Collective effect in an electron plasma system catalyzed by a localized electromagnetic wave

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The possibility of the existence of an essentially single-species plasma state represented by a stable packet of charged particles moving collectively through space-time is examined. The collective plasma state is catalyzed by a localized electromagnetic wave. Condensation to this state is shown to occur on a very short time scale. The model treats the particle packet as a warm electron plasma (fluid) and self-consistently incorporates the resulting electromagnetic field. Predicted characteristics of the localized particle packet and its associated electromagnetic fields compare favorably with recent experimental data.

I. INTRODUCTION

It has been shown recently that localized-wave (LW) solutions can be constructed for a variety of linear hyperbolic partial differential equations.[1–4] For instance, these novel space-time solutions have been constructed for the scalar wave equation, Maxwell's equations, and the Klein-Gordon equation. They are characterized by the maintainence of their initial, localized characteristics over unusually long propagation distances.

Recent ultra-short-time discharge plasma experiments by Shoulders and his co-workers have produced data[5] that indicate the existence of a plasma state representing a freely moving, localized packet of electrons. This collective plasma state has been called an electromagnetic vortex (EV) by that group. They have been described as tightly bound groups of negative charges with extremely high densities. In particular, the EV's have been reported as follows:

(1) to be roughly spherically symmetric with radii on the order of 1.0 μm;

(2) to travel at speeds on the order of $0.1c$;

(3) to have electron densities approaching that of a solid, on the order of 10^{20} to 10^{24} cm^{-3} with negligible ion content;

(4) to have highly localized electromagnetic fields associated with them;

(5) to tend to propagate in straight lines for non-negligible distances on the order of 1.0 to 10.0 mm;

(6) to deflect and accelerate in experiments as though they have only electron characteristics;

(7) to be a highly localized energy state since they release copious amounts of x rays with their sudden destruction;

(8) to transport in some cases (called the black EV state) without emission of electrons or photons;

(9) and to form other quasistable structures by coupling adjacent EV's together.

The principle requirement for generating these EV structures has been reported to be a sudden creation of a very high, uncompensated set of electronic charges in a very small volume of space; i.e., a fast emission process coupled to a fast switching process. The times of creation are noted to be considerably less than 10^{-13} sec. The actual threshold initiation times for the particle packet are believed to be $\tau_i \sim 10^{-15}$ sec. The packet then travels approximately $10^5\tau_i$ before it catastrophically decays. Thus, these localized, long-lived, collective charge states are associated with very short time scales, and it is these short time scales, not the long propagation distances, that dictate their unusual characteristics. Additionally, although ions are present in the creation process, the mass differential between the electrons and the ions precludes the ions from moving at the observed speeds with the electron packets resulting in their absence.

Unfortunately, much of the experimental data is extremely difficult to obtain and exact quantitative numbers for speeds, packet sizes, and their electron densities do not appear to have been obtained. This opens the observations to a variety of explanations—some agreeing with those given in Ref. 5 and others agreeing with conventional wisdom. Nonetheless, the possibilities are intriguing and the ramifications of this work are startling if future efforts confirm the original observations and interpretations.

The similarity of this observed, localized plasma state with the LW solutions prompted our investigation. For discussion purposes, we will take its nominal descriptive parameters to be a threshold initiation time on the order of $\tau_i \sim 10^{-15}$ sec, an electron density on the order of $n_0 \sim 10^{27}$ m^{-3}, a radius on the order of $\rho \sim 1.0$ μm. and a group velocity on the order of $v_g \sim 0.1c$. With these plasma parameters and space-time scales in mind, we will examine the possibility of localized plasma states, discern their characteristics from the resulting model, and compare these results to the experimental data.

We will consider a non-neutral electron plasma and model it as a single component fluid. The associated electromagnetic fields are described by Maxwell's equations. In contrast to conventional analyses, the short time scales considered here necessitate the inclusion of the displacement current effects in our model. With the available LW solutions to the Klein-Gordon equation, we construct LW solutions to the combined Maxwell and fluid equation set. The characteristics of the resulting localized plasma state are discussed, and it will be shown that they are very similar to the ones observed experimentally. This long-lived state, bosonic in nature, can be treated as a collective mode of the plasma whose presence is catalyzed by the localized electromagnetic wave. Detailed descriptions of the actual creation of the packet are beyond the scope of the present work. Rather, we emphasize the possible existence of the localized plasma state given a realizable set of plasma and field parameters.

