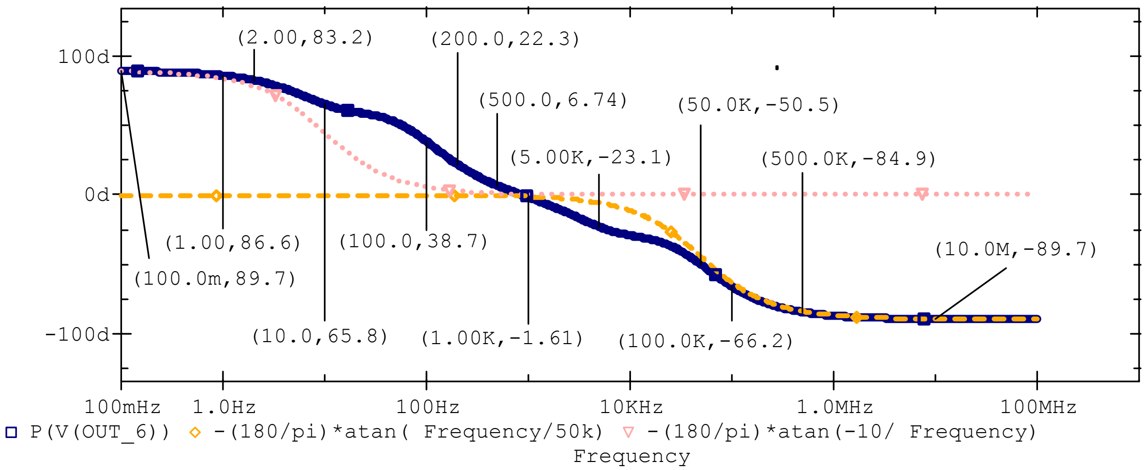


FIGURE 3

Gain phase plot for Problem 1; low- and high-frequency asymptotes are shown (EQ. 1 and EQ. 2)

The hand solution proceeds as follows. First we put the gain in the form of standard factors as shown below.

$$A_\upsilon(f) = 1000\,\frac{200\times 20\times 10k}{100\times 5k\times 50k}\,\frac{j\frac{f}{10}\left(1+j\frac{f}{20}\right)\left(1+j\frac{f}{10k}\right)}{\left(1+j\frac{f}{10}\right)\left(1+j\frac{f}{100}\right)\left(1+j\frac{f}{5k}\right)\left(1+j\frac{f}{50k}\right)}\,.$$

Next we look at the form of the gain at very low frequencies, f << 10 Hz:

EQ. 1

$$A_\upsilon(f) = 1.60j\frac{f}{10}\,,$$

which shows that the gain is initially rising at 20 dB/decade with a phase of 90°.
Then we look at high frequencies, f >> 50 kHz:

EQ. 2

$$A_\upsilon(f) = 1000\,\frac{200\times 20\times 10k}{100\times 5k\times 50k}\,\frac{j\frac{f}{10}\left(j\frac{f}{20}\right)\left(j\frac{f}{10k}\right)}{\left(j\frac{f}{10}\right)\left(j\frac{f}{100}\right)\left(j\frac{f}{5k}\right)\left(j\frac{f}{50k}\right)} = 1000\,\frac{200}{100}\,\frac{1}{\left(j\frac{f}{100}\right)} = 2\times 10^5\,\frac{1}{jf}\,,$$

which shows that the gain at high frequencies drops at 20 dB/decade with a phase of –90°. These asymptotes are shown in Figure 2 and Figure 3.

Next we look at the individual factors in the gain, grouped as one of the standard forms: high-pass, low-pass, or zero, as shown below

$$A_{\upsilon}(f) = 1.60 \circ \left(\frac{j\frac{f}{10}}{1+j\frac{f}{10}} \right) \circ \left(1+j\frac{f}{20}\right) \circ \frac{1}{\left(1+j\frac{f}{100}\right)} \circ \frac{1}{\left(1+j\frac{f}{5k}\right)} \circ \left(1+j\frac{f}{10k}\right) \circ \frac{1}{\left(1+j\frac{f}{50k}\right)}$$

The factors are: high-pass filter, zero, low-pass filter, low-pass filter, zero and low pass filter, arranged in order of increasing zero and pole frequencies.

