ECE 304: Exam 5, Spring 05 Solutions

NOTE: IN ALL CASES
1. Solve the problem on scratch paper
2. Once you understand your solution, put your answer on the answer sheet
3. Follow your answer with an outline of your solution. No points for answer without an outline of the solution. A mish-mash of computation is not an acceptable outline.

PRINT your name at the top of each answer sheet

Assume in all problems V_{TH} = 25.864 mV, $V_{CB}(sat)$ = 0V, and $V_{BE} = V_{TH}\,\ell n\{I_C(V_{CB}=0V)/I_S\}$

Problem 1: FB amplifier comparison

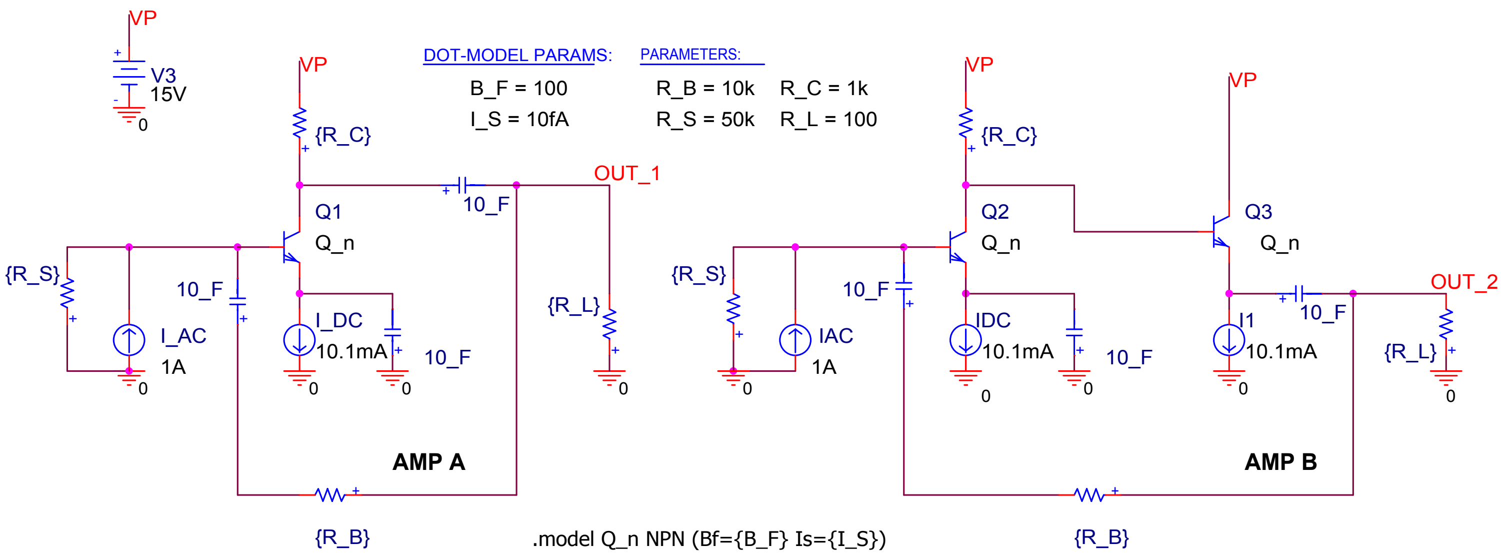

FIGURE 1

Two feedback amplifiers driving a load of 100 Ω

1. In Figure 1, what kind of amplifier (V/V, A/A etc.) is Amp A? Amp B? Explain in your outline.

Answer: Both amplifiers are transresistance amplifiers.

Outline: Both amplifiers can be arranged as shown in Figure 2, so both are shunt-shunt FB amps → current input and voltage output or transresistance gain.

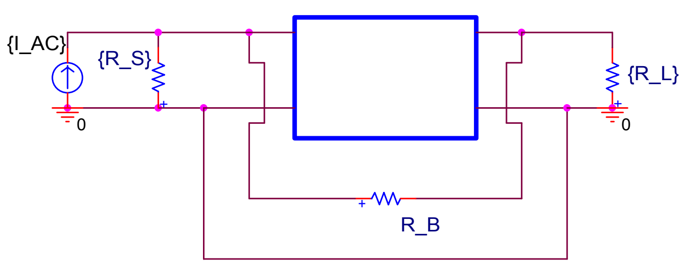

FIGURE 2

Re-arrangement of the FB amplifier to show the shunt-shunt FB connections

2. In words, what is the role of the second stage in Amplifier B (right)?

Answer: The VF increases the resistance as seen from the collector of the CE amplifier, increasing the open-loop amplifier gain. A larger open-loop gain means the amplifier with the VF stage will exhibit a FB gain closer to the ideal $1/\beta_{FB}$ than the single stage amplifier.

3. Set up the two two-port interpretation of both amplifiers, including an explicit two-port for the feedback network. Show your reasoning in your outline.

