Closed-form, localized wave solutions in optical-fiber waveguides: comment

Bernd Lichtenberg and Neal C. Gallagher

Department of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

Richard W. Ziolkowski

Department of Electrical and Computer Engineering, University of Arizona, Tucson, Arizona 85721

Received October 26, 1992; revised manuscript received February 17, 1993; accepted May 12, 1993

The new bidirectional decomposition of the solution to the scalar wave equation introduced by Vengsarkar *et al.* [J. Opt. Soc. Am. A **9**, 937 (1992)] characterizing highly directional pulses is shown to have another solution. If the modulation spectra of the pulses considered satisfy a certain conjugation property, this solution turns out to be the complex conjugate of the solution presented in the above reference.

In Ref. 1 a new bidirectional decomposition of the solution to the scalar wave equation as a basis for localized wave solutions is introduced. These localized wave solutions characterize pulses that have high directionality and slow energy decay. The decomposition is used to describe the propagation of different source modulation spectra in optical fiber waveguides.

To find the limits for the integral of the general solution, Vengsarkar *et al.*[1] used the following two choices [relations (11a) and (11b) of Ref. 1]:

$$\beta_{\text{cl}} \leftrightarrow (\beta - \alpha), \qquad \omega_{\text{cl}} \leftrightarrow -(\alpha + \beta)c, \tag{1}$$

$$\beta_{\text{cl}} \leftrightarrow (\alpha - \beta), \qquad \omega_{\text{cl}} \leftrightarrow (\alpha + \beta)c. \tag{2}$$

We show here that there exists another solution with proper integral limits for choice (1), which was not considered in Ref. 1. For this solution the two parameters α and β in the bidirectional elementary function, introduced by Eq. (2) in Ref. 1, take on negative values. As is demonstrated below, this leads to the complex conjugate of the solution given in Ref. 1.

From Eq. (1) ($\omega_{\text{cl}} > 0$) it follows immediately that $(\alpha + \beta) < 0$. Next, the waveguiding condition with $n_1 = 1$ and $n_2 = n_e$ is written as [relation (22) of Ref. 1]

$$-n_e(\alpha + \beta) < (\beta - \alpha) < -(\alpha + \beta). \tag{3}$$

From the right-hand inequality, one immediately obtains $\beta < 0$. With this kept in mind, the left-hand inequality yields [as relation (27) of Ref. 1, with both α and $\beta > 0$]

$$\alpha\frac{1 - n_e}{1 + n_e} = \frac{\alpha}{s} < \beta \tag{4}$$

or

$$\frac{\alpha}{\beta} > s \qquad (\beta < 0, \qquad s > 0). \tag{5}$$

From inequality (4) it also follows that $\alpha < 0$, and therefore

$$\alpha\beta > 0, \tag{6}$$

which is in full accordance with the result that $\kappa_1^2 = 4\alpha\beta > 0$ [relations (14a) and (19a) of Ref. 1]. As a consequence, there exists a valid solution with the following integral limits:

$$\int_0^\infty \mathrm{d}\kappa_i \int_{-\infty}^0 \mathrm{d}\beta \int_{-\infty}^{\beta s} \mathrm{d}\alpha\, I \tag{7}$$

or

$$\int_0^\infty \mathrm{d}\kappa_i \int_{-\infty}^0 \mathrm{d}\alpha \int_{\alpha/s}^0 \mathrm{d}\beta\, I. \tag{8}$$

To check this solution we look at the limit for $n_e \to 1$ ($s \to \infty$). The solution vanishes as expected, because the cladding has the same index of refraction as the core, and waveguiding is impossible.

The integrand I for the bidirectional decomposition method introduced in Ref. 1 has the form [cf. Eq. (20) of Ref. 1]

$$\begin{aligned} I_1 = {} & (2\pi)^{-2}A(\alpha, \beta, \kappa_1)\kappa_1 J_0(\kappa_1\rho) \\ & \times \exp(-i\alpha\zeta)\exp(i\beta\eta)\delta(4\alpha\beta - \kappa_1^2) \end{aligned} \tag{9}$$

for $\rho \le a$ (core of the fiber), and for the region $\rho \ge a$ (cladding) it looks like [cf. Eq. (21) of Ref. 1]

$$\begin{aligned} I_2 = {} & (2\pi)^{-2}A(\alpha, \beta, \kappa_1)\kappa_1 K_0(\kappa_2\rho)\frac{J_0(\kappa_1 a)}{K_0(\kappa_2 a)} \\ & \times \exp(-i\alpha\zeta)\exp(i\beta\eta)\delta[\kappa_2 - f(\kappa_1)]\delta(4\alpha\beta - \kappa_1^2) \\ & \times \delta[\kappa_2^2 - n_e^2(\alpha + \beta)^2 + (\alpha - \beta)^2]. \end{aligned} \tag{10}$$

By a simple change of integration variables ($\beta \to -\beta$, $\alpha \to -\alpha$) the integral limits become

$$\int_0^\infty \mathrm{d}\kappa_i \int_0^\infty \mathrm{d}\beta \int_{\beta s}^\infty \mathrm{d}\alpha\, \tilde{I}_i, \tag{11}$$

with

$$\tilde{I}_1 = (2\pi)^{-2}A(-\alpha, -\beta, \kappa_1)\kappa_1 J_0(\kappa_1\rho) \times \exp(i\alpha\zeta)\exp(-i\beta\eta)\delta(4\alpha\beta - \kappa_1^2), \tag{12}$$

$$\tilde{I}_2 = (2\pi)^{-2}A(-\alpha, -\beta, \kappa_1)\kappa_1 K_0(\kappa_2\rho)\frac{J_0(\kappa_1 a)}{K_0(\kappa_2 a)} \times \exp(i\alpha\zeta)\exp(-i\beta\eta)\delta[\kappa_2 - f(\kappa_1)]\delta(4\alpha\beta - \kappa_1^2) \times \delta[\kappa_2^2 - n_e^2(\alpha + \beta)^2 + (\alpha - \beta)^2]. \tag{13}$$

If the conjugation property for the modulation spectrum A

$$A^*(\alpha, \beta, \kappa_1) = A(-\alpha, -\beta, \kappa_1) \tag{14}$$

is satisfied, where * represents the operation of complex conjugation, then

$$\tilde{I}_1 = I_1^*, \tag{15}$$

$$\tilde{I}_2 = I_2^*, \tag{16}$$

and the solutions given by relations (7) and (8) are just the complex conjugates of the solutions given in Ref. 1. It should be mentioned that Eq. (14) restricts the possible choices for the modulation spectrum A.

All the spectra used in Ref. 1 are given for $\alpha, \beta > 0$. Their definitions are readily extended to negative values of α and β simply by replacement of those variables in the spectra by their absolute values. This guarantees that those spectra will be finite in the limit of either infinite positive or infinite negative α, β, and therefore they satisfy Eq. (14) and hence are real.

REFERENCE

1. A. M. Vengsarkar, I. M. Besieris, A. M. Shaarawi, and R. W. Ziolkowski, "Closed-form, localized wave solutions in optical fiber waveguides," J. Opt. Soc. Am. A **9**, 937–949 (1992).