BODE GAIN MAGNITUDE PLOT

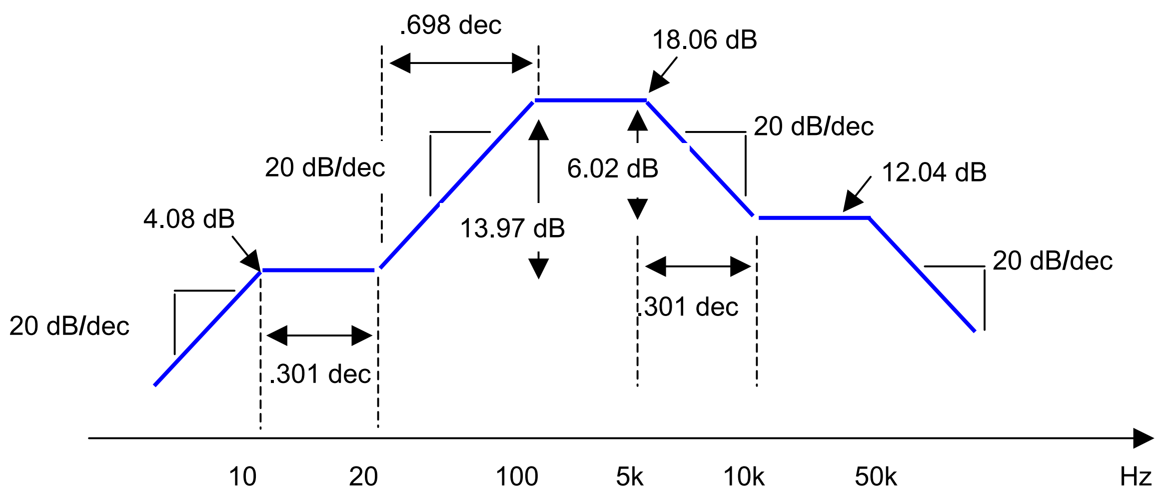


FIGURE 4

Bode magnitude plot

For the phase plot we tabulate the range of each factor with the slope it contributes - Figure 5.

Interval		1 to 2	2 to 10	10 to 100	100 to 200	200 to 500	500 to 1k	1k to 5k	5k to 50k	50k to 100k	100k to 500k	500k and above
Frequency	1	2	10	100	200	500	1000	5000	50000	100000	500000	1.00E+06
High pass		-45	-45	-45								
Zero			45	45	45							
Low pass				-45	-45	-45	-45					
Low pass							-45	-45	-45			
Zero								45	45	45		
Low pass									-45	-45	-45	
Slope (deg/dec)		-45.00	0.00	-45.00	0.00	-45.00	-90.00	0.00	-45.00	0.00	-45.00	0.00
Decades		0.30	0.70	1.00	0.30	0.40	0.30	0.70	1.00	0.30	0.70	0.30
Deg drop in interval		-13.55	0.00	-45.00	0.00	-17.91	-27.09	0.00	-45.00	0.00	-31.45	0.00
Phase at frequency	90	76.45	76.45	31.45	31.45	13.55	-13.55	-13.55	-58.55	-58.55	-90.00	-90.00

FIGURE 5

Tabulation of slope contributions; each standard factor has a slope of ± 45° over a two-decade range; the table makes it clear where each factor changes the overall slope

	Interval		1 to 2	2 to 10		20 to 100	100 to 200	200 to 500	500 to 1k	1k to 5k		5k to 50k	50k to 100k	100k to 500k
	Frequency	1.00	2.00	10.00	20.00	100.00	200.00	500.00	1000.00	5000.00	10000.00	50000.00	100000.00	500000.00
Phase	High pass	90.00	76.45	45.00	31.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Zero	0.00	0.00	31.45	45.00	76.45	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
	Low pass	0.00	0.00	0.00	13.55	45.00	58.55	76.45	90.00	90.00	90.00	90.00	90.00	90.00
	Low pass	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.55	45.00	58.55	90.00	90.00	90.00
	Zero	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	31.45	45.00	76.45	90.00	90.00
	Low pass	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.55	45.00	58.55	90.00
	Phase at frequency	90.00	76.45	76.45	62.91	31.45	31.45	13.55	-13.55	-13.55	-27.09	-58.55	-58.55	-90.00

FIGURE 6

Alternative approach to phase plot using 45°/decade slopes to approximate the phase of factors ($1+jf/f_0$)

Figure 6 shows a different approach to the phase plot, tabulating the phase of each factor of ($1+jf/f_O$). At the bottom of the table, to get the total phase the individual phases are combined by adding or subtracting, depending whether they are zeros or poles.

From either table, it is easy to construct the Bode phase plot, as shown in Figure 7.

