Finally, in a nutshell, we wish to claim that the procedure presented in our paper is much simpler and more straightforward than that in [2]. However, which procedure is *better* is an open question.

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Comments on "Properties of Electromagnetic Beams Generated by Ultra-Wide Bandwidth Pulse-Driven Arrays"

S. N. Samaddar

In the above paper[1] there are some errors which may affect some results and the conclusion of the paper. Equation (46) shows that the open-circuit received voltage across a very small and thin electric dipole is proportional to the first time derivative of the incident electric field. When this result is substituted in (45), the author then concludes (see (47)) that the same received voltage becomes proportional to the third time derivative of the applied input voltage across the terminals of a similar transmitting dipole antenna located at a large distance from the receiving dipole. However, the results in the literature (see [1] and [2] here and [27], [30], [32], and [33] in Ziolkowski's paper) for the same situation show differently.

The expression for the open-circuit received voltage in the frequency domain by a receiving dipole is simply (see [2, eq. (1)] here and [27, p. 180] of the paper in question)

$$V_{\mathrm{oc}} = \vec{E}^i(\vec{r}_t, w) \cdot \vec{h}_r(\vec{r}_r, \omega), \quad (1)$$

where $\vec{r}_t$ and $\vec{r}_r$ are vector coordinates measured with respect to the transmitting dipole and the receiving dipole, respectively. $\vec{E}^i$ is the incident electric field at the receiving dipole and h_r is the vector effective height of the receiving dipole. For a very short and thin dipole (along which current does not vary spatially), parallel to the z axis one finds

$$\vec{h}_j(\vec{r}_j, \omega) = -\hat{\boldsymbol{\theta}}_j \ell_j \sin\theta_j, \quad (2)$$

where $j = t$ for the transmitting dipole, $j = r$ for the receiving dipole, $2\ell_j$ is the total length of the dipole, and $\hat{\boldsymbol{\theta}}_j$ is the unit vector in the θ_j direction from the axis of the j-dipole element. This result can be obtained from [2, eq. (4)] with $w\ell_j/c \ll 1$ (see also eq. (4.24) of [27] in Ziolkowski's paper).

Since the vector effective height for a short diple, as expressed in (2) above, is independent of frequency, the preceding relation (1) implies that in the time domain the time dependence of the incident field is the same as the open-circuit received voltage of a short (Hertzian) receiving dipole. This result makes sense physically also, since a short electric probe (or dipole) does not distort the incident field. Consequently, the received voltage by the electric probe has the same waveform as that of the incident electric field. Equation (4.32) of reference [27] in the paper in question also confirms the observation just made.

In [2] it was state that, "the receiving dipole antenna behaves like an integrating circuit," which is also consistent with the results of references [32] and [33] cited by Ziolkowski. This statement is true when the current varies along the dipole. However, Ziolkowski's equation (46) apparently contradicts this result.

I think the angle ψ, which is related to θ_j in (2) and the associated unit vector implied in (45)–(47) in the paper in question are not the same. The angel ψ in (46) is the colatitude of a distant point (in particular, the center of the receiving dipole) with respect to the orientation of the transmitting dipole. On the other hand ψ in (47) is the colatitude of the direction of the incident field, where ψ is measured with respect to the orientation of the receiving dipole. Therefore, the same notation should not be used, although most papers in the literature fail to mention this point. I have clearly expressed this distinction in my paper [2].

Since the paper in question deals with real signals in the time domain, the imaginary number i has no place in (45) and (46). Incidentally, the relation (45) can also be recovered from [1] and [2] by assuming $\omega h/c \ll 1$ (in the frequency domain) and $t^* \gg h/c$ (in the time domain). Equation (6), (7), or (13) of [1] and (5) of [2] may be used for this purpose.

