Localized Wave Representations of Acoustic and Electromagnetic Radiation

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Novel space-time solutions to general classes of wave equations and their properties will be reviewed briefly. The localized wave (LW) solutions, in contrast to their continuous wave (CW) or monochromatic counterparts, exhibit enhanced localization and energy fluence characteristics. This has led to the analysis and construction of pulse-driven, independently addressable arrays to investigate nonstandard methods of wave energy transmission based upon these LW solutions. Such arrays allow different signals to be transmitted from different locations in the array, thus allowing shading of the spectral features as well as the amplitudes seen by the array. It has been shown experimentally that the beams transmitted by these LW pulse-driven arrays outperform conventional CW driven arrays.

I. Introduction

Large classes of nonseparable space–time solutions of the equations governing many wave phenomena (e.g., scalar wave, [1]–[6] Maxwell's, [3], [7] Klein-Gordon [8] equations) have been reported recently. When compared with traditional monochromatic, continuous wave (CW) solutions such as Gaussian or piston beams, these localized wave (LW) solutions are characterized by extended regions of localization; i.e., their shapes and/or amplitudes are maintained over much larger distances than their CW analogues. This is also true in complex environments such as naturally dispersive media (waveguides) [9] and lossy media. These discoveries have prompted several extensive investigations into the possibility of using these LW solutions to drive finite sized arrays, thereby launching fields having extended localization properties [10], [11]. This paper presents a brief review of the theoretical results and experimental evidence associated with this LW effect.

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The possibility of solutions of Maxwell's equations that might describe efficient, localized transfer of electromagnetic energy in space was first suggested by Brittingham [1]. It has been recently discovered that the original "Focus Wave Modes" (FWM) introduced by Brittingham [1] represent Gaussian beams that translate through space with only local deformations [2]–[5]. Unfortunately, the *FWM* is not focused in the sense originally intended; i.e., it is not a purely localized, translationally invariant solution of the wave equation. The latter is in fact impossible to obtain. A boost solution of the form $\Phi(x, y, z-ct)$ requires $\{\Delta - \partial_{ct}^2\}\,\Phi(\vec{r}_\perp, z - ct) = \Delta_\perp \Phi = 0$; hence, that Φ be a harmonic function. This precludes Φ from having a compact spatial support. Nonetheless, these fundamental Gaussian beams can be used to synthesize other interesting, novel, exact solutions of the wave and Maxwell's equations. As will be discussed below, these LW solutions can be tailored to give localized transmission of wave energy in space and time.

These LW solutions may also have some significance to our basic understanding of elementary particles, such as photons. They allow one to reconcile our intuition of aggregate macroscopic phenomena with a microscopic picture of our world. For instance, a self-consistent photon model has been constructed [12] which encompasses well-known characteristics of photons emitted from atomic transitions and those undergoing Compton scattering with electrons. This model also reproduces [13] the results of Young's two-slit scattering experiment. In particular, it simultaneously incorporates the particle (the solution explains which slit the photons passes through) and the wave (the solution recovers the classic diffraction pattern) nature of the photon interaction with the two-slit screen. While many aspects of this model remain to be explored, the results to date offer a rather tantalizing point of view on the wave-particle duality as well as the possibility of reconciling contemporary ideas of photon localization and causality.

II. Wave Equations

The scalar wave equation

$$\{\Delta - \partial_{ct}^2\}\Phi(\vec{r},t) = 0 \quad (2.1)$$

governs many basic wave phenomena in real, homogeneous, isotropic, lossless media. It is the fundamental equation underlying the physics of wave propagation.

A. FWM Representations

We have shown [4]–[7] that one can construct exact solutions of the scalar wave equation and Maxwell's equations that describe localized, slowly decaying transmission of wave energy in space-time. One such class of fundamental solutions [2] arises naturally if one assumes a solution of the form

$$\Phi_k(\vec{r},t) = e^{ik(z+ct)}G(x,y,z-ct). \quad (2.2)$$

The latter reduces (2.1) to a Schrödinger equation for G in the pulse center or light cone variable $\tau = z - ct$. Setting the transverse distance ρ equal to $\sqrt{\vec{r}_\perp \cdot \vec{r}_\perp} = \sqrt{x^2+y^2}$ and considering specifically an axisymmetric solution, the original wave equation (2.1) has the moving, modified Gaussian pulse

$$\Phi_k(\vec{r},t) = e^{ik\,(z+ct)}\frac{e^{-k\rho^2/V}}{4\pi i V} \quad (2.3)$$