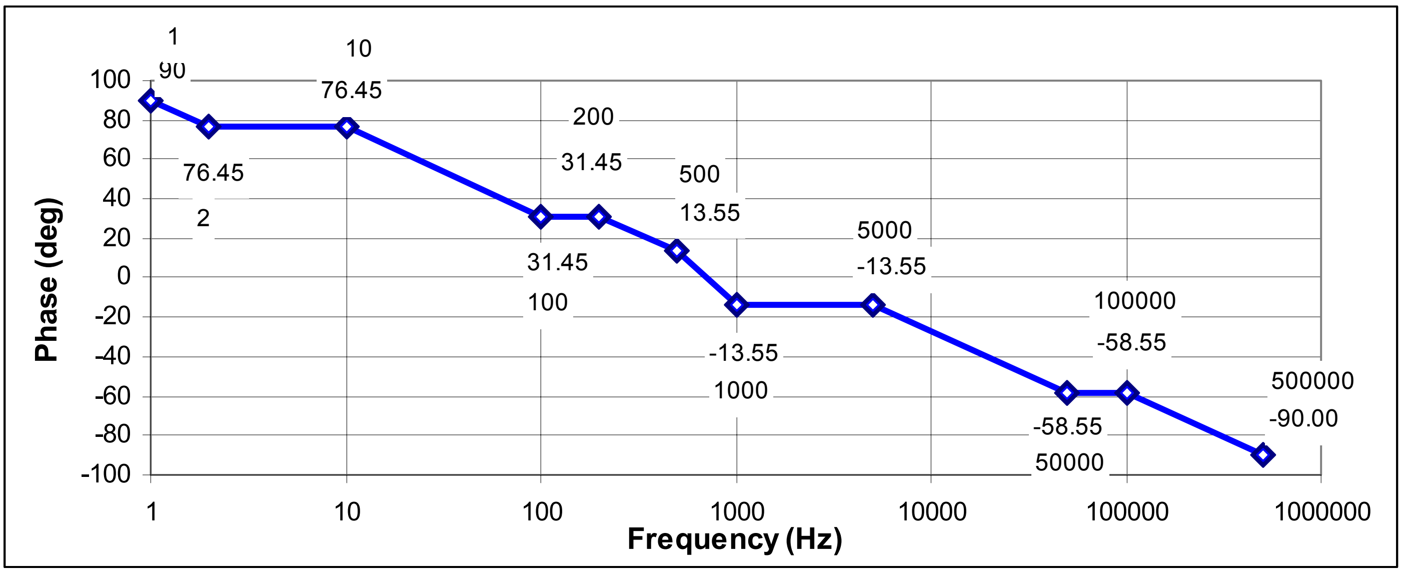


FIGURE 7

Bode phase plot based on Figure 5

Comparison of Figure 3 and Figure 7 shows that the Bode plot is a rather rough approximation to the actual phase plot for the gain. For example, if we keep the Bode approximation that a pole or zero transitions over a decade in frequency above and below its location, but evaluate the inverse tangents exactly, we obtain the results shown in Figure 8.

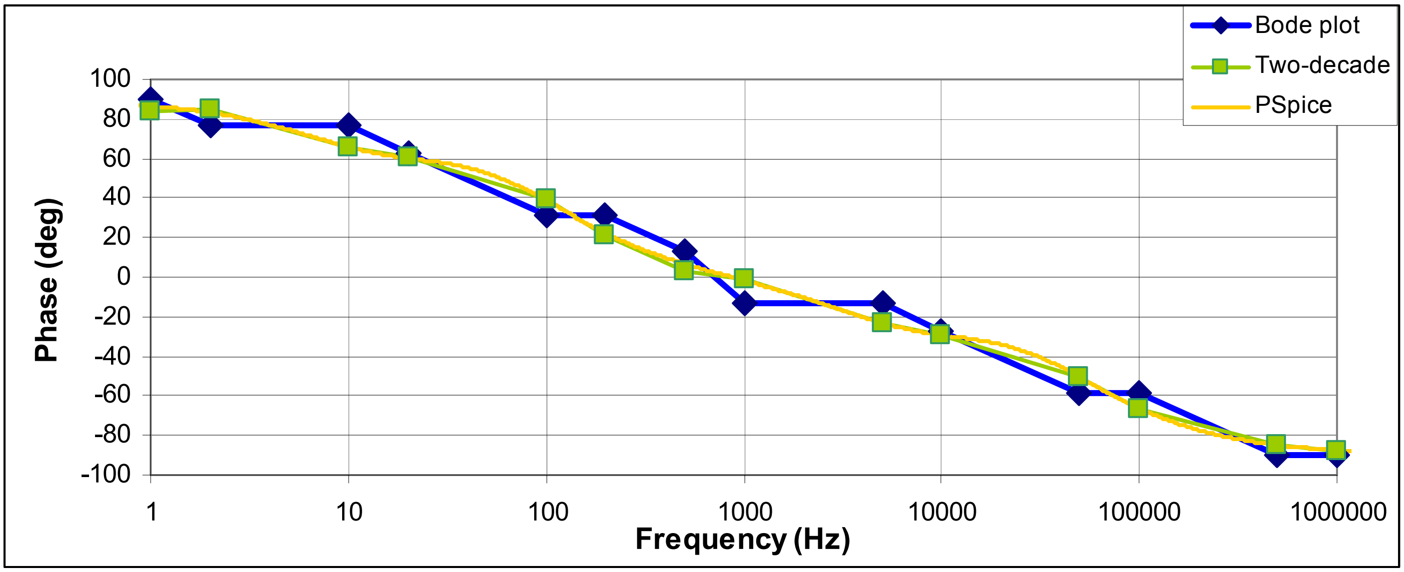


FIGURE 8

Effect of approximating inverse tangents by straight lines in the Bode plot; the curve "two-decade" evaluates the inverse tangents exactly (and agrees closely with PSPICE), while the curve "Bode plot" uses the straight-line approximation of 45°/decade

