Electromagnetic localized waves that counteract Coulomb repulsion to catalyze a collective electron-packet state

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The basic generation parameters for a collective electron-packet state that is catalyzed by an electromagnetic localized wave are considered. Cancellation of the large Coulomb forces required to achieve this electron-packet state is shown to be possible with a properly designed electromagnetic field configuration that is intense and applied over an ultrafast time scale. Experimental realization possibilities are discussed.

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I. INTRODUCTION

The possibility of the existence of an essentially single species plasma state represented by a stable packet of charged particles moving collectively through space-time was examined previously in Ref. [1], hereafter referenced as I. The model in I treated a warm electron plasma as a fluid and self-consistently incorporated all of the electromagnetic fields. The collective electron-packet state (EPS) was catalyzed by an electromagnetic localized wave (ELW). Condensation to this state was shown to occur on a very short time scale (femtosecond) and on a very small distance (micrometer) scale. However, a number of issues, particularly those associated with the ability of the ELW to overcome the resulting Coulomb forces in the EPS have been raised (see, for instance, Ref. [2]). Nonetheless, it appeared in I that the theoretical model satisfactorily reproduced EPS parameters that agreed with experimental observations [3]. It will be shown in this paper that the Coulomb and ELW-generated forces can form an equilibrium state and, hence, that the predicted characteristics of the localized EPS are realizable physically. The requisite experimental parameters will be explored, and the potential experimental validation of the models will be discussed.

II. THE ELECTRON-PACKET STATE SOLUTION

The localized EPS is modeled with the coupled Maxwell and warm electron plasma fluid equations. In particular, if the local electron density is n and the local velocity of this electron fluid is $\mathbf{v}$, then one has the warm plasma equations

$$\partial_t n + \nabla\cdot(n\mathbf{v}) = 0 \quad \text{(continuity equation)}\,, \tag{1a}$$

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B}) - \frac{1}{mn}\nabla p \quad \text{(momentum equation)}\,, \tag{1b}$$

$$p = nkT \quad \text{(equation of state)}\,, \tag{1c}$$

and Maxwell's equations

$$\nabla\times\mathbf{B} = \mu_0 n q \mathbf{v} + \epsilon_0\mu_0 \partial_t \mathbf{E}\,, \tag{2a}$$

$$\nabla\times\mathbf{E} = -\partial_t \mathbf{B}\,, \tag{2b}$$

$$\nabla\cdot\mathbf{E} = \frac{nq}{\epsilon_0}\,, \tag{2c}$$

$$\nabla\cdot\mathbf{B} = 0\,. \tag{2d}$$

The free-space permittivity and permeability are respectively ϵ_0 and μ_0 and satisfy $\epsilon_0\mu_0 = c^{-2}$. The particle current is $\mathbf{J} = nq\mathbf{v}$. The presence of the scalar pressure term p in the momentum equation is synonymous with a warm plasma model (essentially a free-electron ideal gas); closure of the plasma equation system is achieved with the equation of state, Eq. (1c), the temperature being taken to be constant locally. Note that any equation of state in which the pressure p is simply a function of the electron density n will lead to the same results below. The present choice appears to be an adequate representation of the physical system under consideration. A two fluid ion-electron plasma model was also considered in I and was shown to promote the evolution of the electron plasma result treated below.

In I it was demonstrated that an ELW for the joint plasma-fluid and electromagnetic field system, Eqs. (1) and (2), occurs. First, it was shown that this system of equations yields the divergence-free quantity

$$\mathbf{Q} = \nabla\times\mathbf{v} + \frac{q}{m}\mathbf{B}\,, \tag{3}$$

which satisfies the relation

$$\partial_t \mathbf{Q} = \nabla\times(\mathbf{v}\times\mathbf{Q})\,, \tag{4}$$

and hence is preserved along the flow defined by the ve-

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locity field $\mathbf{v}$. This means that $\mathbf{Q}=0$ for all time along this flow if the system is prepared initially with that value. This result is independent of the form of the pressure term as long as $p \propto f(n)$; i.e., if the pressure can be written as a simple function of the density. Next, the electric scalar and magnetic vector potentials in the Lorentz gauge were introduced. The resulting canonical momentum $\mathbf{\Pi}=m\mathbf{v}+q\mathbf{A}$ is such that $\nabla\times\mathbf{\Pi}=m\mathbf{Q}$, and hence that $\mathbf{\Pi}=\hbar\nabla\psi$, ψ being a scalar field whose gradient leads to the local momentum field $\mathbf{\Pi}$ through a constant of proportionality which we set equal to $\hbar$, Planck's constant, over 2π. Consequently, the velocity and the vector potential are related as