as an *exact* solution. This is the scalar counterpart of the original *FWM* . The complex variance $1/V = 1/A - i/R$ yields the beam spread $A = z_0 + \tau^2/z_0$, the phase front curvature $R = \tau + z_0^2/\tau$, and beam waist $W = (A/k)^{1/2}$. Because it can be associated with a source at a moving complex location ($\rho = 0$, $z = ct + iz_0$), (2.3) represents a generalization of earlier work by Deschamps [14] and Felsen [15] describing Gaussian beams as fields radiated from stationary complex-source points. Nonetheless, the solution (2.3) is source-free in real space. One usually specifies the real-part of (2.3) as the desired field since the resulting function has its maximum at ($\rho = 0$, $z = ct$). These results are also related to earlier work by Trautman [16] who considered constructing new solutions of the wave equation by applying complex, inhomogeneous Lorentz transformations to known wave equation solutions. The parameter k is free and represents the lowest radian frequency $\omega_{min} = kc$ contained in the solution, i.e., the plane wave term in (2.3) acts like a high-pass frequency filter. Similarly, the parameter $\omega_{max} = c/z_0$ defines the $1/e$ roll-off point in spectrum of the solution, i.e., it represents the maximum radian frequency in the spectrum.

The fundamental Gaussian pulses [3] have either a transverse plane wave or a particle-like character depending on whether k is small or large. Moreover, for all k they share with plane waves the property of having finite energy density but infinite total energy. However, as with plane waves, this is not to be considered as a drawback *per se*. The above solution procedure has introduced an added degree of freedom into the solution through the variable k that can be exploited, and these fundamental Gaussian pulse fields can be used as basis functions to represent new transient solutions of (2.1). In particular,

$$\begin{aligned} f(\vec{r},t) &= \int_0^\infty \Phi_k(\vec{r},t)F(k)dk \\ &= \frac{1}{4\pi i[z_0 + i(z-ct)]}\int_0^\infty dkF(k)e^{-k\,s(\rho,z,t)} \end{aligned} \quad (2.4a)$$

where

$$s(\rho,z,t) = \frac{\rho^2}{z_0 + i(z-ct)} - i(z+ct) \quad (2.4b)$$

is an exact source-free solution of the wave equation. This representation, in contrast to plane wave decompositions, utilizes basis functions that are more localized in space and hence, by their very nature, are better suited to describe the directed transfer of wave (electromagnetic) energy in space. The resulting pulses have finite energy if, for example, $F(k)/\sqrt{k}$ is square integrable [3].

Solutions to Maxwell's equations follow naturally from the scalar wave equation solutions. Let f be a LW solution of the scalar wave equation (2.1). Defining the electric, $\vec{\Pi}_e = f\hat{n}$, or magnetic, $\vec{\Pi}_h = f\hat{n}$, Hertz potential along the arbitrary direction $\hat{n}$, one readily obtains fields satisfying Maxwell's equations that are TE or TM with respect to $\hat{n}$. For instance, if a *TE Polarized Field* is desired,

$$\begin{aligned} \vec{E} &= -Z_0\nabla \times \partial_{ct}\vec{\Pi}_h \\ \vec{H} &= \nabla(\nabla \bullet \vec{\Pi}_h) - \partial_{ct}^2\vec{\Pi}_h. \end{aligned} \quad (2.5)$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0}$ and $Y_0 = \sqrt{\epsilon_0/\mu_0}$ are, respectively, the free-space impedance and admittance.

B. Modified Power Spectrum (MPS) Solution

Clearly, different spectra $F(k)$ in (2.4) lead to different wave equation solutions. Many interesting solutions of the wave equation are created simply by referring to a Laplace transform table. One particularly interesting spectrum selection is the MPS [3]:

$$F(k) = \begin{cases} [4\pi i\beta/\Gamma(\alpha)](\beta k - b)^{\alpha-1}e^{-a(\beta k - b)}, & k > b/\beta \\ 0, & 0 \le k < b/\beta \end{cases} \quad (2.6)$$

It is so called because it is derived from the power spectrum $F(k) = k^{\alpha-1}e^{-ak}$ by a scaling and a truncation. This choice of spectrum leads to the *(MPS)* pulse

$$f(\rho,z,t) = \frac{1}{z_0 + i(z-ct)}\frac{1}{[(s/\beta) + a]^\alpha}e^{-b\,s/\beta}. \quad (2.7)$$

Much effort has been concentrated on this MPS pulse because it has an appealing analytical form and its pulse shape can be tailored to a particular application with a straightforward change in parameters. The transverse behavior of this MPS pulse at the pulse center is essentially $f(\rho, z = ct, t) \sim e^{-b\rho^2/\beta z_0} f(\rho = 0, z = ct, t)$. The corresponding transverse spatial spectra, that is, the $k_x - k_y$