II. EM-PLASMA SYSTEM

We begin with the equations describing the behavior of an unconfined electron plasma. If its local density is n and its local velocity is $\mathbf{v}$, then one has the *continuity equation*

$$\partial_t n + \nabla \cdot (n\mathbf{v}) = 0 \tag{1a}$$

and the *momentum equation*

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B}) - \frac{1}{mn}\nabla p \ . \tag{1b}$$

The presence of the scalar pressure term p in the momentum equation is synonymous with a warm plasma model. We assume the plasma can be treated locally as a free-electron ideal gas. Thus, closure of the plasma equation system is achieved with the *equation of state*

$$p = nkT \ , \tag{1c}$$

where the temperature is taken to be constant locally. An increase in the particle density will then result in a decrease of the local velocity of the packet. Note that any equation of state in which the pressure p is a function of the electron density n will lead to the same results below. The present choice appears to be an adequate representation of the physical system under consideration.

The equations that describe the associated electromagnetic field behavior are *Maxwell's equations*

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \partial_t \mathbf{E} \ , \tag{2a}$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \ , \tag{2b}$$

$$\nabla \cdot \mathbf{E} = \frac{nq}{\epsilon_0} \ , \tag{2c}$$

$$\nabla \cdot \mathbf{B} = 0 \ . \tag{2d}$$

The free-space permittivity and permeability are given, respectively, as ϵ_0 and μ_0 and satisfy $\epsilon_0\mu_0 = c^{-2}$. The particle current is given simply as the *current equation*

$$\mathbf{J} = nq\mathbf{v} \ . \tag{3}$$

We will demonstrate that localized wave (LW) solutions for the joint plasma-fluid and electromagnetic-field system occur. This is achieved by reducing the equations to Klein-Gordon form and then using known Klein-Gordon LW solutions (see Appendix A for a discussion of these solutions).

III. REDUCTION TO KLEIN-GORDON FORMS

Consider first the electromagnetic-field behavior. We seek a Klein-Gordon equation for the vector potential $\mathbf{A}$, where the fields are defined in terms of the vector potential $\mathbf{A}$ and the scalar potential Φ as

$$\mathbf{E} = -\nabla\Phi - \partial_t \mathbf{A} \ , \tag{4a}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \ . \tag{4b}$$

Maxwell's equations and these expressions lead to the following equations for the potentials:

$$\begin{aligned} &\nabla^2\Phi + \partial_t(\nabla\cdot\mathbf{A}) = -\frac{nq}{\epsilon_0} \ , \\ &\nabla^2\mathbf{A} - \partial_{ct}^2\mathbf{A} - \nabla(\nabla\cdot\mathbf{A} + c^{-2}\partial_t\Phi) = -\mu_0\mathbf{J} \ . \end{aligned} \tag{5}$$

These equations are closed by a choice of gauge. We use the Lorentz gauge

$$\nabla\cdot\mathbf{A} + c^{-2}\partial_t\Phi = 0 \ , \tag{6}$$

unless stated otherwise. This reduces the equations relating the potentials and the sources to the form

$$\begin{aligned} &\nabla^2\Phi - \partial_{ct}^2\Phi = -\frac{nq}{\epsilon_0} \ , \\ &\nabla^2\mathbf{A} - \partial_{ct}^2\mathbf{A} = -\mu_0\mathbf{J} \ . \end{aligned} \tag{5$'$}$$

Now consider combining the momentum equation and the vector identity $(\mathbf{v}\cdot\nabla)\mathbf{v} = \nabla(\frac{1}{2}v^2) - \mathbf{v}\times(\nabla\times\mathbf{v})$. One obtains

$$\begin{aligned} \partial_t\mathbf{v} &= \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B}) + \mathbf{v}\times(\nabla\times\mathbf{v}) - \nabla(\tfrac{1}{2}v^2) - \frac{1}{nm}\nabla p \\ &= \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B}) + \mathbf{v}\times(\nabla\times\mathbf{v}) - \nabla\left[\tfrac{1}{2}v^2 + \frac{kT}{m}\ln n\right] \ . \end{aligned} \tag{7}$$

Applying the curl operator to this relation yields

$$\partial_t(\nabla\times\mathbf{v}) = -\frac{q}{m}\partial_t\mathbf{B} + \frac{q}{m}\nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla\times[\mathbf{v}\times(\nabla\times\mathbf{v})] \ .$$

Introducing the definition for the vorticity

$$\zeta = \nabla\times\mathbf{v} \ , \tag{8}$$

this relation can be rewritten as

$$\partial_t\mathbf{Q} = \nabla\times(\mathbf{v}\times\mathbf{Q}) \ , \tag{9}$$

where the quantity

$$\mathbf{Q} = \zeta + \frac{q}{m}\mathbf{B} \ . \tag{10}$$

Since

$$\nabla\cdot\mathbf{Q} = 0 \ , \tag{11}$$

Eq. (9) means that the quantity $\mathbf{Q}$ is preserved along the flow. Thus, if the system is prepared so that $\mathbf{Q}=\mathbf{0}$, it will retain that value for all time along the flow. We assume this to be the case with the result that

$$\zeta = -\frac{q}{m}\mathbf{B} \ . \tag{12}$$

We note that (9), hence this result, is independent of the form of the pressure term as long as $p \propto f(n)$; i.e., if the pressure can be written as a simple function of the density. The momentum equation can then be rewritten in the form

$$\partial_t \mathbf{v} + \nabla(\tfrac{1}{2}v^2) = \frac{q}{m}\mathbf{E} - \frac{1}{nm}\nabla p \ . \tag{13}$$

Equations (12) and (13) represent a slight generalization of the London equations which are used to describe the basic principles underpinning superconductivity.[6] The following analysis draws on this parallel.