$$\mathbf{v}=-\frac{q}{m}\mathbf{A}+\frac{\hbar}{m}\nabla\psi , \tag{5}$$

the velocity satisfying a slight generalization of the London equations: $\nabla\times\mathbf{v}=-(q/m)\mathbf{B}$ and $\partial_t\mathbf{v}+\nabla(\frac{1}{2}v^2)=(q/m)\mathbf{E}-(1/nm)\nabla p$. Finally, the equation for the vector potential is obtained in the Klein-Gordon form,

$$\left[\partial_{ct}^2-\nabla^2+\frac{\omega_p^2}{c^2}\right]\mathbf{A}=\frac{\hbar}{q}\frac{\omega_p^2}{c^2}\nabla\psi , \tag{6}$$

where the plasma frequency $\omega_p^2=nq^2/\epsilon_0 m$. The behavior of the charge density is now described by the relation

$$\frac{d}{d\tau}\ln n=-\nabla\cdot\mathbf{v}=\frac{q}{m}\nabla\cdot\mathbf{A}-\frac{\hbar}{m}\nabla^2\psi , \tag{7}$$

where the convective derivative $d_\tau=\partial_t+\mathbf{v}\cdot\nabla$.

Incompressible and compressible solutions of these nonlinear equation sets were considered in I based upon known Klein-Gordon LW solutions [4–6]. For discussion purposes, we will consider the compressible ($\psi=0$) solution here; the incompressible ($\psi\neq 0$) solution can be obtained as in I with the appropriate gauge transformation. The resulting equation set is

$$\left[\partial_{ct}^2-\nabla^2+\frac{\omega_p^2}{c^2}\right]\mathbf{A}=0 , \tag{8a}$$

$$\mathbf{v}=-(q/m)\mathbf{A} , \tag{8b}$$

$$\frac{d}{d\tau}\ln n=\frac{q}{m}\nabla\cdot\mathbf{A} . \tag{8c}$$

Even though the vector potential equation has been reduced to a homogeneous Klein-Gordon form, this system is highly coupled and nonlinear due to the presence of n in the plasma frequency ω_p term. General solution schemes for this case include iteration and linearization. For example, an iterative solution was obtained in I if we take $n\sim n_0$, giving the local plasma frequency $\omega_{p0}^2=n_0q^2/\epsilon_0 m$. The desired azimuthally symmetric zeroth order LW solution is

$$\mathbf{A}\sim A_0\Lambda\hat{\mathbf{z}} , \tag{9a}$$

where A_0 is a constant to be determined below and the Klein-Gordon solution is

$$\Lambda=e^{-i\beta\gamma^2(1+v_g^2/c^2)[z-(c^2/v_g)t]}\times j_0(2\beta\gamma\sqrt{\rho^2+\gamma^2(z-v_gt)^2}) , \tag{9b}$$

the wave number parameter β being given by the expression

$$\beta=\gamma\left[\frac{v_g}{c}\right]\left[\frac{\omega_p0}{c}\right]=\gamma\frac{v_g}{c}\left[\frac{e^2}{\epsilon_0 mc^2}\right]^{1/2}\sqrt{n_0}$$
$$=[1.879\times 10^{-7}\frac{v_g}{c}\sqrt{n_0}\ (\mathrm{m}^{-1})] . \tag{10}$$

This solution represents an oscillatory phase term times an envelope which moves at the speed v_g. The phase term, which travels at superluminal speeds, has little effect on the motion of the particles; it oscillates at such a high rate that the particles cannot respond to it. Thus the envelope is responsible for the behavior of the packet; the envelope variations then need only be considered to obtain the corresponding ELW field structures.

