Topographic Mapping with Multiple Antenna SAR Interferometry: A Bayesian Model-Based Approach

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Abstract — Multiple-antenna SAR interferometry involves the use of three or more antennas to reduce the overall phase ambiguities and phase noise in interferometric data. This paper presents a Bayesian approach to topographic mapping with multiple-antenna SAR interferometry. Topographic reconstruction is formulated as a parameter estimation problem in the model-based Bayesian inference framework. An InSAR simulator based on a forward model is developed for simulating SAR data from multiple-antenna InSAR for evaluating the Bayesian topographic reconstruction algorithms. A Bayesian point position algorithm is developed to estimate the height of a point in the image. A measure of the uncertainty in estimated position and height is also defined in terms of the spread of the dominant mode of the posterior distribution. An example demonstrating the performance of the algorithm for a three-antenna InSAR system is reported, and conclusions are drawn regarding the performance and improvements are proposed.

I. Introduction

Synthetic aperture radar (SAR) interferometry has emerged a power tool for generating high resolution topographic maps of the earth surface [1]. Two SAR images taken from slightly different viewing angles are combined to form an interferogram. In conventional interferometric processing phase unwrapping is used to estimate the absolute phase difference from the interferogram phase which is used to generate the topographic map using geometric relationships [2]. However, phase unwrapping requires prior assumptions about the slope of the terrain, and is limited by phase noise in low correlation areas as well as phase 'discontinuities' caused by layover [3]. Additionally, no tractable measure has been formulated for quantifying the uncertainty in unwrapped phase maps [4].

The problems associated with phase unwrapping in conventional two-element SAR interferometry can be reduced with multiple-antenna SAR interferometry (MAInSAR). MAInSAR involves use of an asymmetric array of three or more antennas to form several baselines; one with each antenna pair in the array. Thus MAInSAR provides a diversity of long and short baseline interferometric phase data. The fusion of the multiple ambiguity interferometric data can reduce the overall phase ambiguity and also improve the data-noise characteristics of the system relative to an equivalent conventional two-antenna/dual-pass interferometer [5],[6]. Moreover, as MAInSAR is a single-pass system it avoids the problem of temporal decorrelation and atmospheric effects [7] encountered in repeat-pass interferometry.

In this paper, we present a Bayesian model-based approach to topographic reconstruction with MAInSAR [4],[8],[9]. Topographic reconstruction is treated as an estimation problem. The Bayesian approach [10] provides a formal inference framework for optimal combination of the measured multiple-antenna data and available prior knowledge to estimate the surface topography, as opposed to algorithms which estimate the surface topography as some *'ad-hoc'* function of the measured data. The standard deviation of the posterior distribution in the Bayesian approach may also be used to quantify the uncertainty in the terrain height estimates.

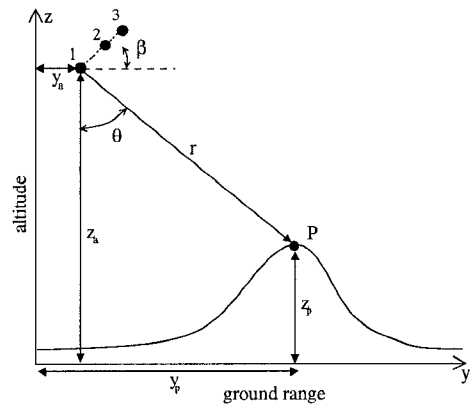

Fig. 1. Geometry showing the relation between the angle of arrival parameter and the position of a point P on ground.

II. The Bayesian Approach

Topographic reconstruction is formulated as a parameter estimation problem, where the estimation parameter is the surface topography. The surface topography is parameterized in terms of position of each point in the image, where the position (z_p, y_p) of a point P in the zero-Doppler plane depicted in Fig. 1 is determined by the angle of arrival parameter θ and the slant range r with the following equations,

$$z_p = z_a - r \cdot \cos(\theta) \quad (1)$$

$$y_p = y_a + r \cdot \sin(\theta) \quad (2)$$

Thus, the topographic estimation problem can now be stated as estimating the angle of arrival parameter θ of all points given the dataset of multiple images V. In the Bayesian inference