Problem 2: Low corner-frequency design

Follow the outline procedure at the top of the exam with headings for each major step in the solution. No points for answer without an outline of the solution.
A mish-mash of calculations is not an acceptable outline.

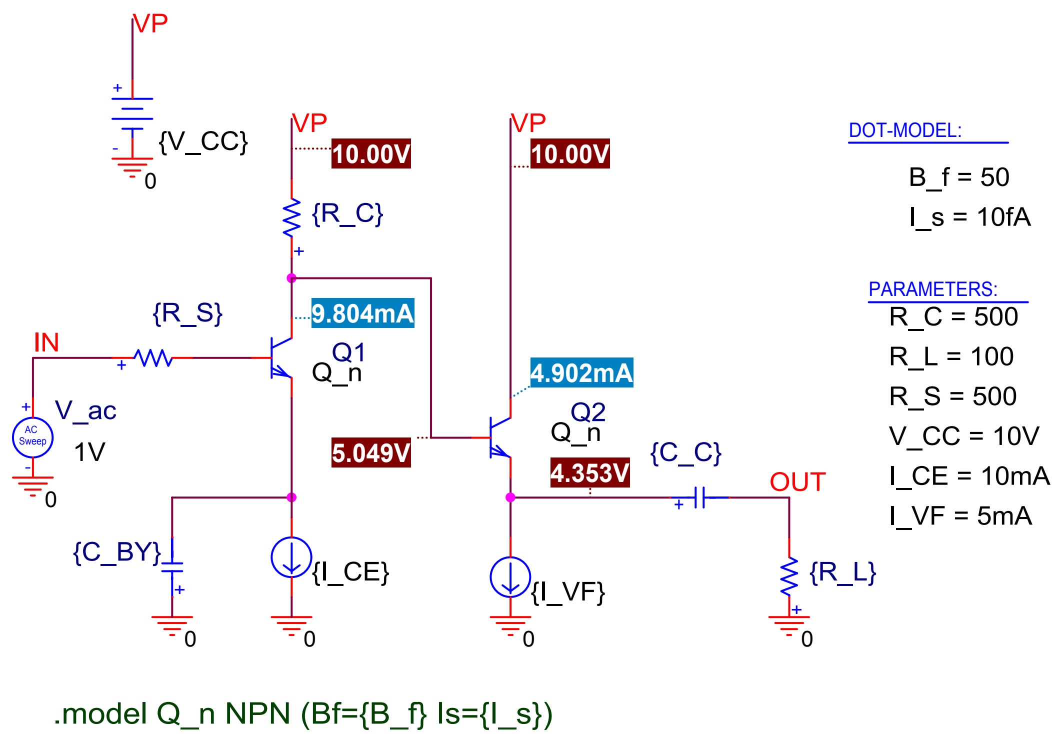


FIGURE 9

Amplifier for Problem 2

Design the amplifier in Figure 9 to have a lower 3 dB corner frequency of f_{3dB} = 130 Hz.
Assume $C_C = C_{BY}$

Answer: $C_C = C_{BY} = 109.45\ \mu F$.
Outline: We use the short-circuit time constant method. We find the resistances seen by the low-frequency capacitors C_{BY} and C_C. According to the dot-model statements, there are no other capacitances in the circuit.