Now consider the vector potential $\mathbf{A}$, in particular, with respect to Eqs. (4b) and (12). We introduce the generalized momentum $\boldsymbol{\pi} = m\mathbf{v} + q\mathbf{A}$ so that $m\mathbf{Q} = \nabla \times \boldsymbol{\pi}$. Moreover, with $\mathbf{Q} = 0$ this generalized momentum can be written in the form: $\boldsymbol{\pi} = \hbar\nabla\psi$, ψ being a scalar field whose gradient leads to a local wave-vector field and $\hbar$ being the constant of proportionality which we set equal to Planck's constant. Consequently, the velocity and the vector potential are related as

$$\mathbf{v} = -\frac{q}{m}\mathbf{A} + \frac{\hbar}{m}\nabla\psi \ . \tag{14}$$

The vector potential is defined up to a gauge transformation; i.e., the vector potential can be modified as $\mathbf{A}' = \mathbf{A} + \nabla f$, for any function f, with no resulting change in the field structure. Note that the vorticity is also unchanged by this gauge transformation even though the velocity field itself would be. This importance of the vector potential on the actual physics is similar to its role in quantum kinematical issues such as the Aharonov-Bohm effect.[7] We note that if $\mathbf{Q} \neq \mathbf{0}$, an additional term would have to be included in $\boldsymbol{\pi}$, hence (14), to reflect this fact.

Now with Eqs. (5′) and (14), we obtain the equation for the vector potential

$$(\partial_{ct}^2 - \nabla^2)\mathbf{A} = -\frac{nq^2}{\epsilon_0 mc^2}\mathbf{A} + \frac{\hbar nq}{\epsilon_0 mc^2}\nabla\psi \ . \tag{15}$$

Introducing the plasma frequency

$$\omega_p^2 = \frac{nq^2}{\epsilon_0 m} \ , \tag{16}$$

the vector potential equation then takes a Klein-Gordon form

$$\left[\partial_{ct}^2 - \nabla^2 + \frac{\omega_p^2}{c^2}\right]\mathbf{A} = \frac{\hbar}{q}\frac{\omega_p^2}{c^2}\nabla\psi \ . \tag{17}$$

Note that if n is locally constant, since $\mathbf{B} = \nabla \times \mathbf{A}$ and $\zeta = -(q/m)\mathbf{B}$, the magnetic field and the vorticity satisfy a homogeneous Klein-Gordon equation, *irrespective of any gauge transformation:* e.g., Eq. (17) gives

$$\left[\partial_{ct}^2 - \nabla^2 + \frac{\omega_p^2}{c^2}\right]\mathbf{B} = \mathbf{0} \ . \tag{18}$$

Equation (17) must be combined with Eqs. (1a) and (14) to describe the behavior of the charge density. The latter give

$$\frac{d}{d\tau}\ln n = -\nabla\cdot\mathbf{v} = \frac{q}{m}\nabla\cdot\mathbf{A} - \frac{\hbar}{m}\nabla^2\psi \ , \tag{19}$$

where the convective derivative

$$\frac{d}{d\tau} = \partial_t + \mathbf{v}\cdot\nabla \ .$$

Several cases are possible that lead to the desired localized wave results. First, consider the case where $\psi = 0$. the resulting equation set is

$$\mathbf{v} = -(q/m)\mathbf{A} \ , \tag{20a}$$

$$\left[\partial_{ct}^2 - \nabla^2 + \frac{\omega_p^2}{c^2}\right]\mathbf{A} = \mathbf{0} \ , \tag{20b}$$

$$\frac{d}{d\tau}\ln n = \frac{q}{m}\nabla\cdot\mathbf{A} \ . \tag{20c}$$

Even though the vector potential equation has been reduced to a homogeneous Klein-Gordon form, this system is highly coupled and nonlinear due to the presence of n in the plasma frequency ω_p term. General solution schemes for this case include iteration and linearization. For example, an iterative solution is obtained if we take $n \sim n_0$ and introduce the local plasma frequency

$$\omega_{p0}^2 = \frac{n_0 q^2}{\epsilon_0 m} \ , \tag{21}$$

in (20b). The desired LW solution $\mathbf{A}_0$ then follows from the discussion in Appendix A. Details of this solution will be discussed in Sec. IV. Formally one can then obtain the density solution of (20c) in terms of a LW solution of (20b)