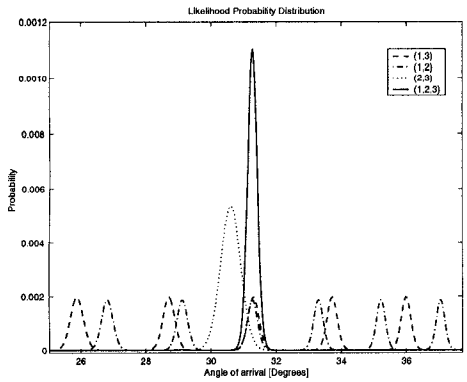

Fig. 2. Likelihood distributions for different data combinations in a three-antenna InSAR system.

framework this is expressed as follows,

$$P(\theta|V) = \frac{P(\theta) \cdot P(V|\theta)}{P(V)} \tag{3}$$

where, $P(\theta|V)$ is the *posterior probability distribution*, $P(V|\theta)$ is the *likelihood probability distribution*, $P(\theta)$ is the *prior probability distribution* and $P(V)$ is called the *evidence* (a normalization constant). The likelihood distribution $P(V|\theta)$ represents the forward data model that completely defines the statistical characteristics of the multiple SAR images as function of surface topography, surface reflectivity and the imaging geometry. Here we define the forward model as a joint complex circular Gaussian probability distribution [9] for a set of N images as,

$$P(V|\theta) = \frac{1}{\pi^N |K|} \exp\left(-V^\dagger K^{-1} V\right) \tag{4}$$

where the data vector $V = (V_1, V_2 \ldots, V_N)$ is the set of N images. $V^\dagger$ is the conjugate transpose and $K = E\left\{V \cdot V^\dagger\right\}$ is the complex covariance matrix. The complex covariance matrix K consists of variance and complex covariance terms representing the correlation between images. This joint likelihood distribution in (4) combines the multiple- antenna data to reduce the ambiguity and data-noise. This *'data-fusion'* is shown in Fig. 2 with plots of the likelihood distributions of all the interferometric (i,j) pairs which combine to produce the joint likelihood distribution of all three antenna data, in a three-antenna InSAR system.

If all the images are accurately focused, registered, and sampled at the Nyquist rate, each pixel in an image can be assumed to be statistically independent of every other pixel and the joint likelihood distribution can be written as a product [9],

$$P(V|\theta) = \prod_{i=1}^{M} P(V_i|\theta_i) \tag{5}$$

$$P(V_i|\theta_i)) = \frac{1}{\pi^N |K_i|} \exp\left(-V_i^\dagger K_i^{-1} V_i\right) \tag{6}$$

where M is the number of pixels in each image, $V_i = (V_1(i), \ldots, V_N(i))$ is the vector of i^{th}pixel in each image and $K_i = cov\,(V_1(i), \ldots V_N(i))$, is the covariance matrix for i^{th}pixel-set. This implies that (3) can be rewritten as,

$$P(\theta|V) = \prod_{i=1}^{M} P(\theta_i|V_i) \tag{7}$$

$$P(\theta_i|V_i) = c \cdot P(\theta_i) \cdot P(V_i|\theta_i)) \tag{8}$$

where c is the normalization constant. Thus the angle of arrival of each pixel in the image can be estimated separately with (8). The Bayesian estimation algorithm is designed to estimate the position of each pixel in the image separately. It involves the following steps [8],

1. Define a uniform prior distribution for the angle of arrival over one unambiguous interval as determined by the system angular ambiguity.
2. Calculate the likelihood probabilities with (6) using the measured data and pre-estimated values of coherence and variance in the covariance matrix, i.e. standard maximum likelihood estimators employing data within a rectangular window centred on the point of interest.
3. Compute the angle of arrival posterior distribution using from the prior and likelihood and normalize it to ensure that $c = \int P(\theta_i|V_i) \cdot d\theta = 1$.
4. Obtain the angle of arrival estimate as the MAP point of the posterior and work out the corresponding height, z_p.
5. Calculate standard deviation of the dominant mode of the posterior distribution and compute the corresponding height standard deviation.