III. CANCELLATION OF THE COULOMB FORCES

The physical implications of the ELW-based EPS are considered in this section. First the specific parameter regime of interest is bounded. Next the cancellation of the Coulomb forces is developed. This treatment leads to the parameters that would be needed to physically realize the ELW-catalyzed EPS.

As shown in I, the vector potential solution (9) results in an ELW which causes a displacement current that can be made to be on the same order in magnitude as the particle current. This means the collective effect of interest here occurs on a very short time scale. In particular, the particle and displacement currents are comparable if the time scale for change, Δt, is on the same order as the inverse of the plasma frequency. The time scale for the desired effect is then $\Delta t\sim 2\pi/\omega_{p0}=1/[56.4\sqrt{n_0}]$. Furthermore, the localized vector potential defined by (9) moves along the z axis with a local group speed $v_g<c$, and its center occurs at $z=v_gt$. For the experimentally observed speeds [3], the local group speed $v_g\sim 0.1c$ and hence $\gamma\sim 1$. The degree of localization of the particle and electromagnetic wave packets is determined from the spherical Bessel function term. The location of the first zero of j_0 occurs at a distance d, either along the transverse, ρ, or longitudinal, $\gamma(z-v_gt)$, coordinate, given by the expression

$$d=\frac{\pi}{2\gamma\beta}=\frac{\pi}{2\gamma^2}\frac{c}{v_g}\frac{c}{\omega_{p0}} . \tag{11}$$

We take the EPS radius equal to d. The number of electrons in the EPS is then $N_e=n_0(4\pi d^3/3)$. Consequently, the EPS radius is directly proportional to the EPS particle number, i.e.,

$$d=3.416\times10^{-15}\left[\frac{v_g}{c}\right]^2 N_e\approx3.416\times10^{-17}N_e\ . \quad (12)$$

Assuming that the number of electrons in the EPS is equal to the claimed experimental value [3], $N_e=2.0\times10^{10}$, the EPS radius $d=0.683$ μm. This corresponds to an electron density $n_0=1.497\times10^{28}$ and hence to a plasma frequency $\omega_{p0}=6.901\times10^{15}$. The time scale for the desired effect is then on the order of $\Delta t\sim9.11\times10^{-16}$ s ~1.0 fs. Thus, if the electron plasma is formed on this time scale with sufficient density in the presence of the prescribed ELW, an EPS may be generated. Note that if $N_e=2.0\times10^{11}$, the EPS particle density $n_0=1.50\times10^{26}$ is less because its radius $d=6.83$ μm is larger. This would lead to a slower creation time $\Delta t\sim9.11\times10^{-15}$ s ~10.0 fs.

Consider now the time and spatial derivatives of the LW solution (9) near the packet enter, $\gamma z=\gamma v_g t+\zeta$ and $\rho\sim\delta$, where $\delta\ll1$ and $\zeta\ll1$. As discussed above, the time derivative near the EPS center along the axis of propagation is governed by its envelope behavior. Introducing the parameters $\eta=\beta\gamma^2(1+v_g^2/c^2)$ and $\xi=[\rho^2+\gamma^2(z-v_g t)^2]^{1/2}$, this time derivative is approximately

$$e^{i\eta}\partial_t\Lambda(\rho\sim0,z\sim v_g t+\zeta/\gamma)\sim\left[2\beta\gamma v_g\frac{\gamma^2(z-v_g t)}{\xi}j_1(2\beta\gamma\xi)\right](\rho\sim0,z\sim v_g t+\zeta/\gamma)\approx+\frac{(2\beta\gamma)^2}{3}\gamma v_g\zeta\ , \quad (13a)$$

since $j_1(x)\sim x/3$ for $x\ll1$. Similarly the z derivative is

$$e^{i\eta}\partial_z\Lambda(\rho\sim0,z\sim v_g t+\zeta/\gamma)\approx-\frac{(2\beta\gamma)^2}{3}\gamma\zeta\ . \quad (13b)$$

This also means that near the packet's center $\partial t\sim-(1/v_g)\partial_z$. Correspondingly, the transverse spatial derivative is

$$\begin{aligned}e^{i\eta}\partial_\rho\Lambda(\rho\sim\delta,z=v_g t)&\\ \sim\left[-2\beta\gamma\frac{\rho}{\xi}j_1(2\beta\gamma\xi)\right]&(\rho\sim\delta,z=v_g t)\\ \approx-\frac{(2\beta\gamma)^2}{3}\delta\ .& \quad (13c)\end{aligned}$$

Note that the spatial derivative is negative ahead of the pulse center where $z>v_g t$ so that $\zeta>0$ and positive behind it where $z<v_g t$ so that $\zeta<0$. Also note that the size of the longitudinal and transverse derivatives differs by a factor of γ as one would expect from relativistic considerations.