$$n = n_0 \exp\left[+\frac{q}{m}\int_{\mathcal{L}} d\tau\, \nabla\cdot\mathbf{A}_0\right] \ , \tag{22}$$

where $\mathcal{L}$ is a flow line of the flow defined by $\mathbf{v}$. Since $\mathbf{A}_0$ is highly localized near the LW center, $z = v_g t$, n will be very small in comparison to n_0 everywhere except near the flow lines connected to the initial peaks of the particle packet. This solution could then be inserted back into (20b) for the next iteration and so on. However, because of the highly localized nature of the solutions, the zeroth-order solution $\mathbf{A}_0$ of the iteration sequence is quite good. We see immediately that a localized plasma state, defined by the initial distribution and a local perturbation, results if a localized electromagnetic wave is excited.

Note that the values of $\nabla\cdot\mathbf{A}$, hence $\nabla\cdot\mathbf{v}$ are not restricted in this case. Thus the flow is compressible, and we can take the vector potential to be along the z axis

$$\mathbf{A} \approx \Lambda\hat{\mathbf{z}} \ , \tag{23}$$

where Λ is a LW solution of the Klein-Gordon equation

$$\left[\partial_{ct}^2 - \nabla^2 + \frac{\omega_{p0}^2}{c^2}\right]\Lambda = 0 \ . \tag{24}$$

The magnetic field

$$\mathbf{B}=-\partial_r\Lambda\hat{\boldsymbol{\theta}}\ , \tag{25a}$$

is transverse to the direction of propagation. The associated electric field can be obtained from integrating the curl equation (2a)

$$\mathbf{E}=c^2\int_0^t dt\left[(\partial_z\partial_r\Lambda)\hat{\mathbf{r}}+\left[\frac{1}{r}\partial_r(r\partial_r\Lambda)+\frac{\omega_{p0}^2}{c^2}\Lambda\right]\hat{\mathbf{z}}\right]\ . \tag{25b}$$

It has both longitudinal and transverse components. The resulting vector structure of the electromagnetic field is characteristic of a moving electric dipole. The size of this dipole is, of course, $\sim n_0 q$. At the speeds the particle packet is moving, the resulting dipole field will be peaked near the axis of propagation.

Next, consider the case where $\psi\neq 0$. LW solutions to a forced Klein-Gordon equation such as (17) have been achieved recently by Besieris, Ziolkowski, and Shaarawi, and will be reported elsewhere. These new forced equation solutions are not as easy to characterize as those given in Appendix A. However, we do not have to abandon the latter because the homogeneous Klein-Gordon form can be recovered if we resort to a gauge transformation. Let $\mathbf{A}'=\mathbf{A}+\nabla\chi$ and $\Phi'=\Phi-\partial_t\chi$, where the gauge potential χ is defined by $\chi=-(\hbar/q)\psi$ and $(\nabla^2-\partial_{ct}^2)\chi=0$. This results in the following relations which are expressed in terms of the gauge transformed potentials:

$$\begin{aligned}&\mathbf{v}=-\frac{q}{m}\mathbf{A}'\ ,\\ &\left[\partial_{ct}^2-\nabla^2+\frac{\omega_p^2}{c^2}\right]\mathbf{A}'=\mathbf{0}\ ,\\ &\frac{d}{d\tau}\ln n=\frac{q}{m}\nabla\cdot\mathbf{A}'\ .\end{aligned} \tag{26}$$

One can then proceed immediately as we did above. On the other hand, we can now force

$$\nabla\cdot\mathbf{A}'=0\ , \tag{27}$$

without coming in conflict with the original potential equations (5′) and (6). Then $\nabla\cdot\mathbf{v}=0$ results and the flow field will be incompressible. The continuity equation reduces to the simple form

$$\frac{d}{d\tau}\ln n=0\ , \tag{28}$$

which means the particle density n will be constant along the flow lines, i.e., $n=n_0$, the initial particle density. The Klein-Gordon equation for the vector potential $\mathbf{A}'$ again provides the desired LW solution (see Sec. IV below). Note that the constraint (27) means that

