# Compressed Sensing with Phase Constrained Partial Radial k-space

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**Introduction:** The Compressed Sensing (CS) theory illustrates that a large class of signals can be accurately reconstructed from a small number of linear measurements [1,2], and has successfully been applied to MRI [3-7]. While different acquisition strategies have been proposed for CS MRI, it has been demonstrated that radial acquisition works particularly well due to its variable sampling density and incoherent undersampling artifacts in commonly used sparsity bases [5-7]. In addition, radial trajectories offer robustness to motion and the possibility of using partial k-space data to manipulate contrast [8-10].

To achieve higher time efficiency, partial Fourier acquisition and reconstruction methods have also been proposed for radial trajectories [11-13]. In these methods, partial radial views are collected during acquisition and the missing k-space points in each radial view are estimated using homodyne detection. In this work, we illustrate that the partial Fourier acquisition methods can be combined with CS techniques to yield further acceleration.

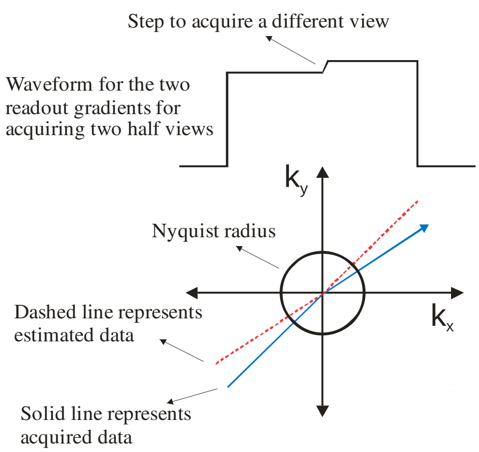
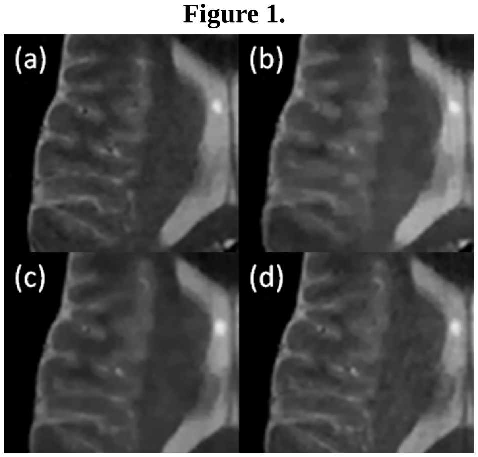



**Figure 1.**




**Methods:** A radial fast spin-echo (RAD-FSE) sequence was modified to acquire two separate half radial views for each excitation as illustrated in Figure 1. The resulting dataset is processed as described in [14]: The uncollected k-space points that are within the Nyquist radius are calculated using interpolation. The missing k-space points that are outside the Nyquist radius are calculated using homodyne detection in projection domain. In homodyne detection, a low-resolution phase map is used to correct field inhomogeneties such that the symmetry of k-space about the origin can be exploited. In this case, the phase of each projection is estimated from the phase of a low resolution projection computed using only the k-space points within the Nyquist radius. The phase corrected projection is then transformed back into Fourier space to calculate the missing k-space points outside the Nyquist radius. Let $\mathbf{f}$ denote the object being imaged and $\mathbf{M}$ denote the undersampled Fourier matrix corresponding to the acquired half radial views such that the collected data $\mathbf{g}$ can be expressed as $\mathbf{g}=\mathbf{Mf}$ . If we let $\mathbf{\Psi}$ denote a sparsity transform such that $\mathbf{\Psi f}$ is sparse, the CS reconstruction problem using the acquired half views can be stated as

$$\min_{\mathbf{f}} \left\|\mathbf{\Psi f}\right\|_1 \text{ subject to } \left\|\mathbf{g}-\mathbf{Mf}\right\|_2 < \varepsilon \tag{1}$$

where $\varepsilon$ accounts for noise. If we then let $\hat{\mathbf{g}}$ denote the full view k-space dataset obtained after the above homodyne detection procedure and $\hat{\mathbf{M}}$ denote the corresponding Fourier matrix, we can restate the CS reconstruction problem as

$$\min_{\mathbf{f}} \left\|\mathbf{\Psi f}\right\|_1 \text{ subject to } \left\|\hat{\mathbf{g}}-\hat{\mathbf{M}}\mathbf{f}\right\|_2 < \varepsilon \tag{2}$$

To compare these two CS reconstruction problems, we acquired data using a half-view RAD-FSE sequence with acquisition parameters: echo train length=8, receiver bandwidth=±32 kHz, number of sampled points along a view=256, TR=3sec, FOV=24 cm, and slice thickness = 5mm. In addition, we also acquired data using a full-view RAD-FSE sequence with the same parameters. The half-view RAD-FSE dataset contained 128 half-views. With the full-view sequence, two datasets with 128 and 64 full views were acquired. Note that the acquisition time of 128 half-views is equal to that of 64 full views.

**Results and Discussion:** Figure 2 illustrates the images obtained using different reconstruction algorithms and different number of radial views. Finite differences (Total Variation) was used to enforce sparsity and the regularization parameters were selected using L-curve analysis. Figures 2a and 2b illustrate the images reconstructed using CS from 128 and 64 full radial views, respectively. Figure 2c illustrates the CS reconstructed image using 128 half radial views according to Equation (1) (without homodyne detection). This half-view dataset was then processed as using the homodyne detection approach described above to create 128 full radial views and the image reconstructed according to Equation (2) is illustrated in Figure 2d. Note that the data acquisition time for the images in Figures 2b, 2c, and 2d is equal and is half the acquisition time for the image in Figure 2a. By comparing Figures 2b and 2c, we can see that acquiring half or full radial views does not result in significant difference in image quality. However, if the half-view dataset is pre-processed using homodyne detection to create full radial views, the CS reconstruction can be improved as illustrated in Figure 2d. In this case, the reconstructed image is similar to an image obtained by acquiring the full radial views (Figure 2a).

**Conclusion and Future Directions**: We have illustrated that phase constrained reconstruction can be combined with CS techniques. While the proposed technique was illustrated in 2D, it can easily be extended to 3D. The proposed method is particularly attractive for ultrashort TE (UTE) pulse sequences [15] which are designed to detect signals from tissues with very short T2s and usually have center-out radial trajectories. Furthermore, phase constrained CS reconstruction can easily be extended to other sampling trajectories.

**Acknowledgements**: AZ Alzheimer Disease Core Center, AZ Cancer Center, and NIH (HL085385).

**References:** [1] Candes E, IEEE Transactions on Information Theory. Feb. 2006. [2] Donoho DL, IEEE Transactions on Information Theory. Apr. 2006 [3] Lustig M, Proc. ISMRM (2005). [4] Lustig M, Proc. ISMRM (2006). [5] Chang TC, Proc. ISMRM (2006). [6] Bilgin A, Proc ISMRM (2007). [7] Block KT, Proc. ISMRM (2007). [8] Song HK, MRM, 44, 825 (2000). [9] Altbach MI, JMRI, 16, 179 (2002). [10] Gmitro AF, ISMRM, 2003. [11] Noll DC, Proc. SPIE, 1443, 29 (1991). [12] Block WF, ISMRM, 659, (1999). [13] Barger AV, MRM, 48, 297, (2002). [14] Bilgin A, Proc ISMRM (2005). [15] Tyler DJ, JMRI, 25, 279, (2007).