The solution to Eq. (8c) gives the density of charges in the plasma:

$$n=n_0\exp\left[+\frac{q}{m}\int_{\mathcal{L}}d\tau\,\nabla\cdot\mathbf{A}\right]\ , \quad (14)$$

where $\mathcal{L}$ is a flow line of the flow defined by $\mathbf{v}$. Since $\nabla\cdot\mathbf{A}=A_0\partial_z\Lambda$, the electron charge density along the flow is approximately

$$\begin{aligned}n&=n_0\exp\left[-(e/m)A_0\int_{\mathcal{L}}d\tau\,\partial_z\Lambda\right]\\ &\approx n_0\exp\left[-2\frac{eA_0}{mv_g}\frac{\zeta}{\gamma}\partial_z\Lambda\right]\\ &\sim n_0\exp\left[-\frac{8}{3}\frac{eA_0}{mv_g}\beta^2\gamma^2\zeta^2\right]\ , \quad (15)\end{aligned}$$

which is highly localized near the EPS and ELW centers as it should be. Thus the electron charge density n will be very small in comparison to n_0 everywhere except near the flow lines connected to the initial peaks of the EPS.

The Coulomb force field of one electron in the packet is (at least to first order) a linear superposition of all of the forces arising from pairwise interactions. This force field is linear and repulsive; its components have the approximate magnitudes [7]

$$\begin{aligned}F_{\text{Coulomb},z}&=\frac{1}{\gamma^2}\frac{N_e e^2}{4\pi\epsilon_0(d/2)^2}\\ &=\frac{1}{\gamma^2}\frac{[n_0(4\pi d^3/3)]e^2}{4\pi\epsilon_0(d/2)^2}=\frac{4n_0e^2d}{3\gamma^2\epsilon_0}\ , \quad (16a)\end{aligned}$$

$$F_{\text{Coulomb},\rho}=\gamma^3F_{\text{Coulomb},z}=\frac{4\gamma n_0e^2d}{3\epsilon_0}\ , \quad (16b)$$

where the radius of the packet d has been taken to define the number of electrons and $d/2$ has been taken to be the average distance between the charges within the EPS. For equilibrium to occur within the EPS this force must be balanced by an attractive force arising from the ELW.

The electromagnetic field generated by the localized wave is given by the following expressions. The magnetic induction field is transverse to the direction of propagation and has the form

$$\mathbf{B}=\nabla\times\mathbf{A}=-A_0\partial_\rho\Lambda\hat{\theta}\ . \quad (17a)$$

With (8b) the electric field [note the sign typographical error in Eq. (25b) of I] becomes

$$\mathbf{E}=c^2\int dt[\nabla\times\mathbf{B}-\mu_0 nq\mathbf{v}]=c^2\int dt\left[\nabla\times\mathbf{B}+\mu_0\frac{nq^2}{m}\mathbf{A}\right]$$
$$=c^2A_0\int dt\left\{(\partial_z\partial_\rho\Lambda)\hat{\boldsymbol{\rho}}+\left[-\frac{1}{\rho}\partial_\rho(\rho\partial_\rho\Lambda)+\frac{\omega_{p0}^2}{c^2}\Lambda\right]\hat{\mathbf{z}}\right\}$$
$$=c^2A_0\int dt\{(\partial_z\partial_\rho\Lambda)\hat{\boldsymbol{\rho}}+[\partial_z^2\Lambda-\partial_{ct}^2\Lambda]\hat{\mathbf{z}}\}\ . \tag{17b}$$