Furthermore, I cannot agree with the statement made below (47), namely, "A three time derivative response is also realized with electrically small conical, cylindrical, and loop antennas." For instance, when a small current loop is used as a transmitting antenna, which is fed by a voltage $V(t)$ across its terminals, the radiated electric field is proportional to $dV(t)/dt$ and not $d^2V(t)/dt^2$. Interestingly enough, when a small loop is used as a receiving element, the open-circuit received voltage becomes also proportional to dE^{inc}/dt, which is independent of the manner in which the incident field was transmitted. Equation (5.1), p. 85, of reference [30] cited by Ziolkowski also implies what is just stated above.

Finally, I would like to point out also that the modeling of an ultra-wide-band signal-detecting device by a very short and thin dipole (along which current does not vary spatially) is not appropriate, since such devices show only the low-frequency effect.

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Authors' Reply[2]—R. W. Ziolkowski[3]

Dr. Samaddar's analysis is unfortunately incorrect and does not reproduce experimentally known results. The error, which is at the heart of any treatment of transmitting and receiving systems—particularly in the time domain—results from an incorrect model of the receiver corresponding to the matched transmitter. Care must be exercised to self-consistently assign voltage or current sources in the complete

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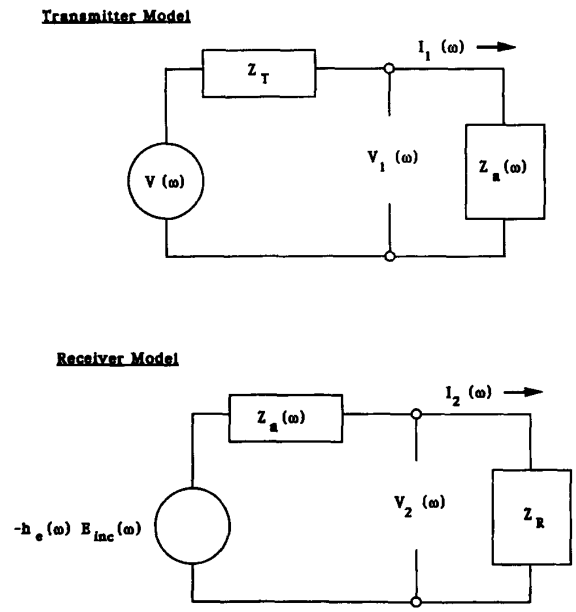

Fig. 1. Equivalent circuits for the transmitter and receiver systems.

model of the transmitting–receiving system. While it is recognized that there are many possible models of these systems, one must validate the appropriateness of any model by its reproduction of known experimental results.

Consider Fig. 1. The transfer function for the transmitter, which connects the radiated electric field in the far zone, E_{ff}, to the source voltage, $V(\omega)$, is related to the effective height, $h_e(\omega)$, of its antenna as

$$T(\omega) = \frac{E_{ff}(\omega)}{V(\omega)} = -i2Z_0\frac{e^{ikr}}{4\pi r}\frac{\omega}{c}\frac{h_e(\omega)}{Z_T + Z_a(\omega)}, \qquad (1)$$

where Z_0 is the characteristic impedance of free space, Z_T is the characteristic impedance of its source which is taken to be a frequency-independent constant over the frequency band of interest, and the antenna impedance is $Z_a(\omega)$. Similarly, the transfer function of the receiver, which connects the measured voltage, $V_{\rm meas}(\omega) = V_2(\omega)$, to the incident electric field strength at the antenna, $E_{\rm inc}(\omega)$, is

$$S(\omega) = \frac{V_{\rm meas}(\omega)}{E_{\rm inc}(\omega)} = \frac{-h_e(\omega)Z_R}{Z_R + Z_a(\omega)}. \qquad (2)$$

These expressions are characteristic of any antenna system.