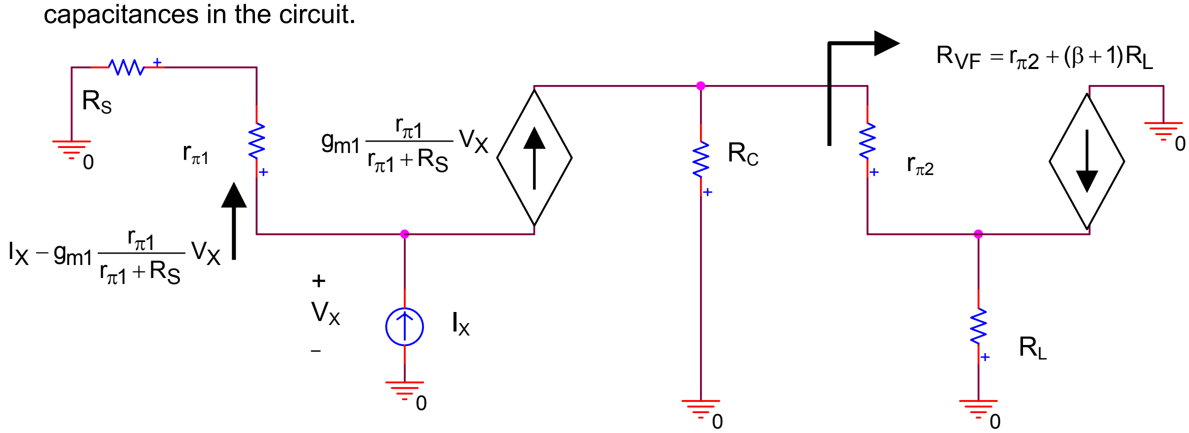


FIGURE 10
Circuit to find the resistance seen by the bypass capacitor C_{BY}

By voltage division, the voltage across $r_{\pi 1}$ is given by EQ. 3 below.
EQ. 3

$$V_{\pi 1} = \frac{r_{\pi 1}}{r_{\pi 1}+R_S}V_X\ ,$$

which determines the current in the dependent source, $g_{m1}V_{\pi 1}$. Therefore the current in $r_{\pi 1}$ is
EQ. 4

$$I_{b1} = I_X - g_{m1}\frac{r_{\pi 1}}{r_{\pi 1}+R_S}V_X = \frac{V_X}{r_{\pi 1}+R_S}\ .$$

Therefore, the resistance seen by C_{BY} is ($g_{m1}r_{\pi 1} = \beta$)
EQ. 5

$$R_{C\_BY} = \frac{V_X}{I_X} = \frac{r_{\pi 1}+R_S}{\beta + 1}$$

RESISTANCE SEEN BY C_C

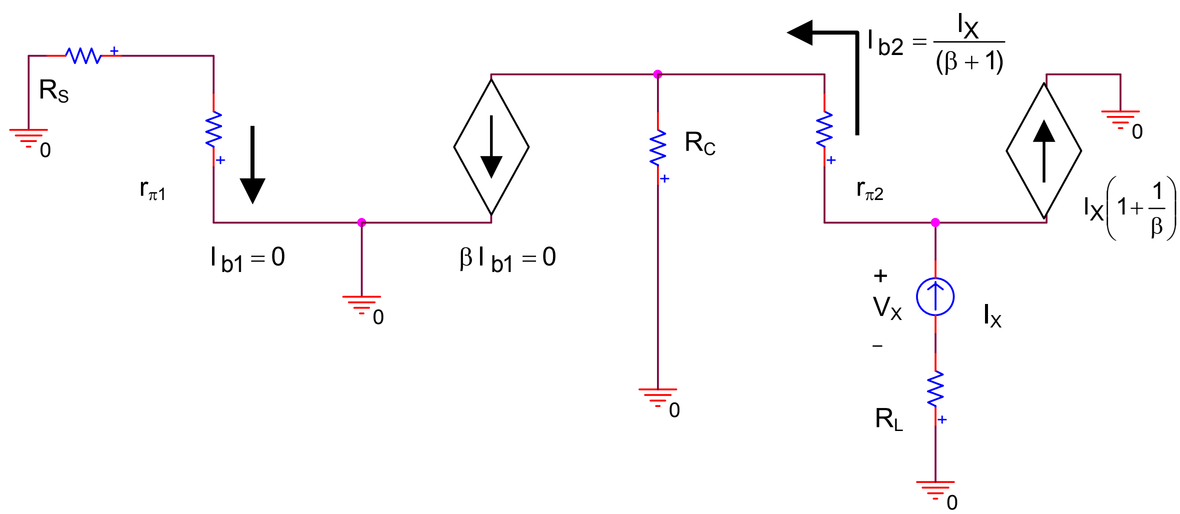


FIGURE 11
Determining the resistance seen by the coupling capacitor C_C

By KVL across $r_{\pi 1}$, $I_{b1} = 0$, so the CE dependent source is also zero. Therefore, all the base current in the VF goes through the collector resistor, R_C, and we find
EQ. 6

$$R_{C\_C} = \frac{r_{\pi 2} + R_C}{\beta + 1} + R_L \, .$$

FINDING C_C AND C_{BY}

The 3dB corner frequency is given by
EQ. 7

$$f_{3dB} = \frac{1}{2\pi}\left(\frac{1}{C_C R_{C\_C}} + \frac{1}{C_{BY} R_{C\_BY}}\right) = \frac{1}{2\pi C_C}\left(\frac{1}{R_{C\_C}} + \frac{1}{R_{C\_BY}}\right),$$

where the second form uses the specification that $C_C = C_{BY}$. Solving for C_C we find
EQ. 8

$$C_C = \frac{1}{2\pi f_{3dB}}\left(\frac{1}{R_{C\_C}} + \frac{1}{R_{C\_BY}}\right).$$