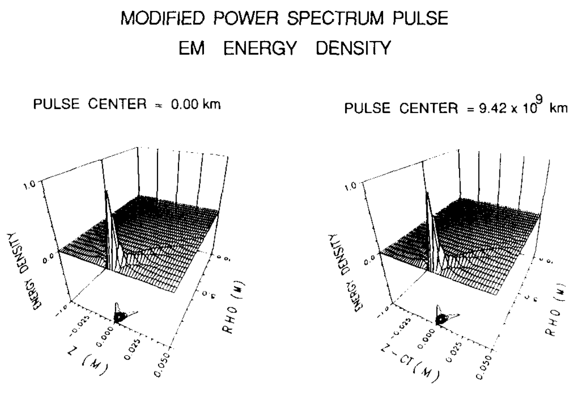

Fig. 1. The field energy density of an electromagnetic MPS pulse remains localized even at extremely large distance from its initial location.

spectrum at various distances $z = ct$, have been shown to remain nearly invariant as the MPS pulse propagates. Along the direction of propagation z and away from the pulse center the MPS pulse decays as $f \sim 1/[z_0^2 + (z-ct)^2]$. Hence, it is localized along the direction of propagation as well.

The MPS pulse can be optimized so that it is localized near the direction of propagation and its original amplitude is recovered out to extremely large distances from its initial location. Its component waveforms, and, therefore, their broad bandwidth spectra, are strongly correlated to each other, a self-similarity property inherent to the LW solutions. The uniqueness of such solutions is their intrinsic space-time nature: they are completely nonseparable. The MPS pulse can be recovered approximately from a finite array of radiating elements by specifying both their spatial and their temporal distributions [3], [10], [11].

The causal, localized nature of these solutions is demonstrated in Fig. 1. It shows surface plots and the corresponding contours plots of the electromagnetic energy density U of a TE electromagnetic MPS pulse relative to the pulse center locations at $z = 0.0$ km and $z = 9.42 \times 10^9$ km. The MPS parameters are $a = 1.0\,\mathrm{m}$, $\alpha = 1.0$, $b = 1.0 \times 10^{14}\,\mathrm{m}^{-1}$, $\beta = 6.0 \times 10^{15}$, and $z_0 = 1.0 \times 10^{-2}$ m. U is normalized to its maximum value at $t = 0$. The transverse space coordinate ρ is measured in meters; the longitudinal space coordinate, $z - ct$, is the distance in meters along the direction of propagation away from the pulse center $z = ct$. These results definitively show the localization of the field near the direction of propagation over very large distances. For the present choice of parameters, the MPS pulse becomes a $1/z$ field when $z \sim \beta a/2 = 3.0 \times 10^{15}\, m$.

C. Bidirectional Representations

A new decomposition of these exact, scalar wave equation solutions into bidirectional, forward and backward, traveling plane solutions is also possible [4], [9]. In particular, it has been shown that:

$$f(\vec{r},t) \equiv \Psi(\rho,\phi,\zeta = (z-ct), \eta = (z+ct))$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int_0^{\infty} du \int_0^{\infty} dv \int_0^{\infty} d\chi \quad \chi J_n(\chi\rho) e^{in\phi} \times \delta(uv - \chi^2/4) G_n(u,v;\chi) e^{+iv(z+ct)} e^{-iu(z-ct)}$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int_0^{\infty} d\chi \int_0^{\infty} dv \frac{1}{v} \quad \chi J_n(\chi\rho) e^{in\phi} \times G_n\Big(\frac{\chi^2}{4v}, v; \chi\Big) \times e^{+iv(z+ct)} e^{-i[\chi^2/(4v)](z-ct)} \tag{2.8a}$$

is actually a generalization of the representation (2.4). This *bidirectional representation* has the inverse:

$$G_n\Big(\frac{\chi^2}{4v}; v; \chi\Big) = \frac{\sqrt{\pi}}{2} \int_{-\pi}^{+\pi} d\phi \int_{-\infty}^{\infty} d\zeta e^{-\zeta^2/16v^2} \times \int_{-\infty}^{\infty} d\eta \int_0^{\infty} d\rho\rho \times \Psi(\rho,\phi,\zeta,\eta) J_n(\chi\rho) \times e^{-in\phi} e^{+i[\chi^2/(4v)]\zeta} e^{-iv\eta}. \tag{2.8b}$$