It has both longitudinal and transverse components. The total force arising from the ELW is thus

$$\mathbf{F}_{\rm LW}=q\mathbf{E}+q\mathbf{v}\times\mathbf{B}=-eE_z\hat{\mathbf{z}}-e[E_\rho-v_zB_\theta]\hat{\boldsymbol{\rho}}$$
$$=-ec^2A_0\int dt[\partial_z^2\Lambda-\partial_{ct}^2\Lambda]\hat{\mathbf{z}}-eA_0\left[c^2\int dt\ \partial_z\partial_\rho\Lambda+v_g\partial_\rho\Lambda\right]\hat{\boldsymbol{\rho}}\ . \tag{18}$$

Consequently, with Eqs. (13) one finds that the ELW-induced fores have approximately the following amplitudes.

Longitudinal component.

$$F_{{\rm LW},z}\sim -ec^2A_0\int dt\ \partial_t\left[-\frac{1}{v_g}\partial_z-\frac{1}{c^2}\partial_t\right]\Lambda=-ec^2A_0\left[-\frac{1}{v_g}\partial_z-\frac{1}{c^2}\partial_t\right]\Lambda$$
$$\sim +e\frac{c^2}{v_g}\gamma^{-2}\partial_z\Lambda\sim\left[e\frac{c^2}{v_g}\gamma^{-2}\right]\left[-\frac{(2\beta\gamma)^2}{3}\gamma\frac{d}{2}\mathrm{sgn}(\zeta)\right]A_0$$
$$=-\frac{\pi}{3}e\gamma\omega_{p0}A_0\mathrm{sgn}(\zeta)\ . \tag{19a}$$

Transverse component.

$$F_{{\rm LW},\rho}\sim -eA_0\left[-\frac{c^2}{v_g}\int dt\ \partial_t\partial_\rho\Lambda+v_g\partial_\rho\Lambda\right]=+eA_0\left[\frac{c^2}{v_g}-v_g\right]\partial_\rho\Lambda$$
$$=+e\frac{c^2}{v_g}\gamma^{-2}\partial_\rho\Lambda\sim\left[e\frac{c^2}{v_g}\gamma^{-2}\right]\left[-\frac{(2\beta\gamma)^2}{3}\frac{d}{2}\right]A_0$$
$$=-\frac{\pi}{3}e\omega_{p0}A_0\ . \tag{19b}$$

These expressions indicate that the longitudinal force is along the $-\hat{\mathbf{z}}$ direction ahead of the packet center and along the $+\hat{\mathbf{z}}$ direction behind it; hence the ELW force is opposite to the Coulomb force in these locations. Similarly, the transverse off-axis force is in the $-\hat{\boldsymbol{\rho}}$ direction; hence it is also attractive and opposite to the Coulomb force.

To compare the Coulomb and ELW force fields, the magnitude of the constant A_0 must be estimated. For the EPS to exist, the ELW force associated with the vector potential in the region of the EPS must be of sufficient magnitude to cancel the Coulomb forces there. The magnitude of the vector potential in the EPS region is approximately

$$A_0\approx\frac{\mu_0}{4\pi}\int\int\int\frac{|J(r')|}{|r-r'|}d^3r'\sim\frac{\mu_0}{4\pi}\frac{n_0ev_g}{d}\left[\frac{4\pi}{3}d^3\right]$$
$$=\frac{\mu_0}{4\pi}\frac{N_eev_g}{d}$$
$$=\tfrac{1}{3}\mu_0n_0ev_gd^2$$
$$=1.602\times10^{-26}\frac{N_ev_g}{d}\ . \tag{20}$$

For the assumed EPS parameters ($N_e=2.0\times10^{10}$, $v_g=0.1c$, and $d=0.683$ μm), the vector potential constant $A_0=0.0141$ Wb/m.