Now, if the transmitter's generator is treated as a current source, the source impedance, Z_T, is much larger than the antenna impedance, $Z_a(\omega)$; i.e., $Z_T \gg Z_a(\omega)$. The current across the antenna, $I_1(\omega)$, is then simply related to the source voltage, $V(\omega)$, as $I_1(\omega) = V(\omega)/[Z_T + Z_a(\omega)] \sim V(\omega)/Z_T$. For example, in the case of an electrically short, center-fed, linear electric dipole of length L_1 that is x oriented and that is driven with the sinusoidal current distribution at the frequency $\omega = kc$:

$$I(x,\omega) = I_1(\omega)\sin\left[k\left(\frac{L_1}{2} - |x|\right)\right], \qquad (3)$$

the resulting electric field in its far zone is given by the well-known expression [31, p. 44]

$$\vec{E}_{ff}(\vec{r},\omega) \approx -iZ_0 I_1(\omega)G(kr)\mathcal{P}(\psi,\phi,kL_1)\hat{\psi}, \qquad (4)$$

where the angle ψ is measured from the positive x axis; i.e., $\psi = \pi/2 - \theta$ and $\hat{\psi} = -\hat{\theta}$, the free-space frequency-domain Green function

$$G(kr) = \frac{e^{ikr}}{4\pi r}, \qquad (5)$$

the pattern of this antenna

$$\mathcal{P}(\psi,\phi,kL_1) = 2\,\frac{\cos[(kL_1/2)\,\cos\psi] - \cos[(kL_1/2)]}{\sin\psi}, \qquad (6)$$

and $r = |\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$ is the distance from the center of the transmitting dipole to the observation point (i.e., to the center of the receiving dipole). The electrically small condition $(kL_1/2) \leq 1$ reduces the pattern to the approximate form

$$\mathcal{P}(\psi,\phi,kL_1) \sim (kL_1/2)^2 \sin\psi. \qquad (7)$$

This is a very good approximation for the main beam direction (where $\psi \sim \pi/2$); the approximation begins to break down for k values near $kL_1/2 \sim 1$ far outside the main beam. Since the pattern is assumed negligible there and since we are interested in maximal coupling of the dipole antennas, (7) is a quite suitable model. The electric field in the far zone of the transmitting dipole thus has the form

$$\vec{E}_{ff}(\vec{r},\omega) \approx -iZ_0 I_1(\omega)\left(\frac{kL_1}{2}\right)^2 G(kr)\sin\psi\hat{\psi}$$
$$\equiv -i\frac{Z_0}{Z_T}V(\omega)\left(\frac{kL_1}{2}\right)^2 G(kr)\sin\psi\hat{\psi}, \qquad (8)$$

which has now been expressed in terms of the source voltage. The corresponding time-domain response of this antenna to the driving (voltage) signal $v(t)$ (the Fourier transform of $V(\omega)$) would then be

$$\vec{e}_{ff}(\vec{r},t) \propto \frac{Z_0}{Z_T}\frac{L_1^2}{4}\frac{1}{4\pi r}\partial_{ct}^2 v\left(t - \frac{r}{c}\right)\sin\psi\hat{\psi}. \qquad (9)$$

This result agrees with those in [27] (after carefully sorting out the current models being used), [28], and [29], and it specifically shows that the radiated electric field of a center-fed linear dipole antenna in the far zone under the above assumptions is proportional to the second time derivative of the input voltage signal.

The Rayleigh–Carson reciprocity theorem requires that the effective height of the antenna, $h_e(\psi,\phi,\omega)$ of length L be related to the pattern $\mathcal{P}(\psi,\phi,kL)$ as

$$kh_e(\psi,\phi,\omega) = \mathcal{P}(\psi,\phi,kL), \qquad (10)$$

which, under the assumption on the electrical size of the current driven dipole, yields

$$h_e^{\rm current}(\psi,\phi,\omega) \approx \omega\frac{L_1^2}{4c}\sin\psi. \qquad (11)$$

These approximations result in the transmitter transfer function

$$T(\omega) \sim -i2Z_0\frac{e^{ikr}}{4\pi r}\frac{\omega}{c}\frac{h_e^{\rm current}(\omega)}{Z_T}$$
$$= -2i\frac{Z_0}{Z_T}\frac{L_1^2}{4}\frac{e^{ikr}}{4\pi r}\sin\psi\left(\frac{\omega}{c}\right)^2, \qquad (12)$$

which means the far field is proportional to two time derivatives of the driving voltage signal: $e_{ff}(t) \propto \partial_{ct}^2 v(t - R/c)$. This reconfirms the presence of the two time derivative far-field time-domain behavior reported in the original manuscript.