The representation (2.4) is obtained by taking the lowest order azimuthal mode, introducing the spectrum

$$G_0(u,v;\chi) = \frac{F(v) e^{-u z_0}}{8\pi i} \tag{2.9}$$

and making the identification of the parameter v with k. Like (2.4), the representation (2.8) is complete and has a well-defined inverse. The more conventional forward and backward propagating plane wave representation can be extracted from (2.8) in a straightforward fashion. In particular, by introducing the variables $k_z = u - v$ and $\omega = u + v$ into (2.8), the basis functions $J_n(\chi\rho)\exp(in\phi)\exp(+iv\sigma)\exp(-iu\tau) \to J_n(\chi\rho)\exp(in\phi)\exp(-i(k_z z - \omega t))$, the constraint relation $uv = \chi^2/4 \to \omega^2 = (k_z^2 + \chi^2)c^2$, and the representation (2.8) becomes

$$f(\vec{r},t) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d(k_z c) \int_0^{\infty} d\omega \int_0^{\infty} d\chi \times \chi J_n(\chi\rho) e^{in\phi} \quad \delta(\omega^2 - [k_z^2 + \chi^2]c^2) \times G_n\Big(\frac{(\omega + k_z c)}{2c}, \frac{(\omega - k_z c)}{2c}; \chi\Big) e^{-i(k_z z - \omega t)} \tag{2.10}$$

a superposition of forward and backward propagating Bessel Beams [17]. The conventional forward and backward propagating plane wave representation follows immediately:

$$f(\vec{r},t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int_0^{\infty} d(k_z c) \int_0^{\infty} d\omega \int_0^{\infty} d\chi \times \chi J_n(\chi\rho) e^{\pm in\phi} \quad \delta(\omega^2 - [k_z^2 + \chi^2]c^2)$$

$$\times \left[G_n^+(k_z,\omega;\chi)e^{-i(k_z z-\omega t)} + G_n^-(k_z,\omega;\chi)e^{+i(k_z z+\omega t)} \right]$$

$$= f_z^+(\vec{r},t) + f_z^-(\vec{r},t). \quad (2.11)$$

Note that these results imply that the bidirectional representation does not replace the standard Fourier synthesis, but rather complements it, especially for the LW solutions. Moreover, the bidirectional representation allows for the intriguing result that locally in a very small region of space-time, one can have causal and acausal components coexisting, but that globally only a causal solution persists. In particular, a number of concerns about the causal nature of the original focus wave mode have been raised in [18]. As mentioned previously, the "tweaked-up" MPS pulse should not suffer from the same pathologies. This can in fact be demonstrated analytically with the bidirectional representation. Integrating over ω to remove the delta function constraint in the lowest order form of the azimuthally symmetric version of (2.10), the MPS pulse is recovered from the expression:

$$f(\vec{r},t) = \frac{1}{\pi}\int_0^\infty d\chi\chi\, J_0(\chi\rho)\int_{-\infty}^{\infty} dk_z \, F(\chi,k_z)e^{-i(k_z z-\omega t)} \quad (2.12)$$

where $\omega = +\sqrt{\chi^2+k_z^2}c$ and the "bidirectional spectrum" [3], [4]:

$$F(\chi,k_z) = \frac{\beta\pi^2}{\Gamma(\alpha)\sqrt{\chi^2+k_z^2}} \times \left[\frac{\beta}{2}(\sqrt{\chi^2+k_z^2}-k_z)-b\right]^{\alpha-1} \times e^{ba}e^{-[\sqrt{\chi^2+k_z^2}+k_z]z_0/2} \times e^{-[\sqrt{\chi^2+k_z^2}-k_z]\beta a/2} \quad \text{for} \quad k_z > \frac{\beta}{b}\frac{\chi^2}{4}-\frac{b}{\beta}$$

$$F(\chi,k_z) = 0 \quad \text{for} \quad k_z < \frac{\beta}{b}\frac{\chi^2}{4}-\frac{b}{\beta}. \quad (2.13)$$

Note that the complicated form of this expression reveals the difficulty in developing the LW solutions from a conventional Fourier point of view. Equation (2.13) gives the spectrum for the forward and backward propagating form of the MPS pulse. When $b/\beta \ll 1$, there clearly is no backward propagating component. If b/β is not small, the ratio of the spectrum $F(\chi,+k_z)$ and $F(\chi,-k_z)$ is such that:

$$\frac{F(\chi,+k_z)}{F(\chi,-k_z)} \sim e^{|k_z|(\beta a - z_0)}.$$

Therefore, for the "tweaked-up" MPS pulse considered above where $\beta \gg 1$, one has $F(\chi,+k_z) \gg F(\chi,-k_z)$ for nonnegligible k_z; as a consequence, only the forward propagating component contributes to the MPS pulse. On the other hand, if $\alpha = \beta = 1.0$, $b = 0$, and $a = z_0$ one recovers the Splash Pulse reported in [2] from the MPS pulse. For that case this ratio yields $F(\chi,+k_z) \sim F(\chi,-k_z)$; i.e., there are about equal amounts of forward and backward propagating waves, as illustrated by Figs. 3 of [2]. The same analysis applied to the focus wave mode reproduces known conclusions [12], [18]: the *FWM* consists mainly of backward propagating components for $k z_0 \gg 1$, and forward propagating components for $k z_0 \ll 1$.