$$\nabla^2\psi=+\frac{q}{\hbar}\nabla\cdot\mathbf{A}\ . \tag{29}$$

Thus in this case, the term ψ represents a quantum-mechanical potential associated with the plasma that compensates for the presence of the electromagnetic vector potential to maintain the initial particle density of the packet. For instance, if $\mathbf{A}=A_z\hat{\mathbf{z}}$ so that $\nabla\cdot\mathbf{A}=\partial_z A_z$, the potential ψ arises from the longitudinal variation in the vector potential: $\nabla^2\psi=(q/\hbar)\partial_z A_z$. Furthermore, if A_z is specified as the localized-wave solution (34), it will be symmetric and uniformly decreasing about its center, which is located at $z=v_g t$. One then has $\partial_z A_z<0$ ahead of this center and $\partial_z A_z>0$ behind it so that from (14) the term $\partial_z\psi\sim(q/\hbar)\int(\partial_z A_z)dz$ compensates for any local changes in the velocity from the longitudinal (static) component of $\mathbf{A}$ to keep the charges moving with the pulse center, i.e., near the center of the packet $v_z=-(q/m)[A_z-(\hbar/q)\partial_z\psi]\sim$const. Therefore, with $\mathbf{A}$ being defined by a localized-wave solution, one finds that the source of ψ, hence, ψ itself moves with the packet and compensates locally for any expansion forces to sustain a constant particle density along the flow.

The constraint (27) then is equivalent to enforcing a Coulomb gauge for the transformed potentials. Equation (6) also yields $\partial_t\Phi'=0$. The fields in this case can then be constructed from the relations

$$\begin{aligned}&\mathbf{A}'=\nabla\times(\Lambda\hat{\mathbf{z}})\ ,\\ &\mathbf{E}=-\partial_t\mathbf{A}'\ ,\\ &\mathbf{B}=\nabla\times\mathbf{A}'\ .\end{aligned} \tag{30}$$

This gives an electric field that will be transverse to the particle packet propagation direction. The associated magnetic field has components along all of the coordinate axes.

IV. LOCALIZED-WAVE EV SOLUTIONS

In contrast to standard arguments describing many collective effects associated with plasmas, we have assumed in deriving our relations that the displacement current is of the same order as the particle current and, hence, is not negligible. This can occur only if the process, which causes the collective effect of interest here, occurs on a very short time scale. In particular one has

$$\left|\frac{\mathbf{J}(=nq\mathbf{v})}{\partial_t(\epsilon_0\mathbf{E})}\right|\sim\frac{n_0q^2}{\epsilon_0 m}\left|\frac{\mathbf{A}}{\partial_t\mathbf{E}}\right|\sim\omega_{p0}^2\left|\frac{\mathbf{E}}{\partial_t^2\mathbf{E}}\right|\sim(\omega_{p0}\Delta t)^2\sim 1\ ,$$

so that the particle and displacement currents are comparable if the time scale for change Δt is on the same order as the inverse of the plasma frequency

$$\Delta t\sim\frac{1}{\omega_{p0}}\ . \tag{31}$$

Since we are dealing with phenomena where the density may be quite large, i.e., on the order of $n_0\sim 10^{27}\ \mathrm{m}^{-3}$, the time scale for the desired effect is on the order of

$$\Delta t\sim\frac{1}{56.4\times\sqrt{n_0}}\sim 10^{-15}\ \mathrm{sec}\ .$$

Thus if the plasma is formed on this time scale, a localized field described by (24) generates the localized-particle state according to either (22) or (28).

Following the LW Klein-Gordon analysis in Ref. 2 and in Appendix A, Eq. (24) has a general LW solution

$$\Lambda_{lm}=e^{-i\beta\gamma^2(1+v_g^2/c^2)[z-(c^2/v_g)t]}\cos(m\theta)\times j_l(2\beta\gamma[\rho^2+\gamma^2(z-v_gt)^2]^{1/2})\times P_l^m\left[\frac{\gamma(z-v_gt)}{[\rho^2+\gamma^2(z-v_gt)^2]^{1/2}}\right], \quad (32)$$

where the wave-number

$$\beta=\gamma\left[\frac{v_g}{c}\right]\left[\frac{\omega_{p0}}{c}\right]. \quad (33)$$

This solution assumes a cylindrical coordinate system (ρ,θ,z). We restrict the discussion only to the azimuthally symmetric zeroth order Klein-Gordon solution

$$\Lambda=\text{Re}\{e^{-i\beta\gamma^2(1+v_g^2/c^2)[z-(c^2/v_g)t]}\times j_0(2\beta\gamma[\rho^2+\gamma^2(z-v_gt)^2]^{1/2})\}. \quad (34)$$