I believe the major issue is that *for reciprocity to hold the receiver must now be treated as a voltage source.* The measurement is taken across a resistive load, Z_R, a frequency-independent constant over the frequency band of interest, which is much larger than the antenna impedance, $Z_a(\omega)$, i.e., $Z_R \gg Z_a(\omega)$. This corresponds to the assumptions made at the transmitter. The measured voltage $V_{\text{meas}}(\omega) = V_2(\omega)$ is then related to the voltage at the antenna, V_{rec}, as $V_{\text{meas}}(\omega) = I_2(\omega)Z_R = V_{\text{rec}}Z_R/[Z_R + Z_a(\omega)] \sim V_{\text{rec}}$. In general the received voltage is related to the effective height of the receiving antenna as $V_{\text{rec}} = -h_e(\omega)E_{\text{inc}}(\omega)$, where $E_{\text{inc}} = re^{ikr}E_{ff}$; hence $e_{\text{inc}}(t) \propto e_{ff}(t)$. Therefore if L_2 is the length of the receiving dipole and the receiver's antenna is assumed to be oriented for maximal coupling, the measured voltage is

$$V_{\text{meas}}(\omega) = -h_e^{\text{current}}(\omega)E_{\text{inc}}(\omega) \sim -\omega\frac{L_2^2}{4c}\sin\psi E_{\text{inc}}(\omega). \quad (13)$$

Consequently the corresponding time-domain response is

$$v_{\text{meas}}(t) \propto \frac{L_2^2}{4}\sin\psi\,\partial_{ct}e_{\text{inc}}(t). \quad (14)$$

Combining (14) with (9) yields the overall three time derivative response of the system:

$$v_{\text{meas}}(t) \approx -\frac{Z_0}{Z_T}\frac{L_1^2}{4}\frac{L_2^2}{4}\frac{1}{4\pi r}\partial_{ct}^3 f\left(t - \frac{r}{c}\right)\sin^2\psi \quad (15)$$

derived in the original manuscript. Note that I agree with Dr. Samaddar's observation that the i has no place in either of the expressions (45) or (46); the equations (45) and (46) should read, respectively, without the i and with the "proportional to" symbol as indicated in (9) and (14) above.

Comparing (9) and (14) one finds, modulo some constants and angle factors, that in the far-field region the transient response of the transmitter is the time derivative of the response at the receiver. This basic property of matched transient antennas has been considered theoretically and experimentally by Kanda in [32] and [33]. Moreover, since $Z_R \gg Z_a(\omega)$, the receiver's transfer function is simply

$$S(\omega) \sim -h_e^{\text{current}}(\omega) \propto \omega, \quad (16)$$

which agrees with the experimental results given in [33, fig. 1]. This result also recovers the experimentally known fact that an electrically short dipole makes a good $\dot{D}$-probe.

In the same manner, if the transmitting generator is taken to be a voltage source, then $Z_T \ll Z_a(\omega)$, where for the case of an electrically short, center-fed, linear electric dipole, its impedance is capacitive: $Z_a(\omega) = (-i\omega C_a)^{-1}$. The current across the antenna $I_1(\omega)$ is related to the source voltage $I_1(\omega) = V(\omega)/[Z_T + Z_a(\omega)] \sim V(\omega)/Z_a(\omega) = -i\omega C_a V(\omega)$. Thus, the driving point voltage is the generator voltage: $V_1(\omega) = Z_a(\omega)I_1(\omega) = V(\omega)$. However, this means that if the far field, which is obtained from the driving point current, is written in terms of the voltage, then one must have

$$h_e^{\text{voltage}}(\omega) = 2\frac{\cos[(kL_1/2)\cos\psi] - \cos[(kL_1/2)]}{\sin(kL_1/2)\sin\psi} \sim \frac{L_2}{2}\sin\psi, \quad (17)$$