III. Solutions in Complex Media

For many realistic situations, the wave equation (2.1) is not an adequate model of the underlying physics. Boundary conditions and material behavior must be included. As illustrated below, LW solutions in complex environments exist and are readily constructed from the bidirectional point of view. The following results have been extended recently to plasma environments [19] and to fiber optical waveguides [20].

A. Klein-Gordon Equation Solutions: Bi- and Uni- Directional Cases

Consider the general Klein-Gordon (KG) equation

$$\left(\partial_{ct}^2 - \nabla^2 + \mu^2\right)\Psi = 0. \quad (3.1)$$

The bidirectional form of the LW KG solution is obtained from the ansatz [8]

$$\Psi = G(\rho,z,t)e^{i\beta\eta} \quad (3.2)$$

where the variable

$$\eta = z + \left(\frac{c^2}{v_g}\right)t$$

and the group velocity of the localized wave is

$$v_g = \frac{|\beta|c}{(\beta^2+\mu^2)^{1/2}}. \quad (3.3)$$

This reduces (3.1) to the form

$$i2\beta(\partial_z - v_g^{-1}\partial_t)G(\rho,z,t) + (\partial_z^2 - c^{-2}\partial_t^2) \times G(\rho,z,t) + \nabla_\perp^2 G(\rho,z,t) = 0. \quad (3.4)$$

Introducing the variable

$$\tau \equiv \gamma(z - v_g t)$$

where the relativistic factor $\gamma = [1-(v_g/c)^2]^{-1/2}$, (3.4) becomes a hyperbolized Schrödinger equation

$$i4\beta\gamma\partial_\tau G(\rho,\tau) + \partial_\tau^2 G(\rho,\tau) + \nabla_\perp^2 G(\rho,\tau) = 0. \quad (3.4')$$

A simple substitution

$$G(\rho,\tau) = g(\rho,\tau)e^{-i2\beta\gamma\tau} \quad (3.5)$$

results in the Helmholtz equation

$$\nabla_\perp^2 g(\rho,\tau) + \partial_\tau^2 g(\rho,\tau) + 4\beta^2\gamma^2 g(\rho,\tau) = 0 \quad (3.6)$$

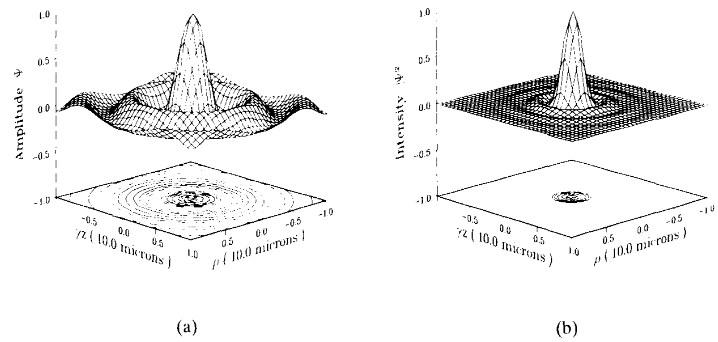

Fig. 2. A localized wave solution in a dense plasma is shown as a function of the spatial variables at $t = 0$. (a) Its amplitude Ψ_{00} and (b) its energy density $|\Psi_{00}|^2$.

which has the known, general solution

$$g_{lm}(\rho,\tau) = j_l(2\beta\gamma\xi)P_l^m(\tau/\xi)\cos(m\theta) \tag{3.7}$$

where

$$\xi = \left[\rho^2 + \gamma^2(z - v_g t)^2\right]^{1/2}.$$

Consequently, the general bidirectional solution has the form

$$\Psi_{lm}(\mathbf{r},t) = j_l(2\beta\gamma\xi)P_l^m\left(\frac{\tau}{\xi}\right)\cos(m\theta)e^{-i2\beta\gamma\tau}e^{i\beta\eta}. \tag{3.8}$$