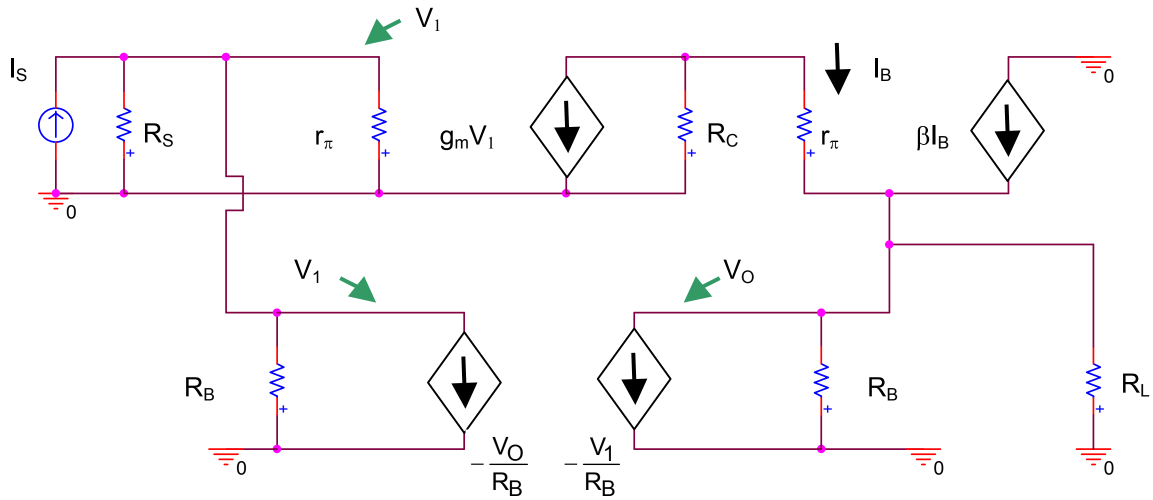

FIGURE 3
Two-two-port arrangement of Amp B

The two two-port arrangement of Amp B is shown in Figure 3; the arrangement for Amp A is the same, except the VF stage is omitted, and the emitter connection to the VF is replaced by an emitter connection to the first stage.

Outline:

The feedback amplifier is shunt-shunt → I_{IN}, V_{OUT} → β_{FB} = VCCS. Doing a two-port analysis of the resistor T-section, we find $\beta_{FB} = -1/R_B = \gamma_{FB}$ and $R_{11} = R_{22} = R_B$.

4. Assuming very large open-loop gains, what formulas describe the small-signal gains of these feedback amplifiers? Explain your reasoning in your outline.

Answer: The gain with feedback is approximately R_B V/A if the open-loop gain is high

Outline: The gain with feedback is $A_{FB} = A_L/(1+\beta_{FB}A_L)$. If the amplifier gain is high enough, $A_{FB} \approx 1/\beta_{FB} = R_B$.

5. Use the two two-port method of FB analysis to find formulas for the loaded gains, performance factors (PF's). Show your algebra in your outline.

Answer:

$$PF_A = 1+\beta_{FB}A_{LA} = 1+\frac{1}{R_B}g_m(R_C//R_B//R_L)(R_S//R_B//r_\pi)$$

and

$$PF_B = 1+\beta_{FB}A_{LB} = 1+\frac{1}{R_B}g_m(R_C//R_{EFF})(R_S//R_B//r_\pi)$$

with

$$R_{EFF} = r_\pi + (\beta+1)(R_B//R_L).$$

Outline:

The loaded gain is found for Amp A using Figure 4. KCL at the base provides

EQ. 1

$$I_S = V_1\left(\frac{1}{R_S}+\frac{1}{R_B}+\frac{1}{r_\pi}\right).$$

Ohm's law at the collector provides

EQ. 2

$$V_O = -g_m(R_C//R_B//R_L)\ V_1.$$

Combining EQ. 1 and EQ. 2 the transresistance loaded gain is found as

EQ. 3

$$\frac{V_O}{I_S} = -g_m(R_C//R_B//R_L)(R_S//R_B//r_\pi).$$

The performance factor for Amp A is then

EQ. 4

$$PF_A = 1 + \beta_{FB} A_{LA} = 1 + \frac{1}{R_B} g_m (R_C // R_B // R_L)(R_S // R_B // r_\pi).$$

where A_{LA} = loaded gain for Amp A, namely, $A_{LA} = -g_m (R_C // R_B // R_L)(R_S // R_B // r_\pi)$.