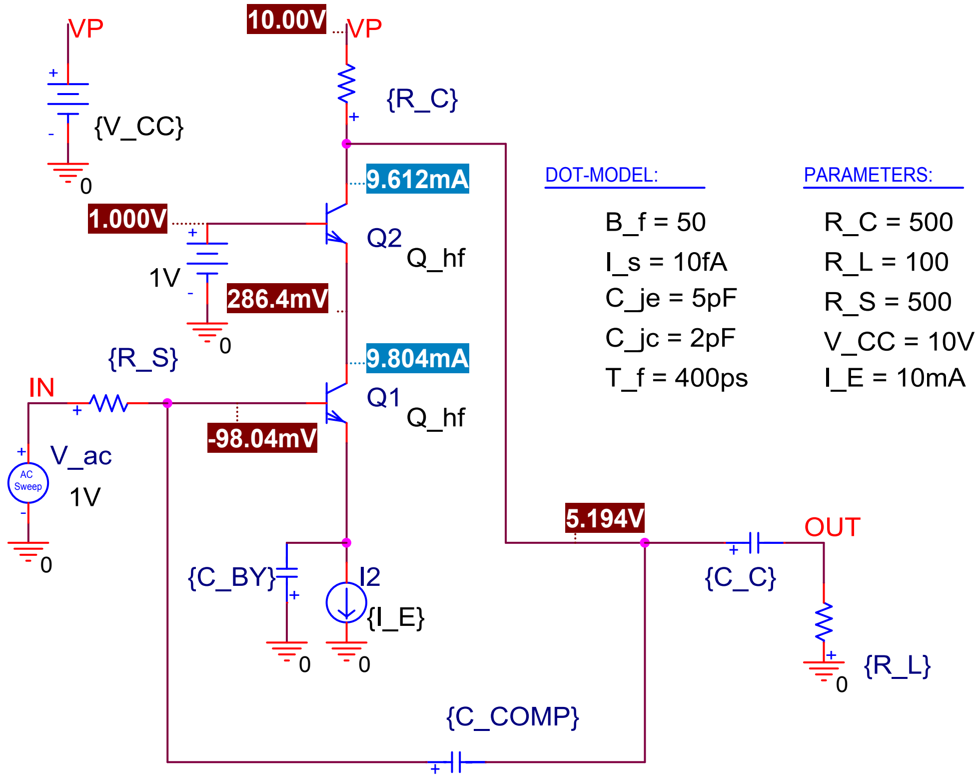
The calculated quantities are tabulated below in Figure 12.

pi	3.1415926		r_pi1	131.9053	beta*V_TH/I_C1
f_3dB	130		r_pi2	263.8107	beta*V_TH/I_C2
beta	50		R_Cc	114.9767	Eq 6
I_C1	0.009804		R_Cby	12.3903	Eq 5
I_C2	0.004902				
R_C	500		C_C (uF)	109.4566	Eq 8
V_th	0.025864		C_BY (uF)	109.4566	
R_S	500				
R_L	100				

FIGURE 12
Evaluation of C_C and C_{BY}

Problem 3: High corner-frequency design

Follow the outline procedure at the top of the exam with headings for each major step in the solution. No points for answer without an outline of the solution.
A mish-mash of calculations is not an acceptable outline.



.model Q_hf NPN (Bf={B_f} Is={I_s} Cje={C_je} Cjc={C_jc} Tf={T_f})

FIGURE 13

Amplifier for Problem 3; notice that the dot-model statement is not the same as for Problem 2

Dot model parameters C_{je} = zero-bias junction capacitance of emitter-base junction, C_{jc} = zero bias junction capacitance of collector-base junction T_f = base transit time.

Select high-frequency capacitor C_{COMP} so the upper 3dB corner frequency for the voltage amplifier of Figure 2 is f_{3dB} = 443 kHz.

Answer: C_{COMP} = 100 pF

Outline: We use the open-circuit time-constant method. We find the resistances seen by the high-frequency capacitors $C_{\pi 1}$, $C_{\pi 2}$ and C_{Comp}. Because this is a cascode circuit, there is a very limited Miller effect for $C_{\mu 1}$ ($C_M \approx 2\ C_{\mu 1}$), and one side of $C_{\mu 2}$ is at AC ground, so there is a very small time constant there too. We expect these capacitors contribute very little to the total time constant. There is a large Miller effect on C_{COMP}, however, because the gain across this capacitor is large, so we expect C_{COMP} to greatly affect f_{3dB}, even for small values of C_{COMP}..