We note that the wavevector of the localized wave is defined from (3.3) simply as

$$\beta = \gamma\left(\frac{v_g}{c}\right)\mu. \tag{3.9}$$

If we hold v_g fixed and change μ, the wavenumber β changes proportionately to μ. This is of interest because β^{-1} determines the length scale of the localization. In the case of a LW propagating at a group velocity $v_g \ll c$ in a plasma consisting of particles with charge q whose density is a constant n_0, the term

$$\mu^2 = \frac{n_0 q^2}{mc^2} = \frac{\omega_{p0}^2}{c^2}$$

ω_{p0} being the associated plasma frequency; and one obtains

$$\beta \sim \sqrt{n_0}v_g. \tag{3.10}$$

Thus the LW solution in a plasma is more localized for larger electron densities and for larger velocities. For an example, consider the $l = 0, m = 0$ solution. The location of the first zero of j_0 occurs at a distance d, either along the transverse, ρ, or longitudinal, $\gamma(z - v_g t)$ coordinate, given by the expression $d \sim [\pi/(2\gamma\beta)] \sim (c/v_g)(c/\omega_{p0})$. For a plasma density $n_0 \sim 10^{21}$ cm^{-3}, the term $\beta = 3.55 \times 10^8$ cm^{-1}; and one then finds that $d \sim 1.0\mu$ m. This micron sized LW solution Ψ_{00} and its energy density $|\Psi_{00}|^2$ are shown in Figs. 2(a) and (b), respectively, for $t = 0$. The dashed contours in both figures indicate areas where the values are less than zero. Other LW solutions are possible. For instance, the unidirectional ansatz

$$\Psi = G(\rho,\tau)e^{-i\alpha[z-(c^2/v_g)t]} \tag{3.11}$$

reduces the Klein–Gordon equation directly to a Helmholtz equation

$$\nabla_\perp^2 G(\rho,\tau) + \partial_\tau^2 G(\rho,\tau) + \chi^2 G(\rho,\tau) = 0. \tag{3.12}$$

where the solution constants, α and χ, are related as

$$\alpha^2\left[\left(\frac{c}{v_g}\right)^2 - 1\right] - \chi^2 = \mu^2. \tag{3.13}$$

Using known solutions to the Helmholtz equation, we arrive immediately at the general unidirectional form

$$\Psi_{lm}(\mathbf{r},t) = j_l(\chi\xi)P_l^m\left(\frac{\tau}{\xi}\right)\cos m\theta e^{\{-i\alpha[z-(c^2/v_g)t]\}}. \tag{3.14}$$

In contrast to the bidirectional representation, note that (3.13) can be rewritten as

$$v_g = \frac{\pm\alpha c}{\sqrt{\alpha^2 + \mu^2 + \chi^2}}. \tag{3.15}$$

Thus the unidirectional form of the KG solutions differs from the bidirectional one in that given the group velocity, there are now two free parameters, α and χ, rather than one, β. Moreover, with (3.15) it allows for positive and negative energy solutions. Superpositions of (3.8) or (3.14) over their free parameters lead to finite energy LW solutions.

B. Waveguide Solutions

Exact LW pulse solutions for a circular acoustic waveguide and a perfectly conducting electric waveguide have also been obtained. For instance, with the bidirectional representation (2.8) restricted to a cylinder of radius R so that the solution is zero on the wall of that cylinder and taking χ_{mn} to be the nth root of the Bessel function J_m, the general, lowest order, axisymmetric, acoustic waveguide solution has the form

$$\Psi(\vec{r},t) = \sum_{m=1}^{\infty} \int_0^{\infty} d\beta G_{0m}(\chi_{0m}/R,\beta) J_0(\chi_{0m}\rho/R) \times \exp\Big(-i\frac{\chi_{0m}^2(z-ct)}{4\beta R^2}\Big)\exp[i\beta(z+ct)] \tag{3.16}$$

Choosing the spectrum

$$G_{0m}(\chi_{0m}/R,\beta) = \frac{1}{2\beta}\exp\Big[-(\frac{\chi_{0m}^2}{4\beta R^2})\,a_1 - \beta a_2)\Big] \tag{3.17}$$

a LW waveguide solution results:

$$\Psi(\vec{r},t) = \sum_{m=1}^{\infty} J_0(\chi_{0m}\rho/R) \times K_0\Big(\frac{\chi_{0m}}{R}\sqrt{[a_1+i(z-ct)][a_2-i(z+ct)]}\Big) \tag{3.18}$$

that can be localized by design and can propagate large distances in the dispersive environment with little variation. In particular, with the constant values: $a_1 \ll 1$ and $a_2 > 1$, one finds that: 1) the energy is forward propagating, 2) the initial pulse is well localized along z axis because $\Psi(\rho,z,0) \sim J_0(\chi_{0m}\rho/R)\exp(-\chi_{0m}\sqrt{a_2 z/2})$, 3) the peak of the initial pulse is large but well-behaved because $\Psi(\rho,0,0) \sim J_0(\chi_{0m}\rho/R)\ln\Big(\sqrt{a_1 a_2}\chi_{0m}/R\Big)$, and (4) the pulse's peak value remains unchanged for $z \ll a_2/2$ and then decays logarithmically for $\chi_{0m}\sqrt{2\,a_2\,z}/R \ll 1$. Thus for $a_1 = 10^{-13}$ m, $a_2 = 10^3$ m and $\chi_{0m}/R = 1000\ \mathrm{m}^{-1}$, the pulse propagates 500 m without any decay and only decays to half its value over the next 50 km.