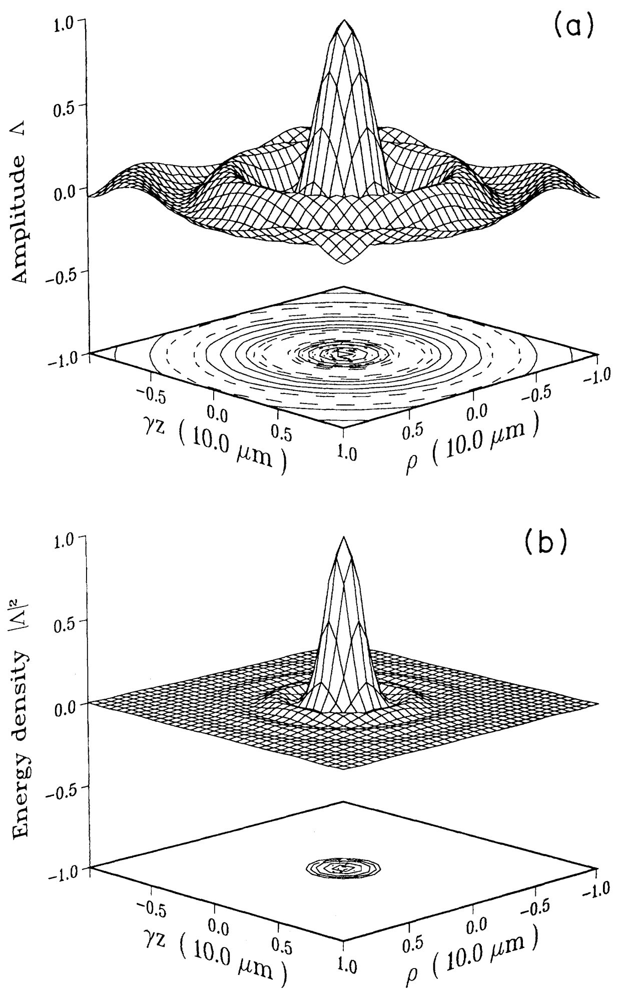

FIG. 1. The electromagnetic vector potential, localized-wave solution Λ is shown as a function of the spatial variables at $t=0$. (a) Its amplitude Λ and (b) its energy density $|\Lambda|^2$.

This localized vector potential moves along the z axis with a local group speed $v_g<c$, and its center occurs at $z=v_gt$. The degree of localization of the particle and electromagnetic wave packets is determined from the spherical Bessel function term. For speeds $v_g\sim 0.1c$ one has $\gamma\sim 1$ and

$$\beta\sim\frac{v_g\omega_{p0}}{c^2}=\left[\frac{q^2}{\epsilon_0 mc^2}\right]^{1/2}\frac{v_g}{c}\sqrt{n_0}.$$

The location of the first zero of j_0 occurs at a distance d, either along the transverse ρ or longitudinal $\gamma(z-v_gt)$ coordinate, given by the expression

$$d\sim\frac{\pi}{2\gamma\beta}\sim\frac{c}{v_g}\frac{c}{\omega_{p0}}. \quad (35)$$

Thus if the time scales are on the order of 10^{-15} sec; i.e., $n\sim 10^{27}\ \text{m}^{-3}$ so that $\omega_{p0}\sim 2\times 10^{15}\ \text{sec}^{-1}$, then

$$d\sim 10\times 3.0\times 10^8/2.0\times 10^{15}\sim 10^{-6}\ \text{m}=1.0\ \mu\text{m}.$$

These packet parameters agree quite nicely with the observed EV states. This micrometer-sized LW solution Λ and its energy density $|\Lambda|^2$ are shown in Figs. 1(a) and 1(b), respectively, for $t=0$. The dashed contours in both figures indicate regions where the values are less than or equal to zero.

V. CONCLUSIONS

In this paper we have shown the possibility of a collective, single-species plasma state that could be created in an ultrashort discharge event and that moves away from its initial region in a stable manner. This plasma state is correlated to a localized electromagnetic wave, particularly its vector potential, through the local velocity field that guides the packet. In one situation, the incompressible flow case, it was shown that a quantum-mechanical potential could arise that locally preserves the packet shape by compensating for the local electromagnetic field forces. For the compressible flow case, a LW state was realized through an iteration of the field and plasma quantities to reduce the coupled, nonlinear vector potential and plasma density equations to a manageable form. The characteristics of all of the resulting localized plasma states agree well with reported experimental data.

We must reemphasize that we have not modeled in any detail the breakdown events that lead to the localized charge states. Nonetheless, we anticipate that during the charge initiation process, there will be many ions present in the source region that would help compensate for the Coulomb repulsion forces of the electrons, hence, the formation of the electron cluster. As we have shown in Appendix B, if one accounts for a two-component plasma made up of ions and electrons, one can establish the existence of a localized state for that plasma. This localized state evolves into an electron cluster away from the source region because of the large difference in mass ratios between the electrons and the ions. Note that the existence of this state was established without the need of any external structure. If one were to introduce a dielectric guiding channel for the electron clusters, which has also been done experimentally,[5] their lifetimes would be enhanced because that structure would provide additional compensation for the Coulomb forces trying to dissolve the cluster. The latter configuration is analogous to laser-induced guiding channels for electron beams in rarefied gases. The localized electromagnetic fields excited by the very short initiation process provided the mechanism which overcame the Coulomb forces and led to the possible existence of the collective state represented by the "free"-electron cluster treated here.