RESISTANCE SEEN BY C_{COMP}

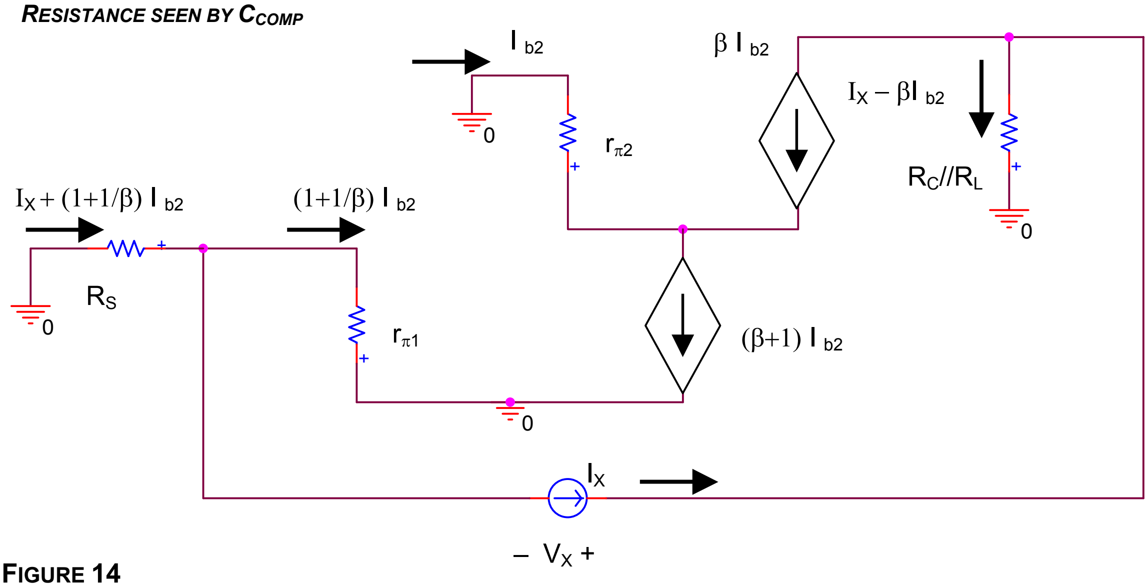


FIGURE 14

Small-signal circuit with test source for finding resistance seen by C_{COMP}

Using KVL from the left-hand ground through R_S and through $r_{\pi 2}$ to ground we find:

EQ. 9

$$I_{b2} = -I_X \frac{R_S}{\left(1+\frac{1}{\beta}\right)(R_S + r_{\pi 1})}.$$

Using KVL from the left-hand ground through R_S and the test source and through $R_C//R_L$ to ground we find:

EQ. 10

$$V_X = I_X(R_S + R_C // R_L) + I_{b2}\left[\left(1+\frac{1}{\beta}\right)R_S - \beta(R_C // R_L)\right].$$

Combining these equations we find the resistance seen by C_{COMP} to be

EQ. 11

$$R_{C\_Comp} = \frac{V_X}{I_X} = R_S // r_{\pi 1} + (R_C // R_L)\left(1 + \frac{\beta R_S}{\left(1+\frac{1}{\beta}\right)(R_S + r_{\pi 1})}\right).$$