C. Lossy Media

Well-behaved solutions for lossy media have also been constructed from a bidirectional representation generalization of the FWM superposition in [4]. A recent argument due to Besieris, Shaarawi, and Ziolkowski will achieve a lossy medium solution that consistently reduces to the FWM solution in the limit where the loss disappears. Moreover, it demonstrates further the utility of the bidirectional approach to finding unusual LW solutions of wave equations. Consider, for instance, the equation governing the behavior of a wave in a lossy medium having permittivity ϵ, permeability μ, and conductivity σ:

$$\{\nabla^2 - \epsilon\mu\partial_t^2 - \mu\sigma\partial_t\}u(\vec{r},t) = 0. \tag{3.19}$$

Set $\nu^2 = (\sigma/2\epsilon c)^2$. Assuming for $t \geq 0$ the solution form $u(\vec{r},t) = e^{-\nu(ct)}\phi(\vec{r},t)$, leads to a Klein–Gordon equation with an imaginary mass term for ϕ: $\{\partial_{ct}^2 - \nabla^2 - \nu^2\}\phi(\vec{r},t) = 0$, which has the general bidirectional solution

$$\phi(\vec{r},t) = \frac{1}{(2\pi)^2}\int_\nu^{\infty} d\chi \int_0^{\infty} d\alpha \int_0^{\infty} d\beta \chi J_0(\chi\rho) \times C(\alpha,\beta,\chi)e^{-i\alpha(z-ct)}e^{+i\beta(z+ct)} \times \delta\Big[\alpha\beta - \frac{1}{4}(\chi^2-\nu^2)\Big] \tag{3.20}$$

where the restriction $\chi \geq \nu$ is made to guarantee satisfaction of the constraint condition in the positive α, β quadrant. The bidirectional spectrum $C(\alpha,\beta,\chi)$ will now be chosen to have the FWM form introduced in [4]: $C(\alpha,\beta,\chi) = (\pi/2)e^{-\alpha z_0}\delta(\beta - k)$. Clearly, modifications of the FWM spectrum appropriate for this lossy medium case are simply introduced, for instance, by including a function $f(\chi)$ in this expression. This will modify the behavior of the terms most closely coupled to the transverse behavior of the solution. Several integral and algebraic manipulations lead to the result

$$\phi(\vec{r},t) = \frac{e^{ik(z+ct)}e^{-k\rho^2/[z_0+i(z-ct)]}}{4\pi[z_0+i(z-ct)]} + \phi_R(\vec{r},t) \tag{3.21}$$

which is the original FWM solution $\phi_{FWM}(\vec{r},t)$ modified by the remainder term

$$\phi_R(\vec{r},t) = \frac{e^{ik(z+ct)}}{4\pi[z_0+i(z-ct)]}\sum_{\ell=1}^{\infty}\frac{1}{\ell!}\Big(-\frac{\rho^2}{4}\Big)^{\ell}\sum_{m=1}^{\infty}\frac{1}{m!} \times \Big(\frac{4k}{[z_0+i(z-ct)]}\Big)^{\ell-m}\nu^{2m} \tag{3.22}$$

the latter, which accounts for the presence of the lossy medium, is bounded and clearly reduces to zero in the limit $\nu \to 0$. On the other hand, in the large loss limit $\nu \to \infty$ the ν^ℓ term dominates the last sum so that $\lim_{\nu\to\infty}\phi_R(\vec{r},t) \sim J_0(\nu\rho)$, giving a well-behaved solution: $\phi(\vec{r},t) = \phi_{FWM}(\vec{r},t) + e^{ik(z+ct)}J_0(\nu\rho)/\{4\pi[z_0+i(z-ct)]\}$. The remainder term is localized near $\rho = 0$ and $z = ct$ as is the FWM solution. Other choices of the bidirectional spectrum C in (3.20) will lead to finite energy superpositions of these FWM-based solutions, possibly a MPS pulse type of solution for the lossy medium case.