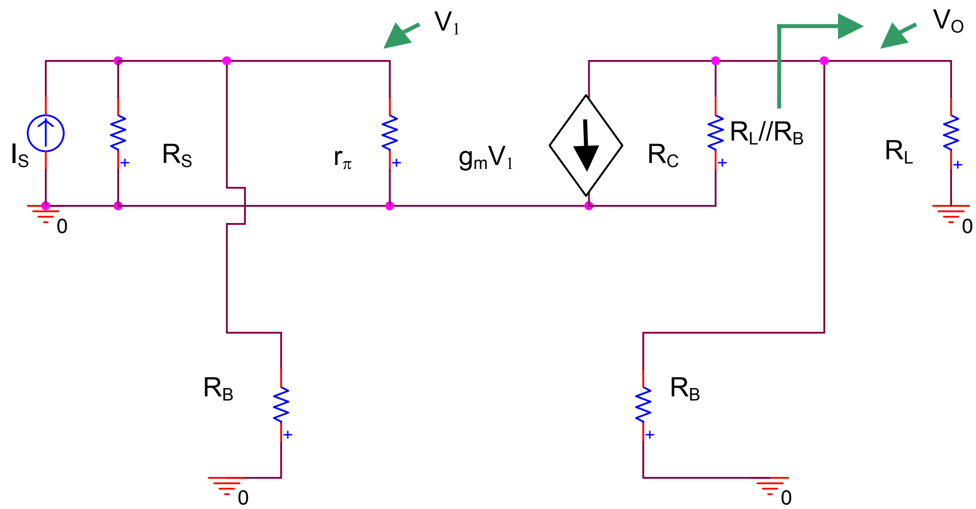

FIGURE 4
Small-signal circuit for Amp A with FB turned off

The results for Amp B are found using the same formulas, but substituting R_{EFF} from Figure 5 below for $R_B//R_L$, which is the load seen in Figure 4.

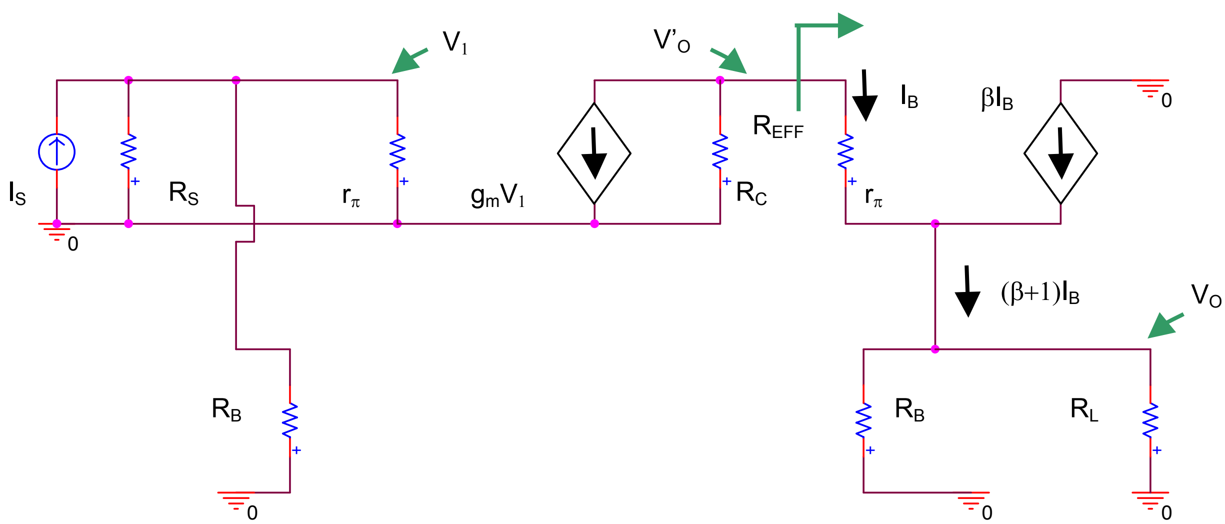

FIGURE 5
Circuit for Amp B with FB turned off, indicating the effective load R_{EFF} seen by the first stage

Putting a test voltage at the position of R_{EFF}, we find

EQ. 5

$$R_{EFF} = r_\pi + (\beta+1)(R_B//R_L).$$

With R_{EFF}, the gain V'_O/I_S is the same as EQ. 2, namely

EQ. 6

$$\frac{V'_O}{I_S} = -g_m(R_C // R_{EFF})(R_S // R_B // r_\pi).$$

The gain V_O/V'_O is just the gain of the VF, which is nearly unity. More exactly,

EQ. 7

$$\frac{V_O}{V'_O} = \frac{R_L // R_B}{R_L // R_B + r_E} \approx 1.$$

Hence, provided $R_L//R_B >> r_E$, we can use EQ. 6 as the gain of Amp B. That is,

EQ. 8

$$\frac{V_O}{I_S} = \frac{V'_O}{I_S}\frac{V_O}{V'_O} \approx -g_m(R_C // R_{EFF})(R_S // R_B // r_\pi)$$

With this approximation the PF for Amp B is

EQ. 9

$$PF_B = 1 + \beta_{FB}A_{LB} = 1 + \frac{1}{R_B}g_m(R_C // R_{EFF})\,(R_S // R_B // r_\pi),$$

with A_{LB} = loaded gain for Amp B, namely, $A_{LB} = -g_m(R_C // R_{EFF})(R_S // R_B // r_\pi)$.

6. Find formulas for the small-signal gains (with feedback) of both amplifiers. Show your algebra in your outline.

Answer: The small signal gains are just A_L/PF in both cases.

7. With words and one or two rough sketches, compare the two amplifiers over a large range of values for R_L, for example, 1 Ω ≤ R_L ≤ 100 kΩ. Justify your conclusions in your outline.