With the possible theoretical existence of an EV state in hand, we will be investigating numerically in the future the full evolution of an EV state: from the initial conditions for their creation in a discharge event to their propagation through and interaction with the local environment. We hope to predict typical discharge parameters and configurations that lead to an EV state so that this phenomena can be investigated experimentally in a very detailed manner.

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APPENDIX A: LOCALIZED-WAVE SOLUTIONS OF THE KLEIN-GORDON EQUATION

Consider the general Klein-Gordon (KG) equation

$$(\partial_{ct}^2 - \nabla^2 + \mu^2)\Psi = 0 \ . \tag{A1}$$

The bidirectional form of the LW KG solution is obtained from the ansatz[4]

$$\Psi = G(\rho, z, t) e^{i\beta\eta} \ , \tag{A2}$$

where the variable

$$\eta = z + \left[\frac{c^2}{v_g}\right]$$

and the group velocity of the localized wave is

$$v_g = \frac{|\beta| c}{(\beta^2+\mu^2)^{1/2}} \ . \tag{A3}$$

This reduces (A1) to the form

$$i2\beta(\partial_z - v_g^{-1}\partial_t)G(\rho,z,t) + (\partial_z^2 - c^{-2}\partial_t^2)G(\rho,z,t) + \nabla_\perp^2 G(\rho,z,t) = 0 \ . \tag{A4}$$

Introducing the variables

$$\gamma = [1-(v_g/c)^2]^{-1/2} \ ,$$

$$\tau \equiv \gamma(z - v_g t) \ ,$$

(A4) becomes a hyperbolized Schrödinger equation

$$i4\beta\gamma\partial_\tau G(\rho,\tau) + \partial_\tau^2 G(\rho,\tau) + \Delta_\perp^2 G(\rho,\tau) = 0 \ . \tag{A4$'$}$$

A simple substitution

$$G(\rho,\tau) = g(\rho,\tau) e^{-i2\beta\gamma\tau} \ , \tag{A5}$$

results in the Helmholtz equation

$$\nabla_\perp^2 g(\rho,\tau) + \partial_\tau^2 g(\rho,\tau) + 4\beta^2\gamma^2 g(\rho,\tau) = 0 \ , \tag{A6}$$

which has the known, general solution

$$g_{lm}(\rho,\tau) = j_l(2\beta\gamma\xi) P_l^m(\tau/\xi)\cos(m\theta) \ , \tag{A7}$$

where

$$\xi = [\rho^2 + \gamma^2(z - v_g t)^2]^{1/2} \ .$$

Consequently, the general bidirectional solution has the form

$$\Psi_{lm}(\mathbf{r},t) = j_l(2\beta\gamma\xi) P_l^m\left[\frac{\tau}{\xi}\right]\cos(m\theta) e^{-i2\beta\gamma\tau} e^{i\beta\eta} \ . \tag{A8}$$

We note that the wave vector of the localized wave is defined from (A3) simply as

$$\beta = \gamma\left[\frac{v_g}{c}\right]\mu \ . \tag{A9}$$

If we hold v_g fixed and change μ, the wave number β changes proportionately to μ. This is of interest because β^{-1} determined the length scale of the localization. In the plasma case where n_0 is a constant particle density and

$$\mu^2 = \frac{\omega_{p0}^2}{c^2} = \frac{n_0 q^2}{\epsilon_0 m c^2} \ ,$$

and where $v_g \ll c$, one obtains

$$\beta \sim \sqrt{n_0} v_g \ . \tag{A10}$$

Thus, the LW solution is more localized for larger electron densities and for larger velocities.

Other LW solutions are possible. For instance, the unidirectional ansatz

$$\Psi = G(\rho,\tau) e^{-i\alpha[z-(c^2/v_g)t]} \tag{A11}$$

reduces the Klein-Gordon equation directly to a Helmholtz equation

$$\nabla_\perp^2 G(\rho,\tau) + \partial_\tau^2 G(\rho,\tau) + \chi^2 G(\rho,\tau) = 0 \ , \tag{A12}$$

where the solution constants α and χ are related as

$$\alpha^2\left[\left[\frac{c}{v_g}\right]^2 - 1\right] - \chi^2 = \mu^2 \ . \tag{A13}$$

Using known solutions to the Helmholtz equation, we arrive immediately at the general unidirectional form

$$\Psi_{lm}(\mathbf{r},t) = j_l(\chi\xi) P_l^m\left[\frac{\tau}{\xi}\right]\cos(m\theta) e^{\{-i\alpha[z-(c^2/v_g)t]\}} \ . \tag{A14}$$

In contrast to the bidirectional representation, note that (A13) can be rewritten as

$$v_g = \frac{\pm \alpha c}{(\alpha^2+\mu^2+\chi^2)^{1/2}} \tag{A15}$$

Thus, the unidirectional form of the KG solutions differs from the bidirectional one in that given the group velocity, there are now two free parameters, α and χ, rather than one, β. Moreover, with (A15) it allows for positive- and negative-energy solutions. Superpositions of (A8) of (A14) over their free parameters lead to further LW solutions.