rather than (11). With the identification of $\ell_2 = L_2/2$, one recovers Samaddar's expression (2). Note that the transmitting transfer function has the same form since now

$$T(\omega) \sim -i2Z_0\frac{e^{ikr}}{4\pi r}\frac{\omega}{c}\frac{h_e^{\text{voltage}}(\omega)}{Z_a(\omega)} = -4\pi Z_0 C_a\frac{L_2}{2}\frac{e^{ikr}}{4\pi r}\left(\frac{\omega}{c}\right)^2, \quad (18)$$

which means we still find that the far field is proportional to two time derivatives of the driving voltage signal: $e_{ff}(t) \propto \partial_{ct}^2 v(t - R/c)$, in agreement with experiments and Dr. Samaddar's results.

However, the receiver must now be treated as a current rather than a voltage source. This means the measurement being taken across the resistive load Z_R, which is now much smaller than the antenna impedance, $Z_a(\omega)$; i.e., $Z_R \ll Z_a(\omega)$. The measured voltage, $V_{\text{meas}}(\omega)$, is then related to the voltage at the antenna, V_{rec}, as $V_{\text{meas}}(\omega) = I_2(\omega)Z_R = V_{\text{rec}}Z_R/[Z_R + Z_a(\omega)] \sim V_{\text{rec}}Z_R/Z_a(\omega)$. This immediately gives

$$V_{\text{meas}}(\omega) = -h_e^{\text{voltage}}(\omega)E_{\text{inc}}(\omega)\frac{Z_R}{Z_a(\omega)} \sim i\frac{\omega}{c}C_a Z_R\frac{L_2}{2}\sin\psi E_{\text{inc}}(\omega), \quad (19)$$

which in the time domain means

$$v_{\text{meas}}(t) \propto \partial_{ct}e_{\text{inc}}(t), \quad (20)$$

i.e., the measured voltage is the time derivative of the incident signal, rather than the integration that Dr. Samaddar has reported. This distinction between the voltage and current generators and the resulting impact on effective heights, voltage and current measurements, etc., in the time domain has also been discussed recently by C. Baum in *Note 330* (23 July 1991) of the *Sensor and Simulation Notes* series. The voltage source model derivation is also recovered with a slight extension of the results reported by Dudley and Casey in [29] (D. G. Dudley, private communications).

One must anticipate that if the frequencies of the driving signals were such that kL were not small but not extremely large, then the driving point current and voltage would be proportional to each other and only one time derivative would appear in the far-field signal from the radiation process. Reciprocity then requires that the receiver reproduce this signal so that the overall time response of the system is one time derivative of the input driving signal. Samaddar's integration of the signal by the receiver can only occur, as noted in [28], in the limit of a very large antenna, such as an electrically long dipole. The overall system response would then be a single integration of the matched transmitter's driving signal. However, it must be recognized that the time derivatives can be imposed on the signals corresponding to these lower order systems after reception through a variety of processing techniques. The desirable three derivative results can then be realized synthetically.

I believe I would have been more precise had I made a statement to the effect that "analogous three-time derivative behaviors can be realized with electrically small conical, cylindrical, and loop antennas." For instance, as shown in section 4.3b of *Electromagnetic Wave Theory*, by J. A. Kong (New York: Wiley, 1986), the far field of a small current loop is proportional to ω^2 times the driving current. With a matched loop receiving antenna, a three time derivative behavior on the input current driving function would then be obtained. This result also agrees with the experimentally known fact that an electrically small loop makes a good $\dot{B}$-probe.

Finally, I most emphatically disagree with Dr. Samaddar that electrically short antennas are inappropriate for ultra-wide-bandwidth transmitting or receiving devices. The only assumption made in the paper was the $kL/2 \leq 1$ over the frequency range of interest in the input driving pulses. This condition is readily achieved at higher frequencies, for instance in the THz regime, as evidenced by their very successful use for photoconductive transmitting and receiving antennas.