Answer: As R_L is made to approach zero, both gains tend to zero. As R_L increases, the gain increases, ultimately reaching the FB value R_B V/A. For a large gain, Amp A requires a large value for $g_m(R_C//R_L//R_B)$, which in turn requires $R_L//R_B >> R_C$. On the other hand, Amp B requires a large value for $g_m(R_C//R_{EFF}) \approx g_m\{R_C//[(\beta+1)(R_B//R_L)]\}$, which in turn requires a large value for $(\beta+1)\,(R_B//R_L)$. Because $(\beta+1)$ is a large factor, Amp B will achieve a large gain much sooner than Amp B. A sketch is shown in Figure 6.

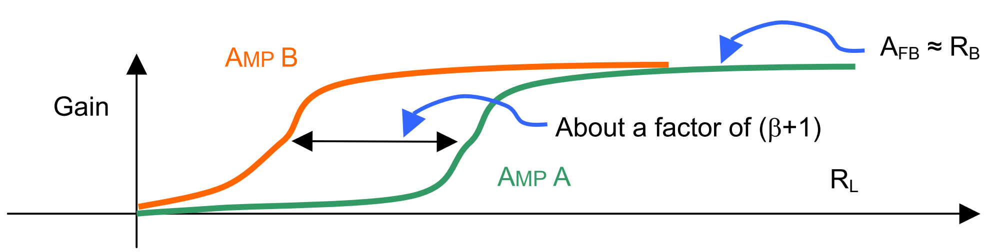

FIGURE 6

Sketch of gain behavior of the two amplifiers

8. From a small-signal standpoint, what are the advantages/disadvantages of the amplifier on the left (Amplifier A) compared to the amplifier on the right (Amplifier B)? Explain your claims.

Answer: The situation already is clear: Amp A works fine if $R_B//R_L$ is large; otherwise, it is necessary to use Amp B.

Note: PSPICE simulations

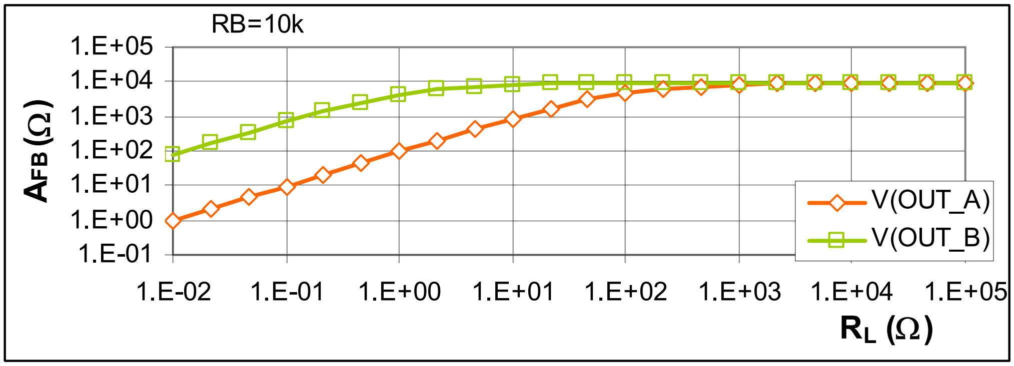

FIGURE 7

PSPICE simulation of feedback gain vs. R_L

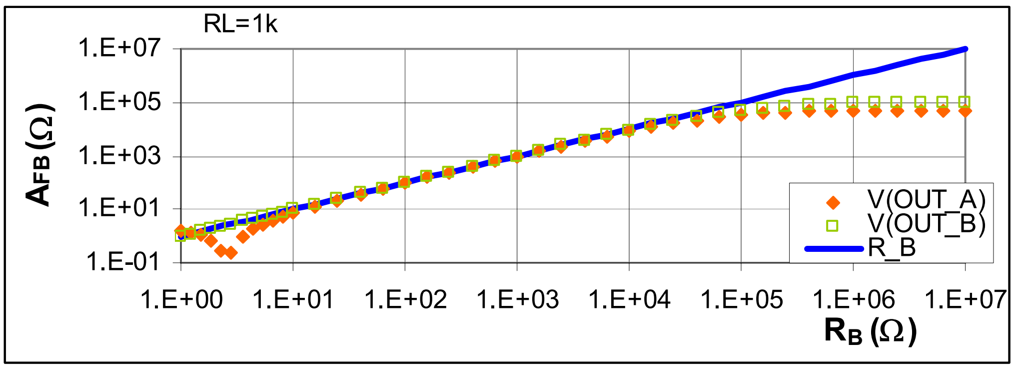

FIGURE 8

PSPICE simulation of feedback gain vs. R_B

Figure 7 shows the behavior described with the sketch in Figure 6. Figure 8 shows how well the two amplifiers follow the FB gain $A_{FB} = R_B$ valid for large open-loop gain A_L. As expected, Amp A is not as good as Amp B at low values of R_B. However, both amplifiers deviate from $A_{FB} = R_B$ at large R_B. Why?[1]

Problem 2: FB amplifier types

A driving signal source can be viewed in terms of its Thevenin or its Norton equivalent circuit with driving resistance R_S. It is desired to drive a resistive load of value R_L.

1. Sketch the feedback amplifiers below with the appropriate form of the driver and the proper location of the load. Place the load so one end is at ground. Clearly mark all the grounded connections. Label the feedback output variable (I_O or V_O). Explain your choices in your outline.