III. MAInSAR Simulator

A MAInSAR simulator was developed for generating the multiple-image data required for evaluating the Bayesian estimation algorithm [8]. It uses the statistical forward model (4) to produce the data with the correct statistical characteristics. The simulator takes the scene digital elevation model (DEM), antenna parameters and side-looking geometry parameters as inputs. It calculates the coherence, interferometric phase and variance (from the radar cross section) for each pixel-pair from the scene model and the side-looking geometry. With these parameters the covariance matrix in (6) is calculated and the Cholesky decomposition method is applied to generate samples from forward model distribution (6) to obtain the complex data for each pixel in the image set with the proper correlation structure.

IV. Results

The simulator is used to simulate an asymmetrical three-antenna airborne InSAR system over a simple Gaussian DEM. The system and imaging parameters are: ($\beta = 90$, vertical baseline) antenna pair (1,3) baseline = 3 m, antenna pair (1,2)

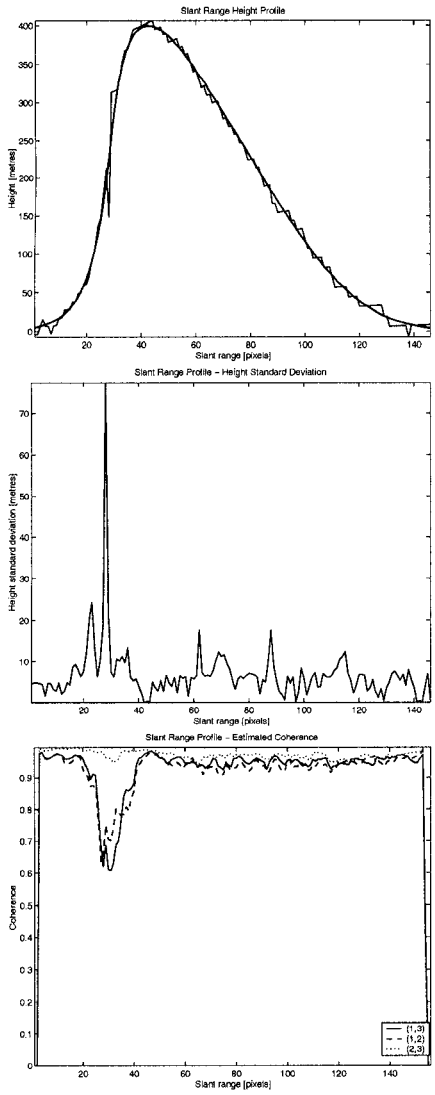

Fig. 3. Typical plots of estimated height, height standard deviation and the estimated coherence for a three-antenna InSAR system.

baseline = 2.5 m, antenna pair (2,3) baseline = 0.5 m , nominal SNR (mid-swath) = 18 dB, antenna height = 9 Km, incidence angle (mid-swath) = 37.5, λ = 5.66 cm (C-Band), slant range resolution = 12.5 m. The reduced height ambiguity of the three-antenna system as determined by appropriate ambiguity relationships [5],[8], is found to be 1241m. Thus the prior is defined as uniform distribution over the range of angles corresponding to the height interval (0-1241m). This implies that surface height can be estimated unambiguously over a range of 1241 m. In this example, the DEM used in simulation, ranges from 0 to 400 m and hence the terrain height can estimated unambiguously. The Bayesian estimation algorithm described in Section II is applied to one azimuth slice of data from the three simulated images. First plot in Fig. 3 shows the real height superimposed on the estimated height obtained from the corresponding angle of arrival estimates. The second plot in Fig. 3 shows the height standard deviation in the estimated height values as a function of the slant range. The last plot in Fig. 3 shows the estimated coherence for the three data pairs. The height standard deviation varies as a function of coherence as expected, since coherence is a measure of the data quality. Lower coherence indicates higher data-noise and hence lower estimation accuracy highlighted by the large height standard deviation values and vice-versa.

V. Conclusions

A Bayesian approach to topographic reconstruction with multiple-antenna interferometry has been presented. The results show the effectiveness of the Bayesian approach for simultaneous reduction of the phase ambiguity and phase noise. The posterior standard deviation provides a quantitative measure of the uncertainty in the estimated height values.

The current problem formulation does not addressed reconstruction in layover areas. Further work would include extension of the existing Bayesian algorithm to include layover regions with the applicable layover data models.

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