We can now compare the Coulomb and ELW forces directly. One has

$$\left|\frac{F_{{\rm LW},z}}{F_{{\rm Coulomb},z}}\right|=\frac{(\pi/3)e\gamma\omega_{p0}A_0}{(4n_0e^2d)/(3\gamma^2\epsilon_0)}$$
$$=\frac{\pi}{4}\gamma^3\frac{\omega_{p0}v_gd}{c^2}=\frac{\pi^2}{8}\gamma\sim1\ , \tag{21a}$$
$$\left|\frac{F_{{\rm LW},\rho}}{F_{{\rm Coulomb},\rho}}\right|=\frac{(\pi/3)e\omega_{p0}A_0}{(4\gamma n_0e^2d)/(3\epsilon_0)}$$
$$=\frac{\pi}{4}\frac{1}{\gamma}\frac{\omega_{p0}v_gd}{c^2}=\frac{\pi^2}{8}\frac{1}{\gamma^3}\sim1\ , \tag{21b}$$

which implies that the Coulomb and ELW forces are in balance within the EPS. The very large spatial and temporal variations within the ELW result in sufficiently large, attractive force components that compensate for the equally large and repulsive Coulomb force components existing within the EPS. In analogy with the basic BCS superconductivity concept of a Cooper pair, which results from the presence of a phonon background field, the ELW acts to provide the collective restoring force that catalyzes and maintains the EPS state.

We note from (21) that it appears that, if the packet velocity, and hence γ were to increase substantially, the EPS would begin to collapse in the z direction due to the ELW force and would expand dramatically in the ρ direction due to the Coulomb force. Thus any external forces on the EPS should cause it to break up in directions transverse to its direction of propagation.

Finally, the side of the ELW's electric field that would be required to achieve this balance is now readily estimated. Since $\mathbf{E}\sim -\partial_t \mathbf{A}$, this critical field value

$$|\mathbf{E}|_{\text{crit}} \approx \frac{(2\beta\gamma)^2}{3}\gamma v_g \frac{d}{2} A_0 \sim 2.369\times 10^{-9}\frac{v_g^2}{c^2}\frac{N_e}{d^2} = 9.923\times 10^{-9}\left[\frac{v_g}{c}\right]^2 n_0 d \ . \tag{22}$$

For the assumed EPS parameters this means $|\mathbf{E}|_{\text{crit}} = 1.02\times 10^{12}$ V/m = 1.02 MV/μm. Consequently, since the intensity and the electric field amplitude are related as $I = E^2/2\sqrt{\mu_0/\epsilon_0}$, the corresponding critical intensity would be $I_{\text{crit}} = 1.38\times 10^{21}$ W/m^2 = 1.38×10^{17} W/cm^2.

IV. EXPERIMENTAL REALIZATION OF THE EPS

It has been shown above that an EPS can result if an ELW is excited in a free-electron gas. Because of the stability exhibited by the general class of localized wave solutions, particularly to experimental uncertainties [8,9], it is anticipated that an excitation field with the rise time and spatial localization demanded by the ELW will be sufficient to excite the EPS. The rise time and distance scales are such that at $v_g = 0.1c$ the electrons move only $0.006\,37d$ for the time Δt. Thus the electrons cannot run out of the local potential well formed by the propagating ELW; hence the charges in the plasma should be captured by the electromagnetic forces generated in the central region of the ELW. This means that because of the extremely short rise times the EPS formation must occur fast enough so that the Coulomb forces do not have the time to prevent it.

The EPS should stably propagate in space as far as the ELW does. This coherence region would be over the extended Rayleigh distance associated with an ultrafast pulsed beam [10]. For the indicated femtosecond time scales, the effective wavelength [10] of the pulsed-laser system would be approximately $\lambda_{\text{eff}} \sim 150$ nm so that from a circular aperture having an $R\sim 5$ μm radius a Rayleigh distance of $\pi R^2/\lambda_{\text{eff}} \sim 500$ μm could be achieved. It then would be possible to measure experimentally the resulting ELW-EPS structures and behaviors. Moreover, this distance suggests several interesting applications including micrometer-sized free-electron or Smith-Purcell lasers driven by a stream of EPS's.