[1] An intuitive explanation is that at large R_B, the resistor R_B approaches an open circuit. Then there is zero feedback, and the amplifiers are just open-loop amplifiers with the open-loop gain. Naturally, the open-loop gain does not depend on R_B, so the curves flatten out at large R_B.

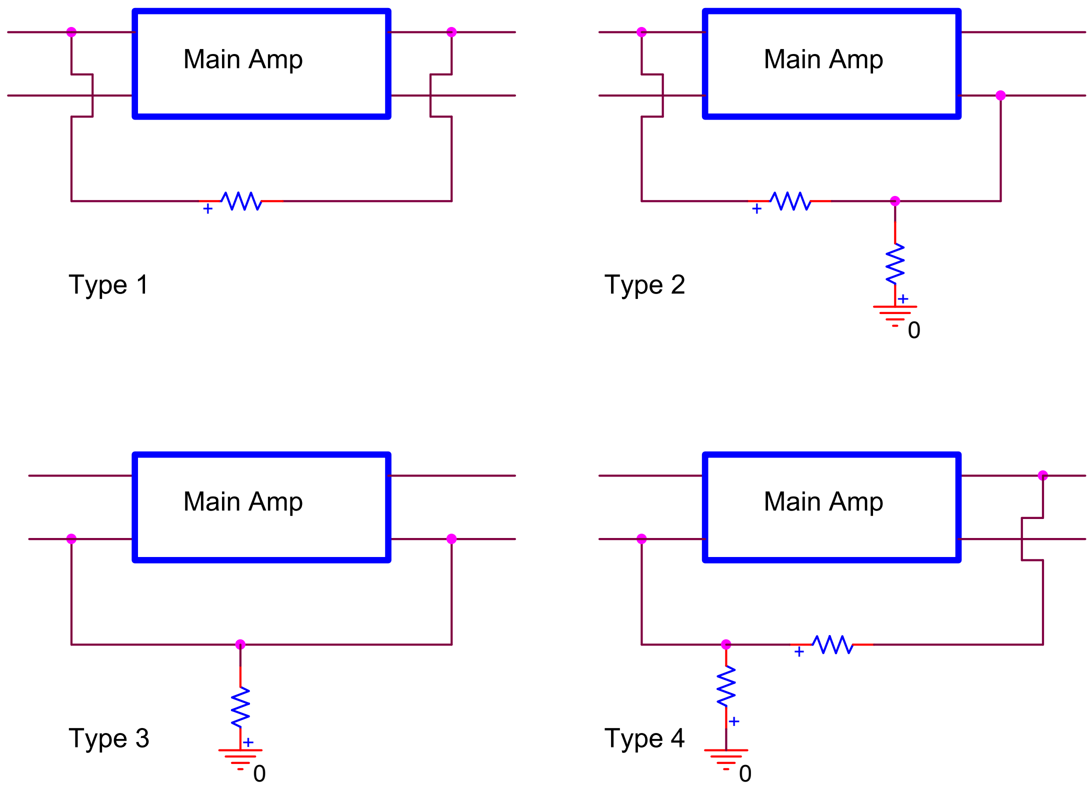

FIGURE 9
Types of feedback amplifier

Answer:

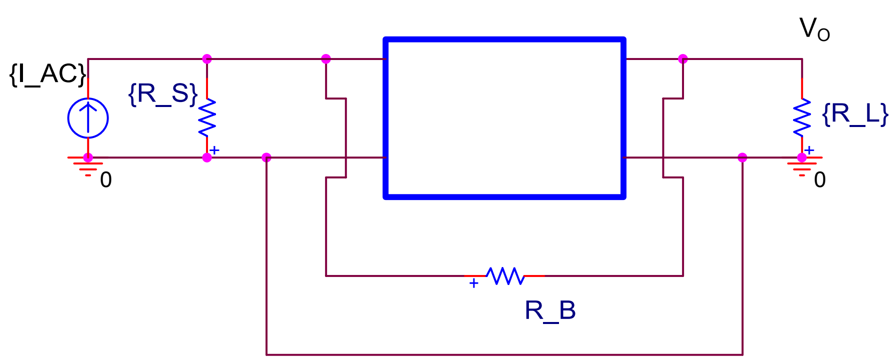

FIGURE 10
Type 1 Amplifier: Trans R Amp

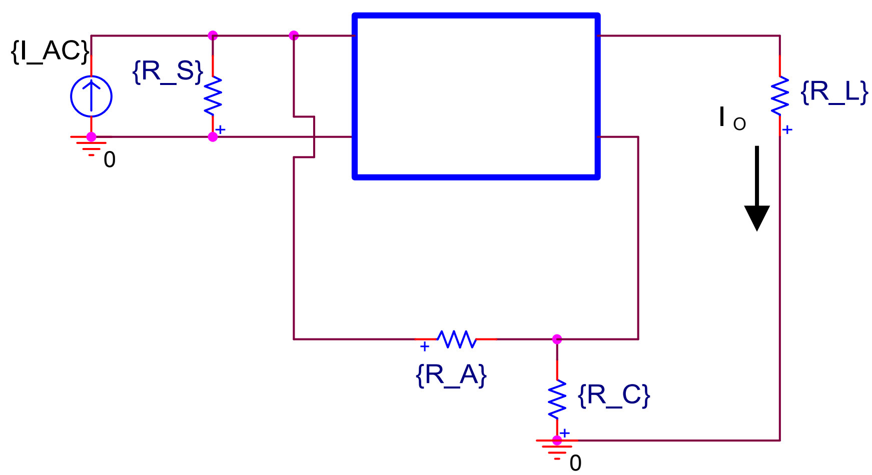

FIGURE 11

Type 2 Amplifier: Current Amp

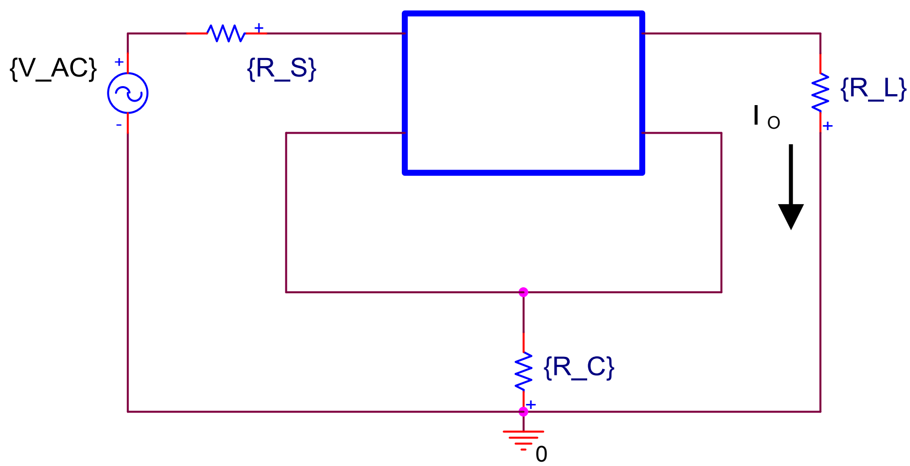

FIGURE 12

Type 3 Amplifier: Trans G Amp

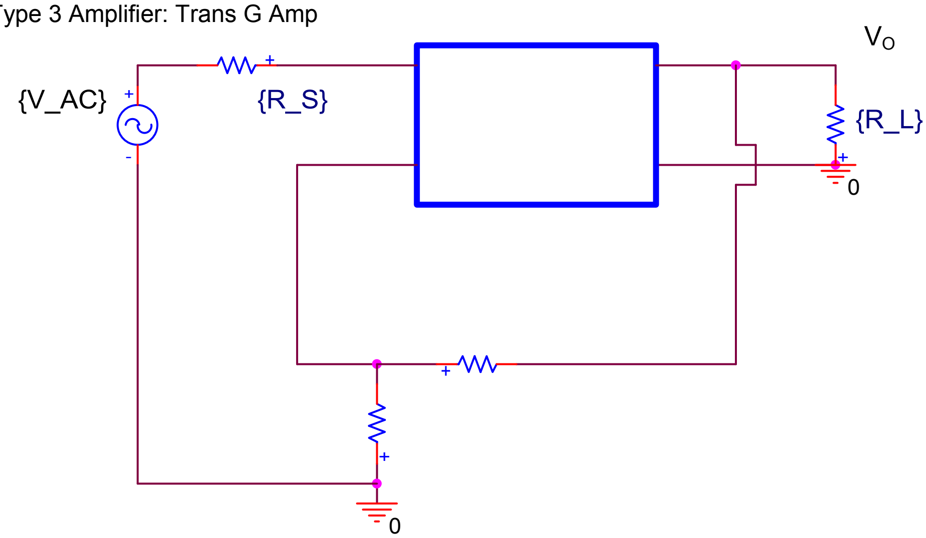

FIGURE 13

Type 4 amplifier: Voltage Amp

Outline: In each case, inspection decides whether the FB hookup is series or shunt. Series input → Thevenin driver; shunt input → Norton driver. Series output → current out; shunt output → voltage output.

2. Which choice of main amplifier (V/V, A/A, etc.) is most likely to be best in each case? Explain why in your outline.

Answer: Type 1: Trans R, Type 2: Current; Type 3: Trans G; Type 4: Voltage

Outline: In each case, the best choice of Main Amplifier is of the same type as the FB amplifier because then the main amplifier already approximates the desired input and output resistance properties, and we do not have to exert the FB network to make very large changes in these properties.

3. For each amplifier type, explain what happens to the voltage across the load V_L and the current in the load I_L if the load resistor value R_L is decreased by a factor of two.

Answer: In those cases where voltage is the output variable, voltage will not change much and current will be doubled. In those cases where current is the output variable, current will not change much and voltage is cut in half.

Outline: The variable that is monitored at the output is maintained constant by the feedback, assuming the open-loop gain is large. The unmonitored variable can change. These observations lead to the above conclusions.

In more detail, if x_I = input variable (either current or voltage, depending on the amplifier), and x_O = output variable, $x_O = x_I / \beta_{FB}$ provided the open-loop gain is large so the feedback gain is $A_{FB} \approx 1/\beta_{FB}$. In the amplifiers shown, β_{FB} is controlled by the FB network and does not involve the load R_L, so x_O does not change with R_L.