APPENDIX B: LOCALIZED CHARGED-PARTICLE STATES FOR A TWO-COMPONENT PLASMA

Consider the fluid equations for a two-component plasma consisting of electrons and ions coupled with Maxwell's equations. Each species will be labeled by a subscript a, where $a=i$ for the ions and $a=e$ for the electrons. Assuming a scalar pressure for each component of the plasma, $p_a = f_a(n_a)$, and assuming charge neutrality so that the densities $n_e = n_i = n$, this equation set takes the form

$$\partial_t n + \nabla\cdot(n\mathbf{v}_a) = 0 \ , \tag{B1a}$$

$$\partial_t \mathbf{v}_a + \nabla\left[\frac{v_a^2}{2}\right] + \frac{\nabla p_a}{n} = \frac{q_a}{m_a}\mathbf{E} + \mathbf{v}_a \times \left[\zeta_a + \frac{q_a}{m_a}\mathbf{B}\right] , \tag{B1b}$$

$$\nabla\times\mathbf{B} = \mu_0 n q(\mathbf{v}_i - \mathbf{v}_e) + \mu_0\epsilon_0\partial_t\mathbf{E} \ , \tag{B1c}$$

$$\nabla\times\mathbf{E} = -\partial_t\mathbf{B} \ , \tag{B1d}$$

where the vorticity $\zeta_a = \nabla\times\mathbf{v}_a$. Taking the curl of (B1b) and setting

$$\mathbf{Q}_a = \zeta_a + \frac{q_a}{m_a}\mathbf{B} \ , \tag{B2}$$

gives

$$\partial_t \mathbf{Q}_a = \nabla\times(\mathbf{v}_a\times\mathbf{Q}_a) \ . \tag{B3}$$

Since $\nabla\cdot\mathbf{Q}_a = 0$, this relation means that $\mathbf{Q}_e$ and $\mathbf{Q}_i$ are preserved along the flows defined by $\mathbf{v}_e$ and $\mathbf{v}_i$, respectively. Now if the vector potential $\mathbf{A}$ is introduced such that $\mathbf{B} = \nabla\times\mathbf{A}$, the equation set for the two-component plasma becomes

$$\partial_t n + \nabla\cdot(n\mathbf{v}_a) = 0 \ , \tag{B4a}$$

$$\partial t \mathbf{Q}_a = \nabla\times(\mathbf{v}_a\times\mathbf{Q}_a) \ , \tag{B4b}$$

$$\partial_{ct}^2 \mathbf{A} - \nabla^2\mathbf{A} = \mu_0 n q(\mathbf{v}_i - \mathbf{v}_e) \ , \tag{B4c}$$

together with the appropriate initial conditions. If the system is prepared initially in the state where $\mathbf{Q}_a(t=0) = \mathbf{0}$, then one has for all times from (B4b):

$$\zeta_a = -\frac{q_a}{m_a}\mathbf{B} \ , \tag{B5a}$$

or equivalently,

$$\mathbf{v}_a = -\frac{q_a}{m_a}\mathbf{A} + \frac{\hbar}{m_a}\nabla\psi_a \ . \tag{B5b}$$

Then (B4c) reduces to the Klein-Gordon form

$$\partial_{ct}^2 \mathbf{A} - \nabla^2\mathbf{A} + \frac{\omega_p^2}{c^2}\mathbf{A} = \frac{\hbar}{m_e}\frac{\omega_{p,e}^2}{c^2}\nabla\psi_e + \frac{\hbar}{m_i}\frac{\omega_{p,i}^2}{c^2}\nabla\psi_i \ , \tag{B6}$$

where the plasma frequencies

$$\omega_{p,a}^2 = \frac{n q_a^2}{\epsilon_0 m_a} \ ,$$

$$\omega_p^2 = \frac{n q_e^2}{\epsilon_0 m_e} + \frac{n q_i^2}{\epsilon_0 m_i} \ . \tag{B7}$$

One can then proceed as was done in Secs. III and IV to obtain localized solutions to the remaining equation set (B4a) and (B6). However, since in general $m_i \gg m_e$, one finds that $v_e \gg v_i$ and $\omega_{p,e} \gg \omega_{p,i}$ so that (B6) reduces essentially to the electron equation (17). Moreover, this also means that even though an extremely rapid discharge event would most probably form a combined electron-ion localized state, that state quickly transforms itself into the related electron localized state described in Sec. III.

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