Several laser systems are becoming available that could achieve the ELW and EPS intensity, distance, and time operating conditions. In particular, there are many systems whose intensity levels surpass 10^{18} W/cm^2 and that can be focused to micrometer-sized dimensions. Moreover, designed ultrafast-pulse focal enhancements [11,12] could readily surpass the necessary intensities with lower power systems. Pulse designs yielding a 20-fold enhancement in the focal plane intensity would boost an $I\sim 10^{16}$ W/cm^2 system above the EPS threshold value I_{crit}. Nonetheless, the requisite rise time of the ELW pulse, ~ 1.0 fs, is an order of magnitude shorter than those associated with most currently available systems. This would be particularly true for a focal enhancement system that would require pulse shaping in the subfemtosecond regime. Fortunately, ultrafast (attosecond) pulsed-laser systems with these characteristics are currently under development. One could envision in the near future an ultrafast pulsed-laser–lens system tailored to develop the necessary ELW field distribution as it propagates into an electron plasma to form the EPS. This is but one of many potential effects that could be studied with future subfemtosecond pulsed-laser systems.

The ultrafast time discharge plasma experimental work [3] by Shoulders and his co-workers can now be evaluated further. Their extremely rapid switching process has to conspire properly with an equally fast or faster emission process to achieve their so-called EV (*electrum validum* or electromagnetic vortex). The plasma evolution events would have to include the formation of a combined electron-ion localized state initially which would then transform itself quickly into the related EPS as described in I. An average EV was reported [2,3] to contain on the order of 2.0×10^{10} electrons in a $d\sim 1.5$ μm radium sphere. This gives $n_0 \sim 1.41\times 10^{27}$; hence $\omega_{p0} = 2.12\times 10^{15}$ and $\Delta t = 2.95$ fs. The corresponding EPS critical field strength is then $E_{\text{crit}} \sim 2.11\times 10^{11}$ V/m. This value appears to be quite different from the electric field strengths obtained from the reported experimental parameters.

The typical EV operating parameters were reported to be a 10.0 kV voltage applied over a 100 μm distance for an electric field strength of 1.0×10^{8} V/m. This is a factor of 2000 times smaller than the calculated E_{crit} for the theoretical ELW-EPS system. The EPS formation corresponding to the EV would require a voltage discharge event having a 200.0 kV peak field applied over a 1.0 μm distance on a femtosecond time scale. These theoretical parameters do not seem to be compatible with the EV experimental ones. However, note that the various EV sources appear to be configured like micrometer-sized Marshall (electron) guns [3] in which the observed breakdown and EV evolution processes actually deal with fine filaments that form in the breakdown region. This means that the effective distances over which the local field strengths are applied are in fact considerably smaller than the anode-cathode separation distance. Taking the effective interaction distance to be a filament size ~ 0.1 μm, one has $|E|\sim 10^{11}$ V/m at a 10.0 kV voltage strength. This would put the experimental parameters in line with the ELW-EPS theoretical values. Yet another possibility is the fact that EV's are frequently generated with tip-sharpened electrodes. If the electrodes were fabricated with micromachining techniques, their tips could have points sharpened to within 1000 Å$=0.1$ μm, easily smaller than the indicated EV size. If such a cathode were positioned within a tip separation distance from the anode (~ 0.1 μm), one would again obtain an electric field strength on the order of $|E| \sim 10^{11}$ V/m with an applied voltage of 10.0 kV. Hence the conditions for an ELW-EPS are possible, in principle, even in the original EV experiments.

Other ELW-EPS solutions of the electromagnetic plasma system (1) and (2) are possible and may conform better to the EV experimental values. The EV may even be formed by an entirely different electromagnetic process. Nonetheless, the claim that EV formation will be associated with an extremely fast, intense discharge event still seems very plausible.

Unfortunately, as noted in I, many of the EV experimental data are extremely difficult to measure. Exact quantitative numbers for speeds, packet sizes, and their electron densities still do not appear to have been obtained since I appeared. This still leaves the claimed EV observations open to a variety of explanations. More extensive experiments continue to be badly needed. Nonetheless, the possibilities remain intriguing, particularly in view of the present calculations that predict the existence of an equilibrium ELW-EPS configuration. Pulsed-laser-induced plasma experiments that could provide the requisite ELW-EPS generation characteristics are nearly available. Improved ultrafast, ultrasmall discharge experiments could be designed and performed with current technologies. The ramifications of an EV or a stable ELW-EPS configuration remain startling and could lead to a number of very practical devices if future efforts confirm their existence and quantify their operational parameters.

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