The time constant contribution of C_{COMP} is then

EQ. 12

$$\tau_{C\_Comp} = R_{C\_Comp}\, C_{COMP}.$$

TIME CONSTANT OF $C_{\pi 1}$

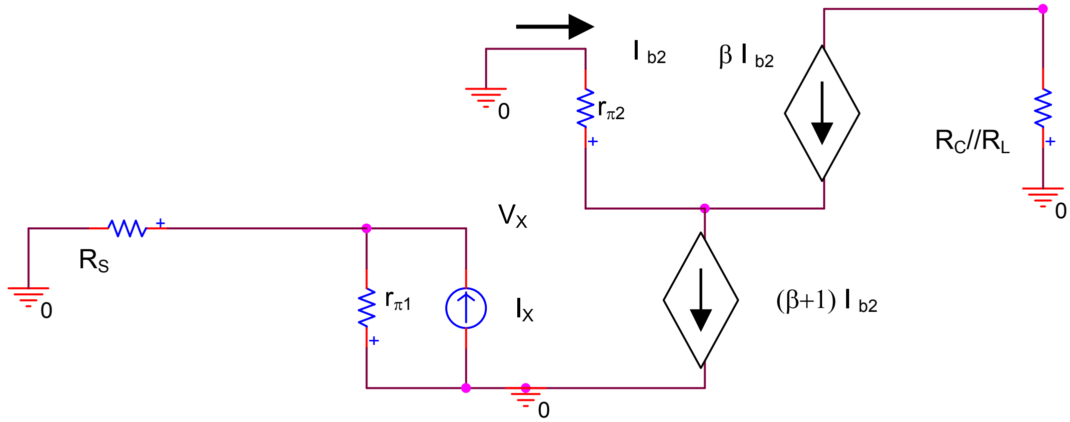


FIGURE 15

Small-signal circuit to find resistance seen by $C_{\pi 1}$

From Figure 15 we find

EQ. 13

$$R_{C\_\pi 1} = \frac{V_X}{I_X} = R_S \,//\, r_{\pi 1} \,.$$

TIME CONSTANT OF $C_{\pi 2}$

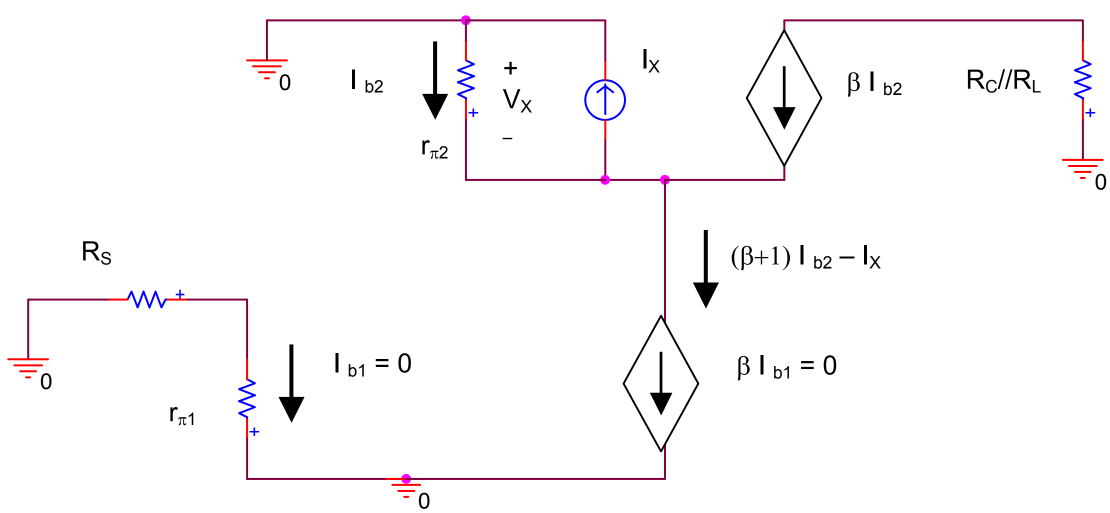


FIGURE 16

Small-signal circuit to find resistance seen by $C_{\pi 2}$

KVL through R_S and $r_{\pi 1}$ shows that $I_{b1} = 0$, so the dependent source of value βI_{b1} also is zero. Consequently, the emitter current of Q2 is zero too, resulting in

EQ. 14

$$I_X = (\beta + 1)\, I_{b2} \,.$$

From Ohm's law,

EQ. 15

$$V_X = I_{b2} r_{\pi 2} = \frac{r_{\pi 2}}{\beta + 1} I_X \,.$$

Thus, $R_{C\_\pi 2} = r_{E2} \equiv r_{\pi 2}/(\beta+1)$.

FINDING C_{COMP}

Combining the time constants we have

EQ. 16

$$\tau = \frac{1}{2\pi f_{3dB}} = C_{COMP}R_{C\_COMP} + R_{C\pi 1}C_{\pi 1} + R_{C\pi 2}C_{\pi 2} \ ,$$

which is rearranged to find C_{COMP} as

EQ. 17

$$C_{COMP} = \frac{\frac{1}{2\pi f_{3dB}} - \left(R_{C\pi 1}C_{\pi 1} + R_{C\pi 2}C_{\pi 2}\right)}{R_{C\_COMP}} \ .$$

The various terms to be evaluated are shown in Figure 17.

	pi	3.1415926		r_pi1	131.9053448	beta*V_th/I_C1
	f_3dB	4.43E+05		r_pi2	134.5401581	beta*V_th/I_C2
	beta	50		C_pi1	1.56624E-10	C_je+I_C1*T_f/V_th
	I_C1	9.804E-03		C_pi2	1.53655E-10	C_je+I_C2*T_f/V_th
	I_C2	9.612E-03		R_Cpi1	104.371126	
	R_C	500		R_Cpi2	2.638042316	
	V_th	0.025864		R_Ccomp	3419.966502	
	T_f	4.00E-10		t_Cpi1	1.6347E-08	
	C_je	5.00E-12		t_Cpi2	4.05347E-10	
	R_S	500		C_Comp	1.00151E-10	
	R_L	100				
R_C//R_L	RC_RL	83.333333				

FIGURE 17

Evaluation of C_{COMP}