IV. LW Transmission Experiments

The physics behind the LW effect is the coupling of the usually disjoint portions of phase space: space and frequency, due to the nonseparable nature of the LW solutions. The component waveforms, and, therefore, their broad bandwidth spectra, are strongly correlated to each other, a self-similarity property inherent to the LW solution. This spatial spectrum correlation leads to different pulses arriving from different locations with different, but correlated, frequency content; i.e., they arrive at the right place at the right time with the frequency components necessary to reconstruct the wave packet. A moving interference pattern forms at enhanced distances as the individual waveforms continue to propagate away from their sources. From a practical point of view, a new type of array is necessary to achieve this effect—each array element must be independently addressable so that the appropriate waveform can be radiated from it.

The LW effect has been verified with a set of three acoustic experiments using ultrasound in water. The first set of acoustic experiments in water was reported in [10]. Successful localization of a transient, pencil-beam of ultrasound launched from a LW pulse-driven array was exhibited. The array was linear, synthetic, and driven with the MPS pulse. The next set of experiments simply extended the previous results to circular and square synthetic arrays. In both cases the pencil-beam generated by the LW pulse-driven array outperformed the corresponding beam transmitted by an array driven with a CW tone burst. This was true when the array was uniformly illuminated (an effective piston which produces a naturally focussed beam) and when it was shaded with a spatial Gaussian taper (an initial transverse Gaussian with the same waist as the MPS pulse). The beam quality was better than the highest frequency Gaussian tested and avoided the inherent near-field variations associated with a piston generated field.

The final experimental series involved an actual array of ultrasonic transducers. This experiment was designed to avoid some of the ambiguities that arise in comparing LW and CW driven arrays. In particular, the LW solutions are composed of broad bandwidth waveforms while traditional performance criteria are based upon CW, narrow-band concepts. There is no special frequency that can be selected to define, for instance, a Rayleigh distance when several different broadband spectra are involved. Nevertheless, performance comparisons are desirable and a specific Rayleigh distance L_R was derived for these comparisons [11].

A 25 element, 5×5, square array was fabricated which is 1.1 cm on a side and has 0.5 mm diameter disc elements (acoustic transducers) spaced on 2.5 mm centers. The small number of elements limits the number of CW configurations; there are too few elements for any effective shading or focusing. Six unique waveforms were designed for this array to achieve a tenfold experiment; i.e., maintaining localization at least to $10 \times L_R$. For the maximum frequency of significance included in these signals, $L_R \sim 2-3$ cm. The signal design was accomplished with a numerical simulation of the experiment which accounted for the effects (time derivatives of the signals) of the receiving transducer as well as those of the transmitting ones. Although the resulting time derivatives have no effect on a CW field other than multiplication by a constant, they greatly impact the result of the LW fields because of the inherent spatial spectrum correlation (coupling of space and time). Comparisons of the energy efficiency (energy received relative to the energy delivered to the array) and beam profile (half width at half maximum of the intensity profile) were made and confirmed more than a ten-fold enhancement of the Rayleigh distance of the beam. As in the synthetic array experiments, the pencil-beam generated by the LW pulse-driven array outperformed the corresponding beam transmitted by an array driven with an equivalent CW tone burst. The sidelobe levels of the LW pencil beam were greatly reduced, especially when compared to beams exhibiting grating lobes which are generated by driving this sparse array with much higher frequency CW tone bursts. The LW pencil-beam is quite robust even with a variety of losses and perturbations inherent in the experimental apparatus. Recent analytical results [21], [22] have extended the meaning of diffraction lengths for the radiated and measured field energies and intensities and of the transverse widths of these beam quantities to the broad bandwidth cases associated with these localized wave pulse-driven arrays. The theoretical and experimental results are in excellent agreement.

V. Conclusions and Future Plans

Some very interesting physics, mathematics, and engineering issues have surfaced because of the many studies on localized wave transmission. These include questions on causality, localization, space–time field representations, complex source interpretations of beam fields, and time domain antenna effects. The macro-photon and macro-particle results suggest that alternate descriptions of photons and particles are possible which have physically interesting and appealing properties. The issue of determining optimal source distributions that could generate fields that at least closely approximate these LW solutions is a very complex one and poses a current challenge to workers in this field. The efficiency and beamwidth limits derived in [21] and [22] suggest that with improvements in the source distributions currently in use, one can realize beams with these enhanced localization properties.

Many of these LW issues are in the formative stages and require further investigation and clarification. Nonetheless, much progress has been made. The studies of the generation (or launching of) LW pulses from finite sized arrays of radiating elements have many potential applications. These include the design of pulses with localized wave transmission characteristics for remote sensing, communications, power transmission, and other directed energy applications.

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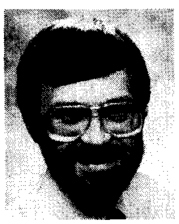

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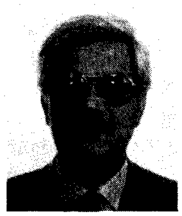

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