Problem 3: FB network design

The open-loop amplifier below is to be hooked up as a closed-loop feedback amplifier satisfying these impedance specifications on the closed-loop input resistance R_{IF} and output resistance R_{OF}.

$$R_{IF} \geq R_{IS} = 19\ k\Omega \qquad R_{OF} \geq R_{OS} = 200\ k\Omega$$

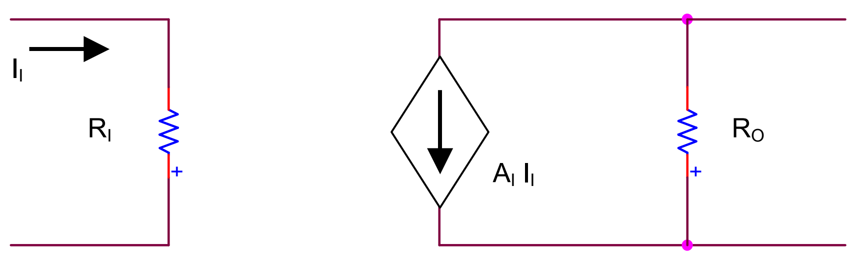

FIGURE 14

Open-loop amplifier; $R_I = 1\ k\Omega$; $R_O = 10\ k\Omega$; gain is 10^3 A/A

1. What is the β_{FB} (with units) for the ideal feedback network (no feedback resistors)? Explain how you find it in your outline.

Answer: $\beta_{FB} = 19\ \Omega$.

Outline: Because both resistances must be increased beyond their present values, the FB connection in both cases is series. On the input side, series → voltage input. On the output side, series → current output. Therefore, the gain is A/V, and the units of β_{FB} are V/A → CCVS. Consequently the ideal FB amplifier is as shown in Figure 15.

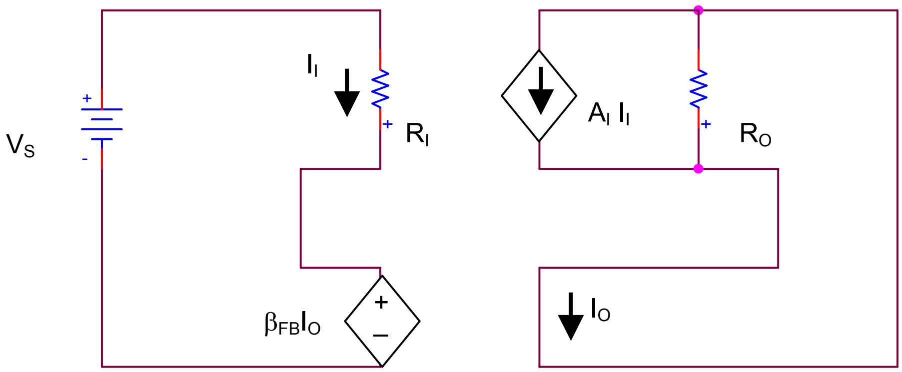

FIGURE 15
Ideal FB amplifier

To determine β_{FB} we need the performance factor. Setting β_{FB} = 0 Ω, the loaded gain is found as

EQ. 10

$$A_L = \frac{I_O}{V_S} = \frac{A_I I_I}{V_S} = \frac{A_I}{R_I},$$

so the PF is

EQ. 11

$$PF = 1 + \beta_{FB}\frac{A_I}{R_I}.$$

The specified R_{SO} and R_{SI} then determine the numerical value of the PF as

EQ. 12

$$PF \geq \frac{R_{OS}}{R_O} = 20 \quad \text{or} \quad PF \geq \frac{R_{IS}}{R_I} = 19;$$

Evidently choosing PF = 20 satisfies both requirements. Therefore, β_{FB} is given by

EQ. 13

$$20 = 1 + \beta_{FB}\frac{A_I}{R_I}, \quad \beta_{FB} = 19\frac{R_I}{A_I} = 19\ \Omega.$$

2. What resistor values are appropriate for a T-section of resistors that provide an amplifier satisfying the specs? Derive your result in your outline.

Answer: $R_A = R_B = 0$, $R_C = 19.41\ \Omega$, where R_C is the vertical resistor in the T-section.
Outline: The two-port with a current-controlled voltage source is shown in Figure 16 below.

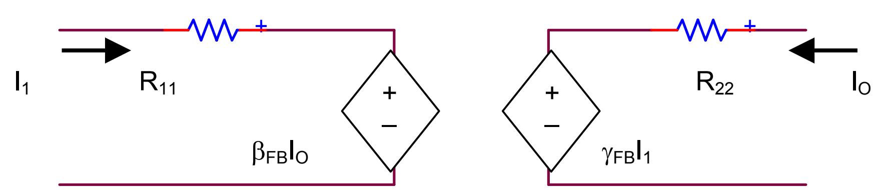

FIGURE 16
Applicable two-port

Open-circuiting the left side and applying a current source to the right side we find Figure 17 below. Comparing voltages at the left of both circuits in Figure 17 we find β_{FB}:

EQ. 14

$$\beta_{FB} = R_C.$$

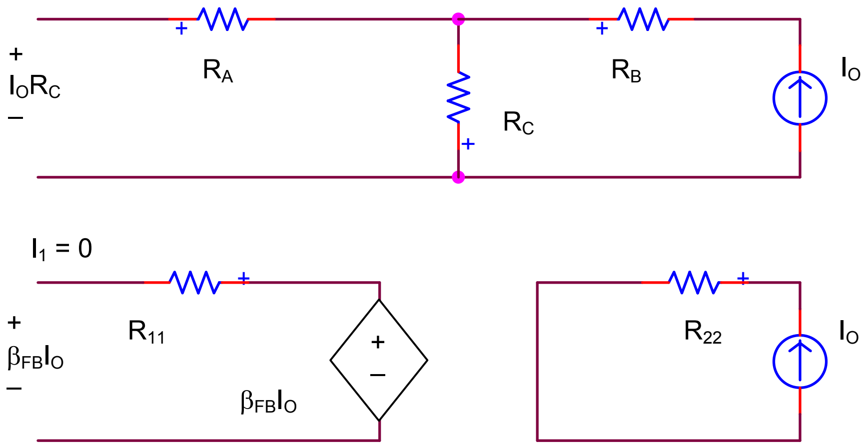

FIGURE 17

Finding β_{FB} for the T-section of resistors in terms of the resistor values

Because β_{FB} can be set without R_A or R_B, we set these resistors to zero to reduce the loading of the gain. Then $R_{11} = R_{22} = R_C$. Next we find the loaded gain with the T-section in place by setting β_{FB} and γ_{FB} to zero.

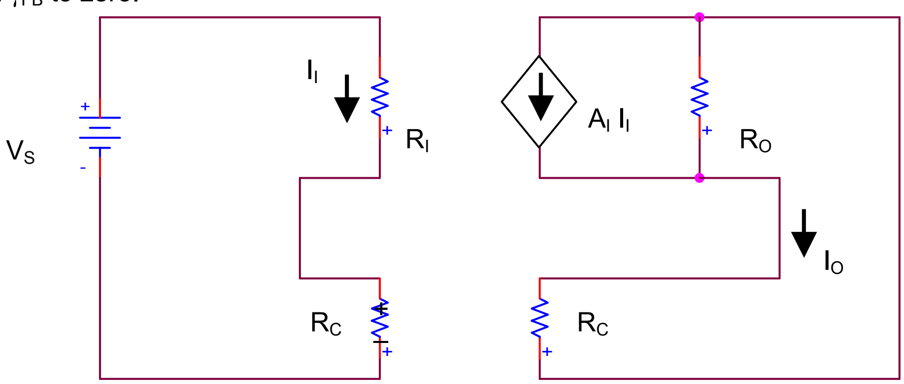

FIGURE 18

Determination of the loaded gain with the T-section resistors

Using Figure 18, the output side provides

EQ. 15

$$I_O = A_I I_I \frac{R_O}{R_C + R_O}\ .$$

The input side provides

EQ. 16

$$I_I = \frac{V_S}{R_I + R_C}\ .$$

Substituting EQ. 16 into EQ. 15, the loaded gain is

EQ. 17

$$A_L = \frac{I_O}{V_S} = \left(\frac{1}{R_I + R_C}\right) A_I \left(\frac{R_O}{R_O + R_C}\right),$$

and the performance factor is

EQ. 18

$$PF = 1 + \beta_{FB} A_L = 1 + \left(\frac{R_C}{R_I + R_C}\right) A_I \left(\frac{R_O}{R_O + R_C}\right) = 20.$$

This equation can be solved by iteration, or it can be solved directly for R_C as EQ. 18 is a quadratic equation in R_C. Either way one finds

EQ. 19

$$\beta_{FB} = 19.41\ \Omega = R_C.$$

The iterative approach is outlined below. Using EQ. 18:

EQ. 20

$$PF = 1 + \beta_{FB} A_L = 1 + \left(\frac{\beta_{FB}}{R_I + R_C}\right) A_I \left(\frac{R_O}{R_O + R_C}\right) = 20\ .$$

Solving for β_{FB} we obtain

EQ. 21

$$\beta_{FB} = 19\left(R_I + R_C\right)\left(\frac{R_O + R_C}{A_I R_O}\right).$$

We substitute the ideal value for R_C from part 1, $R_C = \beta_{FB} = 19$. This leads to a revised estimate for β_{FB}. We set $R_C = \beta_{FB}$ using the revised β_{FB}. We plug this value for R_C into EQ. 21 to obtain the next revised value of β_{FB}, and so forth.

Note: EXCEL implementation

An EXCEL implementation of the first iteration is shown in Figure 19 below. The next iteration is found by copying the value of R_C1 into the cell for R_C0.

R_IS	1.9000E+04
R_OS	2.0000E+05
A_I	1.00E+03
R_I	1.00E+03
R_O	1.00E+04
R_C_ideal	19
R_C0	19
A_L	0.97949323
PF	19.6103714
R_IF	1.961E+04
R_OF	1.961E+05
R_C1	19.3977859

FIGURE 19

EXCEL implementation of iteration