Randomly Perturbed Radial Trajectories for Compressed Sensing MRI

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Introduction: The recently introduced Compressed Sensing (CS) theory illustrates that a small number of random linear measurements can be sufficient to reconstruct sparse or compressible signals [1,2,3]. Since the number of measurements necessary for perfect reconstruction using the CS theory can be drastically smaller than the number of measurements required by the Nyquist sampling theory, CS has the potential to significantly accelerate data acquisition in MRI [4-10]. Although the initial CS theory was based on completely random sampling, it was later suggested that such random sampling may not always be necessary [11]. More specifically, recent results indicate that CS methods can yield exact recovery if the sparsity basis and the measurement basis obey the uniform uncertainty principle and are incoherent [11]. This pertains to MRI since complete random sampling is difficult to achieve with existing hardware. Earlier work in CS for MRI explored strategies for randomizing measurements for spirals [4] and 3DFT trajectories [5]. Recently, reconstructions from CS MRI data acquired on regular radial trajectories were demonstrated [7-10]. While the CS reconstruction of undersampled regular radial data resulted in significant reduction of streaking artifacts, compared to filtered backprojection or regridding, we demonstrate here that further improvements are possible by randomly perturbing the radial trajectories.

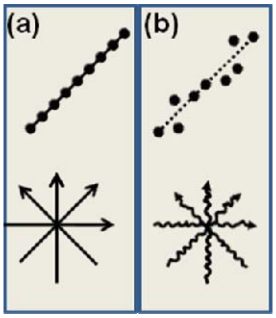

Fig. 1. (a) Regular (b) Randomly perturbed radial trajectory.

Theory: Let $\mathbf{f}$ be an N-dimensional vector representing the object being imaged, $\mathbf{M}$ the KxN measurement matrix, and $\mathbf{g}$ a K-dimensional vector of measurements (where K << N). In MRI, $\mathbf{M}$ is an undersampled Fourier matrix and $\mathbf{g}$ is the measured k-space data such that $\mathbf{g} = \mathbf{Mf}$. Furthermore, let $\mathbf{\Psi}$ denote the NxN transform matrix such that $\mathbf{\Psi f}$ is sparse (i.e. $\mathbf{\Psi f}$ has only S non-zero values where S << N). CS theory suggests that $\mathbf{f}$ can be recovered by solving the convex optimization problem

$$\min_{\mathbf{f}} \left\|\mathbf{\Psi f}\right\|_1 \text{ subject to } \left\|\mathbf{Mf}-\mathbf{g}\right\|_2 < \varepsilon$$

where ε controls the fidelity between the measurements and the reconstruction and is used to account for noise in the measurements. It has also been shown that the number of measurements K necessary for successful recovery of the S sparse coefficients depends on the coherence between the measurement and sparsity bases [11,12]. The more incoherent the measurement and sparsity bases, the fewer measurements are needed for reconstruction. Measurements obtained on regular grids can result in coherent undersampling artifacts in the sparsity domain and, thus, more measurements may be needed for successful reconstruction. Equivalently, between two systems with the same number of measurements, the one with the higher amount of incoherence will recover more sparsity domain coefficients and, thus, will result in higher image quality.

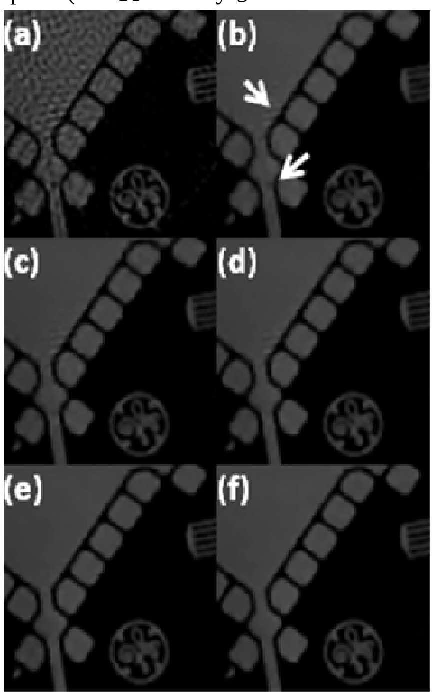

Fig. 2. Reconstructions from 64 radial views. Regular radial data reconstructed with (a) NUFFT (b) CS. Randomly perturbed radial data corresponding to slew rates (c) 150 (d) 375 (e) 750 (f) 1500 T/m/s reconstructed with CS.

Method: We propose randomly perturbed radial trajectories for increased incoherence in radial CS MRI. A regular radial trajectory is illustrated in Fig. 1a, and a randomly perturbed radial trajectory is illustrated in Fig. 1b. The random perturbations occur in the azimuthal direction. The amount of azimuthal deviation is dependent on the gradient slew rate of the MRI system. Higher slew rates enable larger deviations which result in more incoherent measurements. In our experiments, we took an image which was originally acquired using 256 regular radial views and 256 points along each radial view using a FSE pulse sequence. Using this image, we created undersampled datasets by only keeping partial data in k-space. We first took k-space samples along 64 regular radial lines and reconstructed this data set using both non-uniform FFT (NUFFT) and CS. We then created partial k-space data sets with random azimuthal perturbations along 64 radial views. We calculated the radial sampling stepsize Δk_r for FOV = 24 cm, and randomly perturbed the trajectories by $m\Delta k_r$ where m was selected to correspond to different gradient slew rates and BW = 64 KHz. In our experiments, we used orthogonal wavelets as well as finite differences (total variation) for sparsity.

Results and Discussion: Fig. 2 shows images obtained using different reconstruction algorithms and different radial trajectories. Figs. 2a and 2b illustrate the images reconstructed using NUFFT and CS from 64 regular radial views, respectively. Note that while the CS reconstruction has reduced undersampling artifacts significantly, there are some residual streaks in the image (pointed to by the arrows). Images reconstructed using CS from 64 randomly perturbed radial lines are shown in Figs. 2c-2f for slew rates, 150 T/m/s, 375 T/m/s, 750 T/m/s, and 1500 T/m/s, respectively. Notice that the increased azimuthal perturbations result in removal of the residual artifacts without sacrifice of spatial resolution.

Conclusion: We illustrated that randomly perturbed radial trajectories can reduce artifacts in CS MRI. Fast gradients slew rates available in modern MRI systems enable such perturbations.

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